AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Nine Colours collaborative exercise
Teacher guide

Nine Colours

You have 27 small cubes, 3 each of nine colours.

Can you use all the small cubes to make a 3 by 3 by 3 cube so that each face of the bigger cube contains one of each colour?

nrich.maths.org

This task, ‘9 colours’, is found on nrich.maths.org. It is a highly engaging task, best set as a collaborative exercise as more heads are better than one. To complete the initial task successfully 27 small cubes need to be assembled as one large 3 by 3 by 3 cube. There are 3 small cubes of 9 colours and there must be one square of each colour visible on all 6 faces of the large cube.

Learning Intentions

This activity addresses several AC content descriptors but more importantly it develops spatial awareness, particularly when the task is extended beyond simply creating the large cube.

AC Mathematics Content Descriptors

Year 5	Connect three-dimensional objects with their nets and other two-dimensional representations (ACMMG111)
Year 6	Construct simple prisms and pyramids (ACMMG140)
Year 7	Calculate volumes of rectangular prisms (ACMMG160)
	Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)
	Describe translations, reflections in an axis and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)
Year 8	Develop formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume (ACMMG198)
Year 9	Solve problems involving the surface area and volume of right prisms (ACMMG218)
Year 10	Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes (ACMMG244)

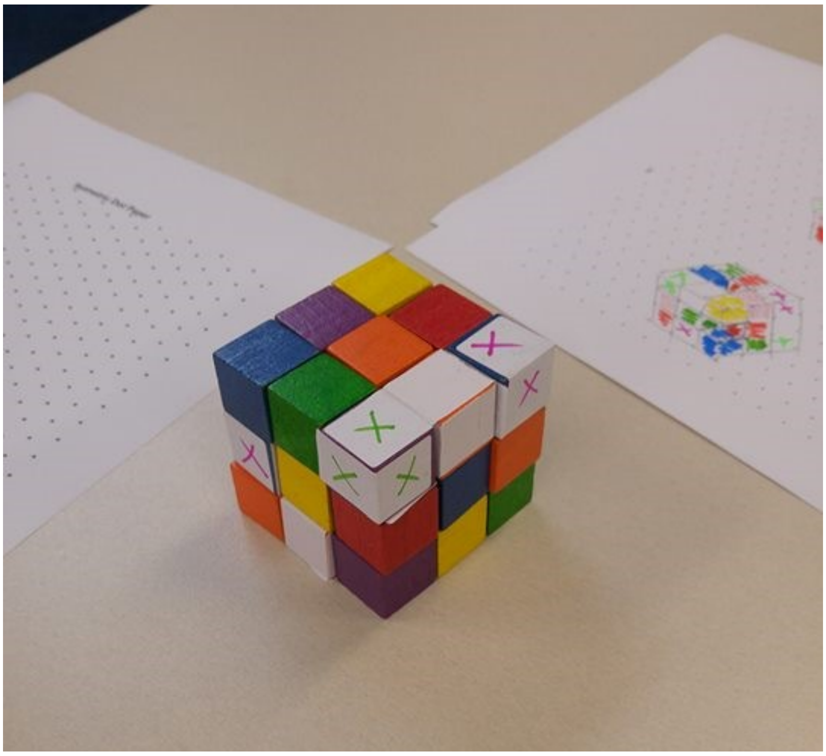
Proficiencies

Problem solving is the ability of students to make choices, interpret, formulate, model and investigate problem situations, select and use technological functions and communicate solutions effectively. Students pose and solve problems when they:

- use mathematics to represent unfamiliar or meaningful situations
- design investigations and plan their approaches
- apply their existing strategies to seek solutions
- verify that their answers are reasonable.

Reasoning refers to students developing an increasingly sophisticated capacity for logical, statistical and probabilistic thinking and actions, such as conjecturing, hypothesising, analysing, proving, evaluating, explaining, inferring, justifying, refuting, abstracting and generalising. Students are reasoning mathematically when they:

- explain their thinking
- deduce and justify strategies used and conclusions reached
- adapt the known to the unknown
- transfer learning from one context to another
- prove that something is true or false
- make inferences about data or the likelihood of events
- compare and contrast related ideas and explain their choices.



Activity

You have 27 small cubes, 3 each of nine colours.

Use the small cubes to make a 3 by 3 by 3 cube so that each face of the bigger cube contains one of every colour.

Once students have completed the large cube, ask them to check the base to make sure that it too has 9 different colours.

From there, give each member of the group a sheet of isometric paper and ask them to draw the cube from four different perspectives, colouring in the drawing accurately.

Focus Questions:

- How many drawings do you need to show all faces?
- Is there more than one way to place each small cube?
- Is yours the same as another group's cube?
- If you gave another group a drawing could they reconstruct your design?
    - What is the minimum amount of information you need to give them?
- Remove a particular colour (3 blocks), then draw the design with missing blocks.
- Is there a pattern to the placement of your cubes?
- Can you predict the colour of the middle block (the one not visible)?
- Can you determine the total number of combinations?
- What can you determine about the number of faces of each colour you will see in the completed cube?
- Can you explain this using numbers/algebra?

Extension questions relating to measurement:

- What is the surface area and the volume?
- How does the surface area and volume change when you remove one colour?
- Is there symmetry in your design?

Other extension questions:

- How many colours would you need for a 4 by 4 by 4 cube?
- How many cubes in a 4 by 4 by 4 cube?
- Do the same rules and patterns exist in a 4 by 4 by 4 cube?
- Is there a point at which the task is impossible? (5 x 5 x 5; 6 x 6 x 6; 7 x 7 x 7...)