Introductory Module: Honeycombs are examples of well-adapted multi-purpose natural habitat structures that incorporate strength, versatility and material efficiency.

LESSON: The Sweet 'Bees-ness' of Honeycomb and the Mathematics of Hexagonal Geometry

Activity Purpose:

1) To develop an understanding that living things have structural features and adaptations that help them to survive in their environment and that growth and survival of living things are affected by physical conditions of their environment.

2) To investigate the properties of regular and irregular polygons and polyhedra and solve problems using geometric and algebraic reasoning.

3) To explore the application of algebraic expressions to describe geometric patterns, and solve problems by extending and applying the laws and properties of arithmetic to algebraic terms and expressions

Student success criteria

Questioning and predicting:
- With guidance, identify questions and problems associated with the geometry and structure of honeycombs that can be investigated mathematically and scientifically; and
- Make modelled predictions about honeycomb structures based on mathematical and scientific knowledge.

Processing and analysing data and information:
- Summarise data from an investigation of honeycomb, use this understanding to identify relationships between natural structures and mathematical reasoning and draw conclusions based on findings

Applying mathematical reasoning:
- Investigate and establish the properties of quadrilaterals and other polygons; regular and irregular tessellations and recurring geometric patterns
- Use algebra to express simple models describing geometric patterns and use algebraic expressions and mathematical reasoning to solve problems

Communicating:
- Students will communicate their ideas and explanations on the geometry and structures of honeycombs using scientific written descriptions, geometric diagrams and simple algebraic models.

Suggested Time: Approx. 190 minutes (4 lessons)

Teaching Notes

A. Common alternative conceptions (misconceptions):

Common misconceptions about beehives and honeycomb might include that honeycomb itself is a food – most children will have encountered the term primarily as the title given to a popular form of crunchy confectionery! Holding a discussion unpacking what students believe honeycomb to (a) be, and (b) be used for, will help clarify any existing misconceptions about honeycomb.

Providing students with the pre-reading (listed below) on 'Bees, Honeycomb and Hexagons' may address some of these scientific misconceptions regarding the role, function and structure of natural honeycomb.

Reinforcing some basic geometric concepts such as 'two-dimensional shape vs. three-dimensional object', 'polygon', 'quadrilateral' and 'polyhedron' will be useful, as will drawing the distinction up front between regular and irregular shapes and objects. Although these are reinforced in the unit, some review of these concepts (hopefully taught previously in Years 3-4) will prove helpful.

Section B (v) involves students adding up the internal angles of a triangle. Although not explicitly required in this section, measuring the angles using a protractor provides a useful proof to students that the sum of internal angles of a triangle is 180^0, and the sum of the internal angles of a hexagon is $120^0 \times 6 = 720^0$ (which is also equal to $180^0 \times 4$). Note here that, if teachers wish to use protractors to demonstrate this proof, students will have needed some pre-exposure to the use of the protractor at least for measuring acute angles.

Finally, specifically for Year 5 and 6 students, holding a discussion on 'what is algebra?' may also help clear up misconceptions about this important mathematical tool. Many younger students believe algebra to be a strange, indecipherable mathematical code or esoteric language or set of hieroglyphs, misunderstanding the notion of using a pronumeral as a *variable* rather than a fixed *value*. Reminding younger students of the symbolism used by their teachers in lower primary when given 'boxes' in which to insert missing value, will help to guide students toward a more accurate understanding of algebra before commencing the unit, eg:

(1) $7 + \square = 10$

(2) $12 - 7 = \square$

} In these simple equations, "$\square$" represents two *different values*, and in that sense is considered an *algebraic variable*.

B. Curriculum Links

Australian Curriculum (Science), Levels 5 to 8	
Biological Sciences	• Living things have structural features and adaptations that help them to survive in their environment (ACSSU043) – **Level 5** • The growth and survival of living things are affected by physical conditions of their environment (ACSSU094) **– Level 6**
Uses and applications of Science	• Scientific knowledge is used to solve problems and make decisions (ACSHE083 **– Level 5**, ACSHE100 **– Level 6**)
Questioning and predicting	• Identify questions and problems that can be investigated scientifically and make predictions based on scientific knowledge (ACSIS232 – **Level 6,** ACSIS124 **– Level 7**, ACSIS139 **– Level 8**)
Planning, investigating and representing	• Construct and use a range of representations to represent and describe patterns or relationships in data (ACSIS090 **– Level 5,** ACSIS144 **– Level 8**) • Summarise data, from students' own investigations and secondary sources, and use scientific understanding to identify relationships and draw conclusions based on evidence (ACSIS130 – **Level 7**, ACSIS145 – **Level 8**)
Communicating scientifically	• Communicate ideas, explanations and processes using scientific representations in a variety of ways, including multi-modal texts (ACSIS110 – **Level 5**, ACSIS133 **– Level 6**, ACSIS148 – **Level 7**, ACSIS174 **– Level 8**)

Australian Curriculum (Mathematics), Levels 5 to 8	
Geometry and Shape	• Connect three-dimensional objects with their nets and other two-dimensional representations (ACMMG111) – **Level 5** • Construct simple prisms and pyramids (ACMMG140) – **Level 6** • Draw different views of prisms and solids formed from combinations of prisms (ACMMG161) – **Level 7** • Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166) – **Level 7** • Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202) – **Level 8**
Patterns & Algebra	• Use equivalent number sentences involving multiplication and division to find unknown quantities (ACMNA121) **– Level 5** • Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133) – **Level 6** • Explore the use of brackets and order of operations to write number sentences (ACMNA134) **– Level 6** • Introduce the concept of variables as a way of representing numbers using letters (ACMNA175) **- Level 7** • Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177) **– Level 7** • Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190) **– Level 8**

C. Resources

Optional:

- **Some 'flipped classroom' pre-reading on Bees, Honeycomb and Hexagons:** http://www.npr.org/sections/krulwich/2013/05/13/183704091/what-is-it-about-bees-and-hexagons
- **Definitions and properties of polygons, polyhedral and internal angles of plane shapes:** Terrific reference site for students exploring the features of quadrilateral plane shapes and polyhedral, including defenitions and explanations of regularity and irregularity, congruence and angle properties of triangles and polygons: http://www.mathsisfun.com/geometry/index.html. Note: some animations require Flash Player (not suitable for iPad).
- **'Warm Up' clip on naturally occurring tessellations:** YouTube Clip: "Tessellations in Nature" - https://www.youtube.com/watch?v=V9uYcnjlAks. A beautifully photographed and soundtracked presentation illustrating a variety of tessellations in art, nature, food and everyday life. Good 'hook' for Section C, 'Terrific Tessellations'.
- **Introductory pre-reading on Algebra: 'Maths is Fun' website** – "An Introduction to Algebra". Great explanatory thinking for Year 5 and 6 students encountering algebra for the very first time: https://www.mathsisfun.com/algebra/introduction.html
- Protractor (per student or shared between two students) *(See Section A in the Teaching Notes, above)*.

Essential:

- Student work booklets / work units – "Adaptive Speciation of Honey Bees: Some Multiplication and Algebraic Thinking"
- Whiteboard or Smartboard for developing students' responses and for explanatory purposes.
- Rulers (per students)
- Scissors (1 pair per student)
- Sticky tape (per student)
- Coloured pencils

D. Teaching & Learning Planner

Section	What students are doing	What the teacher is doing
Intro / Hook ***(5 minutes)***	Listening and participation in class discussion about Honeycomb. Key questions to think about / share responses are: • **What is 'honeycomb' and why do bees build it?** • **In what types of habitats do other insects live in the wild? What purposes do these habitats serve?** • **What important function to bees serve when they collect nectar?**	Leading discussion about natural honeycomb, its functions and purposes. Key questions to ask and share responses are: • What is 'honeycomb' and why do bees build it? • In what types of habitats do other insects live in the wild? What purposes do these habitats serve? • What important function to bees serve when they collect nectar? (Discussion about pollination).
Optional: **Background 1** ***(10 minutes)***	Review pre-reading on 'Bees, Honeycomb and Hexagons' (see above).	Review web resource on Smartboard with students (or on tablets if students have them. See above website article by Robert Krulwich.
Section A - Introduction – What is Honeycomb? ***(10 minutes)***	Read through (independently, in a small group or with the teacher) of Section A in Student work booklet, 'Introduction – What is Honeycomb?' Complete questions (i) and (ii) together or individually and review them with your class.	Lead guided reading / facilitate individual or small group reading of Section A in Student work booklet, 'Introduction – Bees, Honeycomb and Hexagons'. Lead or assist students to complete items (i) and (ii) on page 3 of the booklet. Discuss answers between class members.
Section B – 'Honeycomb Geometry' ***(40 – 60 minutes)***	1. Use a computer (PC – this won't work on Apple devices, including iPads) to open the website 'MathsIsFun': : http://www.mathsisfun.com/geometry/index.html. Use the information in this section to work through parts (i) and (ii) of this section. Work with a pair and listen carefully to your teacher to guide you. 2. Read through part (iii) of this section. Hold a class discussion about why bees would have evolved the use of hexagonal structures for their honeycombs. 3. In part (iv) of this Section, you will need scissors and sticky-tape. Remove page 7 from your booklet; carefully cut out the shape net, construct and stick together the decahedron honeycomb object. See if you can join yours into a honeycomb cluster with those of other students – see page 4 of your booklet. 4. Work through part (v) carefully with a partner or individually. Your teacher may guide you and may also demonstrate the proof of the sum of the internal angles of a hexagon using a *protractor!*	1. On the Smartboard, open up 'MathsIsFun' webpage (see above) for exploring the features of quadrilateral plane shapes and polyhedral, including definitions and explanations of regularity and irregularity, congruence and angle properties of triangles and polygons: http://www.mathsisfun.com/geometry/index.html. Note: some animations require Flash Player (not suitable for iPad). 2. If students have access to computers, use this page to assist them to complete the questions in Section B. Work through Section B with students, either in small groups or as a whole-class guided and prompted reading exercise. 3. Assist students with practical activity in part (iv) of this section, 'Make your own honeycomb cell'. Students will need scissors and sticky-tape. 4. Work through part (v) carefully with students working with a partner or individually. Demonstrate the proof of the sum of the internal angles of a hexagon using a protractor. Note that the teacher answer guide has some additional information for teachers to share with students.

Optional: **Warm-up** ***(5 minutes)***	Watch Youtube clip, "Tessellations in Nature" with the class. Briefly discuss where you might have seen other naturally occurring tessellation patterns in nature. Consider and discuss: Why do you think tessellating patterns occur so often in nature?	Show students "Tessellations in Nature" Youtube clip as a warm up, to demonstrate and open discussion on tessellating patterns in nature. You may want to even discuss why tessellating patterns are more efficient than non-tessellating patterns (Hint: consider the issue of 'wasted spaces' in non-tessellating patterns and relate this to material / energy conservation by organisms.
Section C – 'Terrific Tesselations' ***(30 minutes)***	Read through Section C with your class and complete parts (i) and (ii) in your booklet. Part (i) will require the use of coloured pencils. Construct and complete your own tessellating patterns based on the shapes provided on page 13 of your booklet. In part (ii) of this section you may need to use Google or another search engine to help you answer the questions!	Read through Section C with students and complete parts (i) and (ii) together. Part (i) will require the use of coloured pencils. Guide and assist students as they construct and complete their own tessellating patterns based on the shapes provided on page 13 of their booklet. Students will likely need Google or another search engine to help them answer the questions.
Optional: **Pre-reading on Algebra** ***(15-20 minutes)***	Complete some pre-reading on Algebra using the 'Maths is Fun' website – "An Introduction to Algebra": https://www.mathsisfun.com/algebra/introduction.html (Note that this site may not work well on your iPad)	Have students read through "An Introduction to Algebra": https://www.mathsisfun.com/algebra/introduction.html Students may need access to PCs as some Flash Player features on this site will not work on Apple devices.
Section D – 'Honeycomb Algebra' ***(20 minutes)***	1. Complete Section D (a) with your class and your teacher. You will need to discuss the ideas together and seek clarification to ensure you understand how algebraic expressions describe parts of the recipe. 2. Work in a pair or threes to complete Section D (b), questions (i) and (ii). Review your responses together and with your teacher and correct your work. 3. Work in pairs or small groups to discuss and complete Section D (c), completing questions (iii) and (vi). Review your responses together and with your teacher and correct your work.	1. Work through Section D (a) with students. Some discussion and reinforcement using the whiteboard or smartboard may be useful here. 2. Have students work in pairs or small groups to discuss and complete Section D (a) and (b), completing questions (i) to (iii) as they go. Review student responses together and correct. 3. Have students work in pairs or small groups to discuss and complete Section D (c), completing questions (iv) to (vii) as they go. Review student responses together and correct.
Section E – 'Extension: Algebraic Description of Hexagonal Tessellations ***(30 minutes)***	Section E is best completed with a smaller group of extension students. It is worthwhile working systematically through the explanation and reasoning and completing the algebraic working together, to ensure students understand. At the bottom of Page 22, allow students to work on this individually to see whether they are able to formulate and then solve the algebraic problem. Work through the solutions together.	
Wrap up ***(5 minutes)***	***Wrap up the unit by reiterating that:*** • All living things have structural features and adaptations that help them to survive in their environment. Growth and survival of living things are affected by physical conditions of their environment *and to survive they must conserve energy and materials*. Thus, ***efficient structures like honeycomb tend to evolve as a way of conserving energy and materials***. • Knowing the properties of regular and irregular polygons and polyhedra help us to solve problems in areas such as construction, engineering, architecture and packaging, using geometric and algebraic reasoning. • We can use ***algebraic expressions to describe geometric patterns***, allowing us to solve complex problems, to model and to predict occurrences in science, mathematics, economics and other areas of human endeavour.	

E. Assessment Guide (aligned with curriculum links)

Success Criteria	Possible Assessment
Questioning and predicting: • With guidance, identify questions and problems associated with the geometry and structure of honeycombs that can be investigated mathematically and scientifically; and • Make modelled predictions about honeycomb structures based on mathematical and scientific knowledge. **/ 25**	• Students listen and participate in class discussion about Honeycomb in Section A - teacher anecdotal observation. • Students participate in discussion and class questioning in Section D (a) - teacher anecdotal observation.
Processing and analysing data and information: • Summarise data from an investigation of honeycomb, use this understanding to identify relationships between natural structures and mathematical reasoning and draw conclusions based on findings **/ 25**	• Students participate in guided reading or small group reading of Section A in Student work booklet then complete items (i) and (ii) on page 2 – teacher anecdotal observation and marked student responses. • Students explore the features of quadrilateral plane shapes and polyhedra, including definitions and explanations of regularity and irregularity, congruence and angle properties of triangles and polygons, then use information to complete the questions in Section B – teacher observation and marked student responses.
Applying mathematical reasoning: • Investigate and establish the properties of quadrilaterals and other polygons; regular and irregular tessellations and recurring geometric patterns • Use algebra to express simple models describing geometric patterns and use algebraic expressions and mathematical reasoning to solve problems **/ 25**	• Students work in pairs or small groups to discuss and complete Sections D (a) and D (b), completing questions (i) to (iii) as they go – marked student responses. • Students work in pairs or small groups to discuss and complete Section D (c), completing questions (iv) to (vii) – marked student responses. • ***Extension*** – Students complete Section E, including algebraic formulation and modelling – marked student responses.
Communicating: • Communicate ideas and explanations on the geometry and structures of honeycombs using scientific written descriptions, geometric diagrams and simple algebraic models. **/ 25**	• Students complete practical activity in part (iv) of Section B, 'Make your own honeycomb cell' – marked student responses. • Students complete parts (i) and (ii) of Section C - construct and complete tessellating patterns on page 13 of the booklet, and answer questions - marked student responses.