MET-BEFORE

What's the issue?

Interpreting symbols is not as easy as we perceive. Many students face difficulties in interpreting the meanings of symbols they have **met before**. This sparks an interesting question of why they have such difficulties. In some cases, students' interpretation of symbols is affected by their previous experiences and these can be supportive or problematic in shaping their conceptions about symbols.
There are two distinct types of conceptions; namely supportive conceptions and problematic conceptions.

A **supportive conception** refers to a conception that works in an old context and continues to work in a new context.

A **problematic conception** refers to a conception that works in an old context but doesn't work in a new context

Viewing prior conceptions in this way was proposed in a framework by Chin & Tall (2012) and Chin (2013).

Misconceptions? Perhaps problematic conceptions…

Of 152 first year university mathematics students involved in our study, 75 students claimed that $sin^{-1}x$ is equivalent to $\frac{1}{sinx}$ (or the reciprocal).

Given $f(x) = 3x + 1$, 24 students claimed that $f^{-1}(x)$ is $\frac{1}{3x+1}$.

Some unusual statements from our participants:

> *"-1 means "1 over" for example* $\frac{1}{\sin(x)}$*"*
>
> *"Inverse of function,* $\frac{1}{3x+1}$ *"*
>
> *"x^{-1} means x multiplied by x minus one times"*

This shows that the participants have problematic conceptions that arise from working with negative exponents in Real numbers.

Problematic conceptions in school maths…

Click below to see some more specific examples of students' problematic conceptions at different year levels, which can highlight the effect of problematic conception on new learning.

- *Clues in Years 2-4*
- *Clues in Years 11-12*
- *Clues in Specialist Mathematics*

Ideas from the classroom…

Supportive conceptions and **problematic conceptions** might arise from the contexts in which a student has previously met a symbol. It is important for teachers to help students to build on their supportive conceptions for long-term sense making while becoming aware of the problematic conceptions that impede students' learning. Here are some ideas that you might want to try to help students deal with "**met-befores**":

- Constantly be aware of the possible **problematic conceptions** that might arise, and explicitly relate the new context to the previous contexts.
- Encourage students to share their thoughts in class/group on how to make sense of a particular mathematical symbol or concept.
- Demonstrate the changes of meaning for symbols by using examples.
- Encourage students to speak out and write out the things that don't make sense in thinking of symbols.
- Discuss with students their experience with **supportive conceptions** and **problematic conceptions**.
- Design mathematics tasks that require students to recognise the different meanings of symbols in order to use them successfully.
- Design mathematics tasks that can lead students to the discovery of the different meanings of symbols.
- Encourage students to explore when a particular meaning for a symbol will break down by giving/asking them relevant questions or mathematics tasks.

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Problematic Conceptions in Years 2-4.

Here are some examples of problematic conceptions which are commonly seen in the classrooms at years 2-4:

- Confusion with the varied meanings of symbols. Note that changes in the meaning of a symbol might occur when there is a change of context. Several consequences of this are:
  - Students cannot make sense of new learning due to the changes in meaning of symbols.
  - Students opt for rote-learning over conceptual understanding due to problematic conceptions.
  - Students recognize symbols as actions that need to be performed rather than as meaningful entities.
  - Students are unable to determine a change of context for a particular symbol.
  - Students wrongly interpret a change of context for a symbol.

  As an illustration, the **multiplication symbol** $(\times)$ starts off with the interpretation as "repeated addition" when it involves the multiplication of natural numbers. For instance, $2 \times 3 = 3 + 3$ Therefore, students will have a conception that multiplication is repeated addition. When students learn whole numbers, this conception still holds for the multiplication of whole numbers thus repeated addition is a supportive conception in this new situation.

  However when we are talking about the multiplication of fractions, the conception of repeated addition is problematic because it doesn't make sense to interpret the multiplication symbol $(\times)$ as repeated addition in this new situation. For example, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ means $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$ thus the multiplication symbol $(\times)$ has to be interpreted as "of" in this new situation.

  This shows the subtle changes in meaning of the multiplication symbol $(\times)$ over the longer term.

In summary, a **supportive conception** supports generalization in a new context whereas a **problematic conception** impedes progress.

Problematic Conceptions in Years 11-12.

Here are some examples of problematic conceptions which you might expect to see in the classrooms at years 11-12:

- Confusion with the varied meanings of symbols. Note that changes in the meaning of a symbol might occur when there is a change of context. Several consequences of this are:
    - Students cannot make sense of new learning due to the changes in meaning of symbols.
    - Students opt for rote-learning over conceptual understanding due to problematic conceptions.
    - Students recognize symbols as actions that need to be performed rather than as meaningful entities.
    - Students are unable to determine a change of context for a particular symbol.
    - Students wrongly interpret a change of context for a symbol.

- Problematic conceptions that arise from working with negative exponents in real numbers. Some common mistakes include:
    - Interpreting $sin^{-1}x$ as $\frac{1}{sinx}$.
    - Expressing $f^{-1}(x)$ as $\frac{1}{f(x)}$.
    - Interpreting $\frac{1}{sinx}$ as $\arcsin(x)$.
    - Interpreting the inverse of $f(x)$ as $\frac{1}{f(x)}$.
    - Interpreting -1 as “1 over”, for instance $\frac{1}{sinx}$.

- Problematic conceptions that arise from different contexts in trigonometry. Some examples could include:

    - Students not being able to make sense of a particular trigonometric function when the angle involves is greater than 90° as they are working in the context of triangle trigonometry. Based on Chin & Tall (2012) and Chin (2013), triangle trigonometry is based on right-angled triangles with positive sides, with angles bigger than 0° and less than 90°. For instance, a participant in Chin’s study attempted to describe tangent 90° in the context of triangle trigonometry context (see Figure 1)

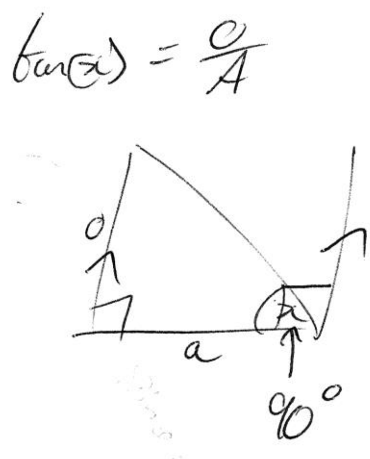

Figure 1: $Tangent$ 90° in the context of triangle trigonometry

- Students not being able to link triangle trigonometry context and circle trigonometry. According to Chin & Tall (2012) and Chin (2013), circle trigonometry involves dynamic angles of any size and sign with trigonometric ratios involving signed numbers and the properties of trigonometry functions represented as graphs. For example, a participant in Chin's study was asked to make sense of $sin200°$ and he responded:

  > *"I think I was trying to make sense of sin 200 being just taking the graph so having the first up to 90 defined by a ratio of the triangle and the rest of it just being a continuation of this (pointing to a sine curve)"*

  Additionally, he also responded that he couldn't visualize a triangle with $sin200°$.

- Students not being able to visualize any triangles that involve angles more than $90°$.

In summary, a **supportive conception** supports generalization in a new context whereas a **problematic conception** impedes progress.

Problematic Conceptions in Specialist Mathematics.

Here are some examples of problematic conceptions which you might expect to see in the classrooms of specialist mathematics:

- Problematic conceptions that arise from the context of real numbers. Some common mistakes include:
  - Comparing two complex numbers. For instance, respondents were asked to respond whether $2 + 3i < 4 + 3i$ was correct or not. Of the 4 participants, 3 responded that the given inequality was correct by giving the following reasons:

    "Assuming that these 3is have the same value, so we don't need to look at these anymore. Then 4 is bigger than 2, that's why I said $2 + 3i < 4 + 3i$ is correct."

    "By cancelling 3i on both sides of the inequality, I can see that 4 is greater than 2."

  - Perceiving the existence of negative complex numbers. Take for instance, given $z = -2 - i$, most of the participants perceived that z was a negative complex number by giving the following reason:

    "because there is a negative sign in front of i."

  - Dividing complex numbers like the way of dividing real numbers. As an illustration, given $\frac{(6-9i)}{(2-3i)} = \frac{3(2-3i)}{(2-3i)}$, most of participants responded that $(2 - 3i)$ can be cancelled out leaving 3 as the final answer.

  - Perceiving all the properties of real numbers will be the properties of complex numbers.

In summary, a **supportive conception** supports generalization in a new context whereas a **problematic conception** impedes progress.