Starter Problems for Problem Solving PD

#	Name	Source
1.	A-mazing	Problems to Solve in Primary School Mathematics, AMT
2.	Lucy Likes Darts	Problems to Solve in Primary School Mathematics, AMT
3.	4 by 4	Problems to Solve in Primary School Mathematics, AMT
4.	Number Tower	Problems to Solve in Primary School Mathematics, AMT
5.	1 – 2 - 3	Problems to Solve in Middle School Mathematics, AMT
6.	Tetrahexes	Problems to Solve in Middle School Mathematics, AMT
7.	Found Their Marbles	United we Solve, eeps media
8.	Point of View 1	United we Solve, eeps media
9.	Wayne's Wheel	United we Solve, eeps media
10.	Build It #1	Get it Together, Equals
11.	Stick Figures 1	Get it Together, Equals
12.	What's the Pattern?	Get it Together, Equals
13.	Mountains and Matchsticks	Maths 300
14.	Leap Frog/Tic Tac Toe	Wooden Puzzle, Hachette Partworks
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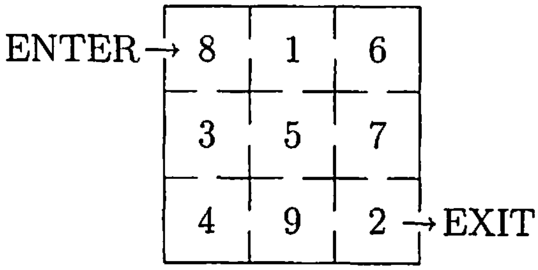
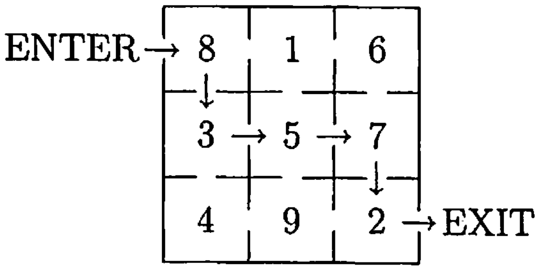
A-mazing

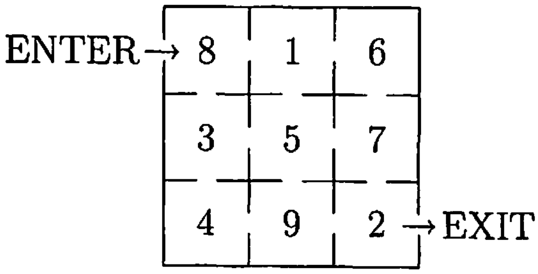
Lee drew this maze. You enter at the top on the left, move either to the right or down and exit bottom right. You add the numbers in the boxes you pass through.



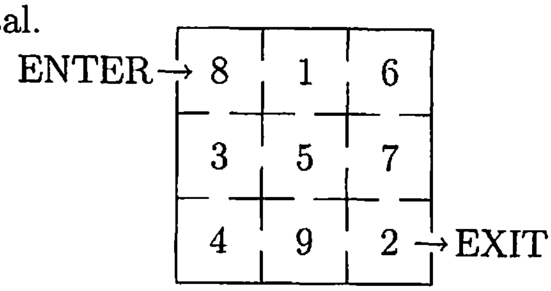
John went this way:
His total was $8+3+5+7+2=25$.



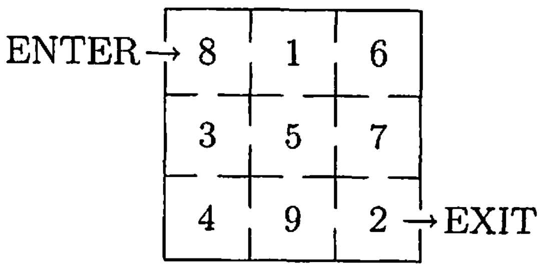
a Find a path which gives you a total of 24.



b Find the path which gives the largest total.



c Find the path which gives the smallest total.



Teacher's Notes – A-mazing

The problem provides practice in addition and in checking all possibilities.

The maze is in fact a magic square of order 3 - all the rows, columns and diagonals add to 15.

Solutions

There are 6 possible paths:

$8 + 1 + 6 + 7 + 2 = 24$

$8 + 1 + 5 + 7 + 2 = 23$

$8 + 1 + 5 + 9 + 2 = 25$

$8 + 3 + 5 + 7 + 2 = 25$

$8 + 3 + 5 + 9 + 2 = 27$

$8 + 3 + 4 + 9 + 2 = 26$

Hence the solutions are:

a $8 + 1 + 6 + 7 + 2 = 24$

b $8 + 3 + 5 + 9 + 2 = 27$

c $8 + 1 + 5 + 7 + 2 = 23$

Extensions

1. Suppose you can go up, down, left or right in the maze, but cannot visit the same square more than once. What is the largest total you can get now?
2. Enter the maze in the original problem as before, travel either right or down. This time, **multiply** the numbers in the boxes you pass through until you go out the exit. What is the largest **product** you can get?

Solutions to Extensions

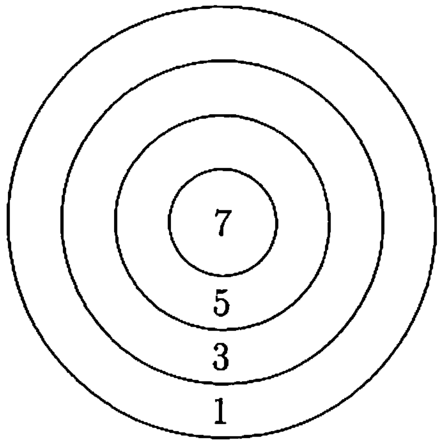
1. $8+3+4+9+5+1+6+7+2 = 45$ OR $8+1+6+7+5+3+4+9+2 = 45$.
   You can visit each square once.
2. For the 6 paths given above, the largest product is $8 \times 3 \times 5 \times 9 \times 2 = 2160$.

Lucy Likes Darts

Lucy throws darts at this target.

She never misses the target.

Her darts always score.



She throws 2 darts. She adds the 2 numbers together. Her score is 8.

a What could her numbers have been?

Is there another way she could have scored?

b After her third throw, her total score was 13. Where did her third dart hit?

c Lucy throws a fourth dart. Explain why her total score is now even.

d Lucy threw 17 darts for a total of 80.

Explain why she must have made a mistake in her adding up.

Teacher's Notes – Lucy Likes Darts

The problem provides practice in addition and subtraction in an unusual setting. It also gives an opportunity to discuss generalisation.

A discussion about odd and even numbers after the problem has been completed is worthwhile.

Solutions

a (3 and 5), (5 and 3), (1 and 7) or (7 and 1).

b 5.

c **Alternative i**

If she hits 1, her score is $13 + 1 = 14$ (even).

If she hits 3, her score is $13 + 3 = 16$ (even).

If she hits 5, her score is $13 + 5 = 18$ (even).

If she hits 7, her score is $13 + 7 = 20$ (even).

So it is always even.

Alternative ii

Each of 1, 3, 5 and 7 is odd, and 13 is odd. But two odd numbers add to an even number, so the total is always even.

d Lucy is adding 17 odd numbers together. The total must be odd.

Extensions

1. What is the smallest total Lucy could have after she has thrown 4 darts and all the darts hit the board?

   What is the largest total she could have after throwing 4 darts and all the darts hit the board?

2. Lucy throws some darts and gets a total of 15. What is the smallest number of darts she could have thrown?

Solutions to Extensions

1. Smallest total $= 1 + 1 + 1 + 1 = 4$.

   Largest total $= 7 + 7 + 7 + 7 = 28$.

2. Largest score from 2 darts is $7 + 7 = 14$, so she must throw at least 3 darts. 3 darts is enough – she might throw $7 + 7 + 1 = 15$, or $7 + 5 + 3$, etc.

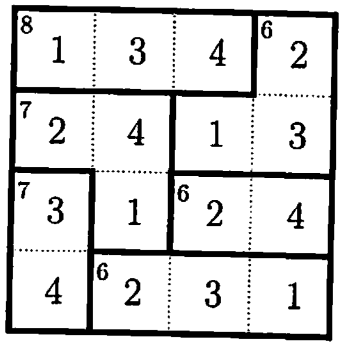
4 by 4

Here is a 4×4 grid filled with the numbers 1, 2, 3 and 4.

Each row has exactly 1 of each number.

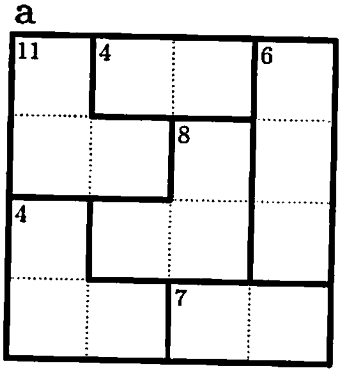
Each column has exactly 1 of each number.

In each of the outlined shapes, the sum of the numbers in the shape is the small number in the shape.



Complete these grids in the same way.

a



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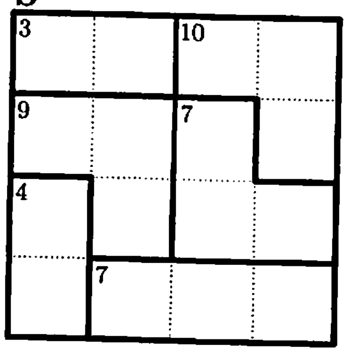
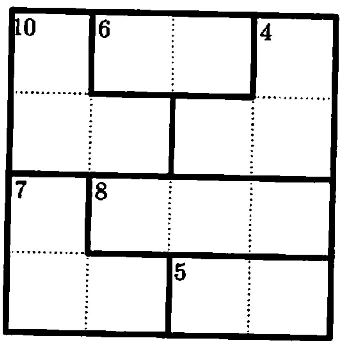
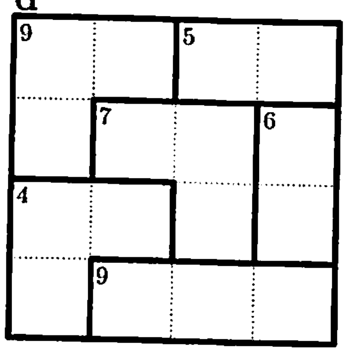


c



d



Teacher's Notes - 4 by 4

The problem gives practice in addition in an unusual way and requires students to use logic.

Go through the rules with the example given so that children know how it works.

Some further help may be needed to get things started on a:

'What 2 nunbers add to 4 for the middle of the top row?'

'Why must they be different?'

'Now what 3 *different* numbers add to 6 for the right-hand column?'

'But we need 3 and 1 in the top row already, so what goes in the top-right corner?'

'Now what goes in the top-left corner?'

Another useful idea is that the sum of the numbers in each row or column is 10. For example, if three of them add to 7, then the fourth one must be $10 - 7 = 3$.

Solutions

a

(11) 4	(4) 3	1	(6) 2
3	4	(8) 2	1
(4) 1	2	4	3
2	1	(7) 3	4

b

(3) 2	1	(10) 3	4
(9) 4	2	(7) 1	3
(4) 1	3	4	2
3	(7) 4	2	1

c

(10) 3	(6) 2	4	(4) 1
4	3	1	3
(7) 2	(8) 1	3	4
1	4	(5) 2	3

d

(6) 1	3	(6) 4	2
2	(8) 4	3	(5) 1
(4) 3	2	1	4
4	(6) 1	2	3

Extension

Try this one:

10			
7			3
6		4	
4	6		

Solution to Extension

(10) 3	1	2	4
(7) 1	2	4	(3) 3
(6) 2	4	(4) 3	1
(4) 4	(6) 3	1	2

Number Tower

The number in each box above the bottom row in this number tower is the sum of the two numbers below it.

				50				
			28		22			
		17		11		11		
	9		8		3		8	
3		6		2		1		7

a Fill in the numbers in this tower.

7		2		6		9		3

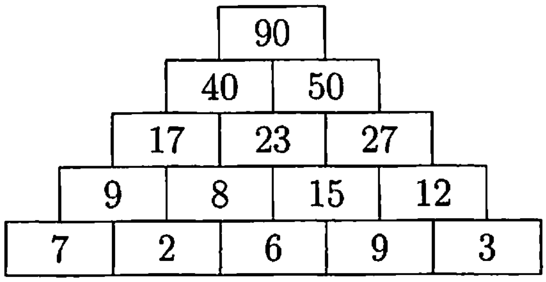
b Use the numbers 1, 2, 3, 4 and 5 in the bottom row of this tower. Place them so that the number at the top of the tower is the largest possible.

Teacher's Notes - Number Tower

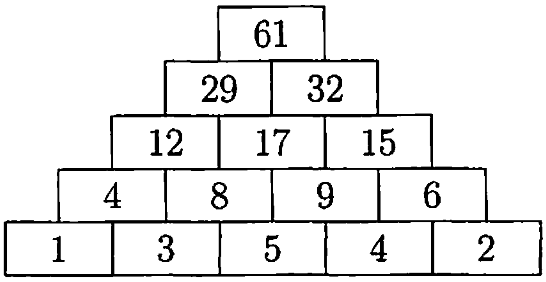
The problem provides practice in addition in an interesting way.

Solutions

a



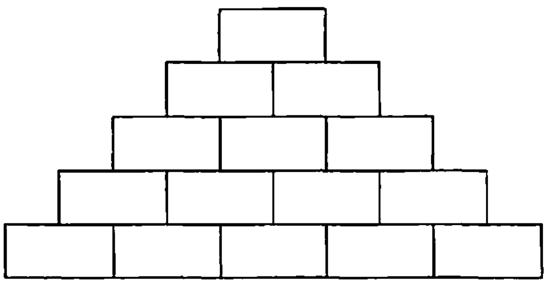
b We get the largest total when the largest number is in the middle of the bottom row (it contributes to the final total six times), and the next two largest numbers on either side of it (they each contribute four times to the final total).



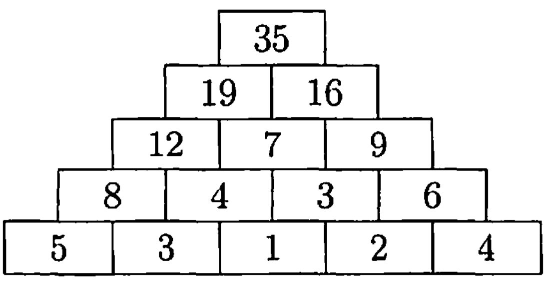
The same total is reached if the 3 and 4 are interchanged and if the 1 and 2 are interchanged.

Extension

Use the numbers 1, 2, 3, 4 and 5 in the bottom row of this tower. Place them so that the number at the top of the tower is the smallest possible.



Solution to Extension



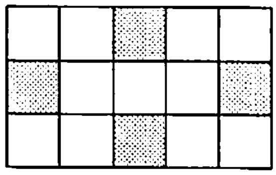
Again, the 5 and 4, or the 3 and 2 can be interchanged.

1 – 2 – 3

Here is a grid where each space contains a 1, 2 or 3. The 2-digit and 3-digit numbers formed across and down (11, 31, 211 and 223) are all prime numbers.

2	1	1
2		1
3	1	

a Fill the spaces in the grid below with 1s, 2s and 3s so that you can read these primes across and down: 11, 13, 23, 31, 113, 131, 211.

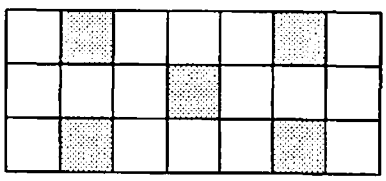


There are other ways that the grid above can be filled with 1s, 2s and 3s so that the seven 2-digit and 3-digit numbers formed across and down are different primes.

b Find one of these ways. (You may use some of the primes given in a, but you will need to use some others.)

c Explain why the prime 223 cannot be one of the numbers formed in this grid.

d The spaces in the grid below can be filled with 1s, 2s and 3s so that the eight 3-digit numbers formed across and down are different primes.



Find one way to complete the grid.

Teachers' Notes – 1 – 2 – 3

1. The problem requires students to make a list of primes, either using a printed list, or an internet site, or from first principles, using a calculator. In the latter case, checking for division by smaller primes is necessary, and a point at which to stop needs to be established.
2. It would be useful to discover with the students which digits can be the last digit of a 2-digit or 3-digit prime.
3. After students have submitted their solutions, it makes a good class exercise to find *all* the solutions to part **b**. Students can make a master list of solutions on a board or poster. To be sure that all solutions have been found, a systematic approach is needed. Note that there are only four 2-digit primes to be placed and four places for them; students can list all the possibilities for these places (24 of them) and then allocate each of these to a student or pair of students to find all the ways to place the remaining digits.

   It is also useful to have students find all the solutions to **d**.

Solutions

a

3	1		2	3
	1	3	1	
1	3		1	1

(This is the only solution.)

b We list the possible primes: 11, 13, 23, 31, 113, 131, 211, 223, 233, 311, 313, 331. The following are the only possible solutions. (Students need show only one of these.)

1	1		3	1
	1	3	1	
2	3		1	3

1	1		2	3
	3	1	1	
3	1		1	3

1	1		2	3
	3	3	1	
3	1		1	3

1	3		1	1
	1	3	1	
2	3		3	1

1	3		2	3
	1	1	3	
1	1		3	1

1	3		2	3
	3	1	3	
1	1		3	1

1	3		2	3
	1	3	1	
3	1		1	1

1	3		2	3
	3	1	1	
3	1		1	1

1	3		3	1
	1	3	1	
2	3		1	1

1	3		3	1
	1	1	3	
2	3		1	1

2	3		1	1
	1	3	1	
1	3		3	1

2	3		1	1
	1	1	3	
3	1		1	3

2	3		1	1
	3	1	3	
3	1		1	3

2	3		1	3
	1	3	1	
1	1		3	1

2	3		1	3
	3	1	1	
1	1		3	1

2	3		1	3
	3	1	3	
3	1		1	1

2	3		1	3
	1	1	3	
3	1		1	1

2	3		3	1
	1	1	3	
1	1		1	3

2	3		3	1
	1	3	1	
1	3		1	1

2	3		3	1
	1	1	3	
1	3		1	1

3	1		2	3
	3	1	1	
1	1		1	3

3	1		2	3
	3	3	1	
1	1		1	3

c If 223 is placed in the first vertical 3-digit column, the 2-digit prime in the top left corner will end in 2, which is not possible.

If 223 is placed in the second vertical 3-digit column, the 3-digit prime across the centre must end in 2, which is not possible.

If 223 is placed in the middle horizontal row, the 3-digit prime in the first vertical column has middle digit 2, which is not possible.

d There are eight 3-digit primes which can be used to fill the grid: 113, 131, 211, 223, 233, 311, 313, 331. As there are eight places to put these primes, all eight must be used. There are four alternative solutions:

2		2	2	3		3
1	1	3		1	3	1
1		3	3	1		3

3		2	2	3		2
1	1	3		1	3	1
3		3	3	1		1

2		2	2	3		3
3	3	1		1	3	1
3		1	1	3		1

2		2	2	3		1
1	1	3		3	1	3
1		3	1	1		1

(Students need only show one of these solutions.)

Extension

1. Fill the grid below using different primes made only from the digits 1, 3 and 6.

2. Fill the grid below using different primes made only from the digits 1, 3 and 7.

Solutions

1. The primes which can be used are 113, 131, 163, 311, 313, 331, 613, 631 and 661.

1		6	3	1		3
6	6	1		1	3	1
3		3	1	3		1

There are other solutions.

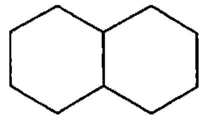
2. The primes which can be used are 113, 131, 137, 173, 311, 313, 317, 331. 337, 373, 733 and 773. As there are 11 primes needed to fill the grid, all but one of these must be used.

3		7	7	3		1	3	1
3	7	3		1	3	7		1
1		3	3	7		3	1	3

There are other solutions.

Tetrahexes

Regular hexagons can be joined edge-to-edge. This is the only way to join two regular hexagons together – others are made from this one by rotation or reflection.



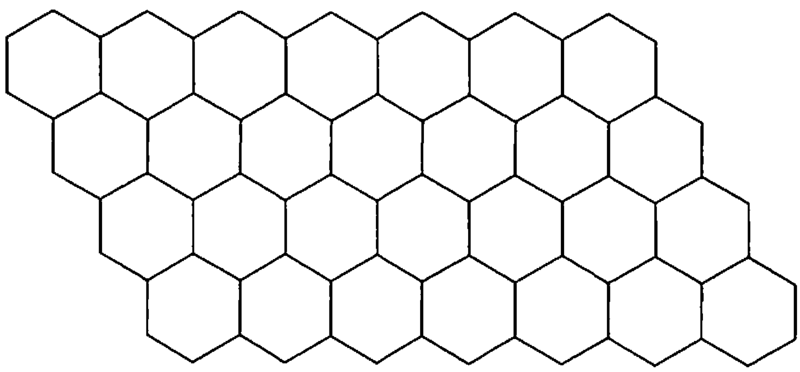
Use isometric dot paper to help you draw these shapes.

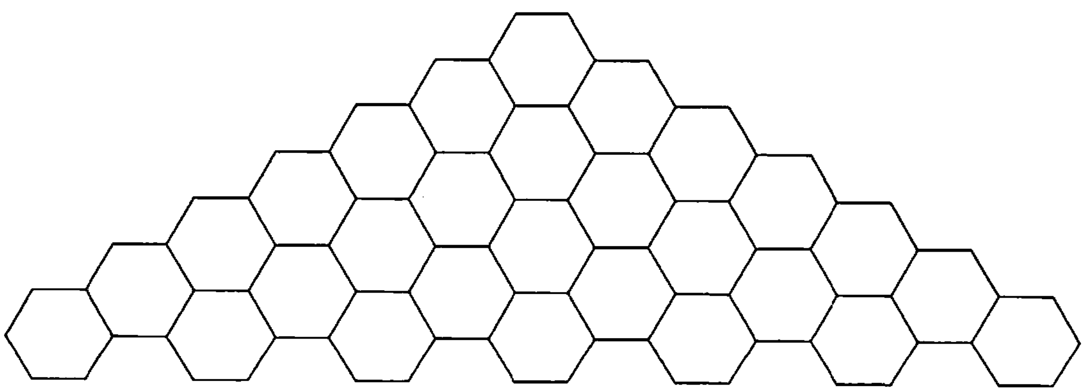
a Draw the three different ways to join three hexagons together.

b There are seven ways to join four hexagons – the shapes are called **tetrahexes.**

Make a set of the seven tetrahexes on isometric paper glued to thin cardboard.

c Use your set of tetrahexes to make these shapes. Show how you have done it by drawing careful diagrams on isometric dot paper.



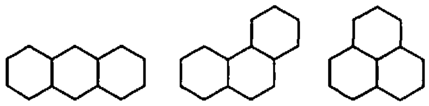


Teachers' Notes – Tetrahexes

1. The problem gives students excellent practice in spatial visualisation and an opportunity to explore symmetry.
2. There are many puzzles associated with trihexes (three hexagons joined together) and tetrahexes. Having made a set, students should keep it, perhaps in an envelope glued inside the back cover of a book.

Solutions

a



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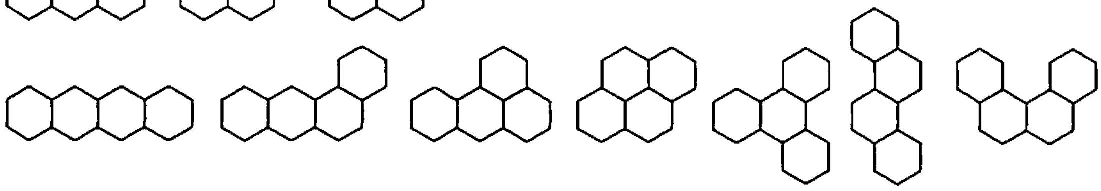
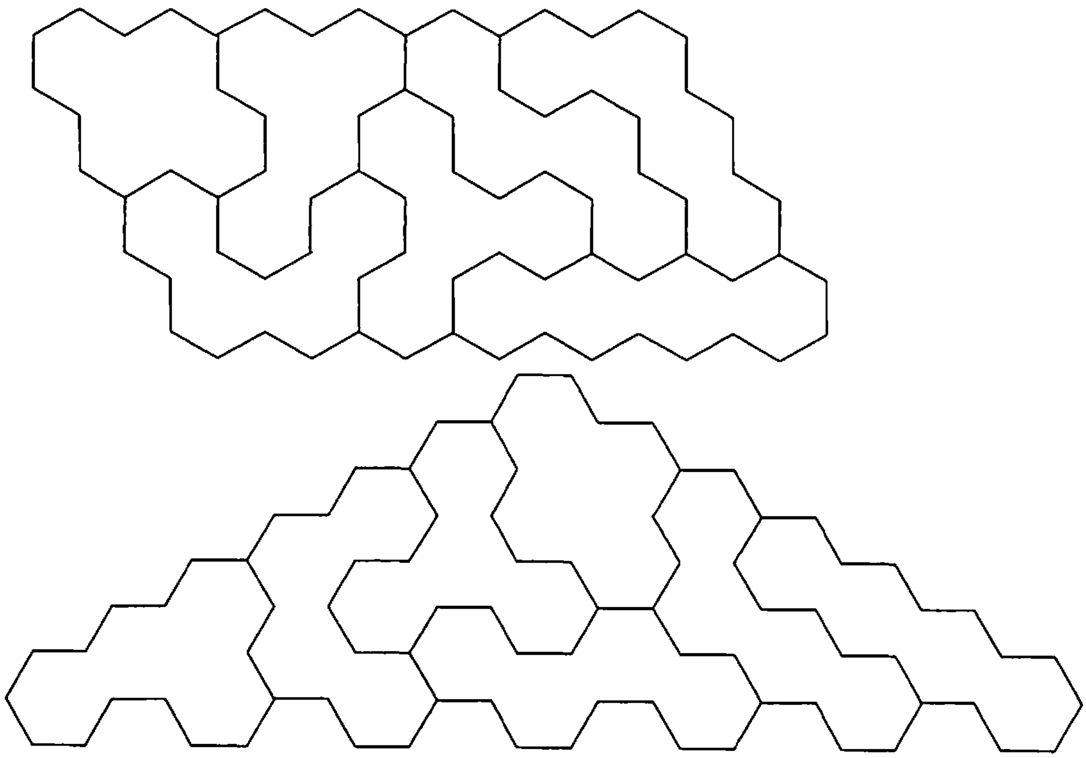
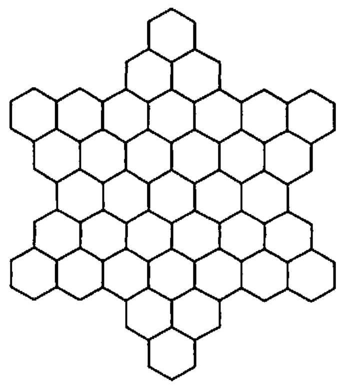


c There are other solutions.



Extension

Use all seven tetrahexes and all three trihexes to fill this shape.



(There are a multitude of ways to do this. A useful class project is to collect solutions from everyone, sort them into classes and see how they are related to each other by the change of position of a few pieces. For example, two solutions might only differ by the interchange of two pieces.)

▲ Found Their Marbles

Every day in April, Alice found a number of marbles equal to ten times the number of the day.

Calvin found two marbles on April 6 (for a total of four). That day, Alice found 60 marbles.

▼ Found Their Marbles

Every day in April, Xavier found 100 marbles.

Calvin found one marble on April 4. On that day, Xavier found a hundred marbles for the fourth time—for a total of 400. On that day, Alice found 40 for a total of only 100.

students! send your solutions to answers@eeps.com

read student solutions at www.eeps.com

◄ Found Their Marbles

On April 1, Calvin found no marbles at all. In fact, he found no marbles on any odd-numbered day.

Here's your group's problem: On which days did Alice have the most marbles altogether? During the first part of April, Xavier had the most.

► Found Their Marbles

On April 2, Calvin found one marble. After that, he found marbles every other day. Strangely, whenever he found some, he found exactly as many marbles as he already had.

At the end of the month, Calvin had the most marbles of any of the kids.

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students! send your solutions to answers@eeps.com

○ Found Their Marbles

There are 30 days in April.

Hint: It might help to make a table as you solve this problem. You might also think about what a graph of it would look like.

△ Found Their Marbles

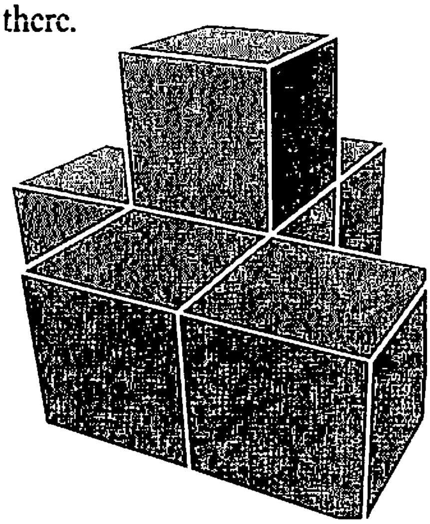
Hint: In this problem, you have to keep track of how many marbles each kid finds on each day.

You also have to keep track of the total number of marbles each kid has found.

Linear vs. exponential vs. quadratic, functions. Organizing. **Cluster: Growth**

▲ Point of View 1

Each person has a different drawing of the same structure. Work together to build it with blocks. Then figure out where the "point of view" of *this* clue is. Place this card there.



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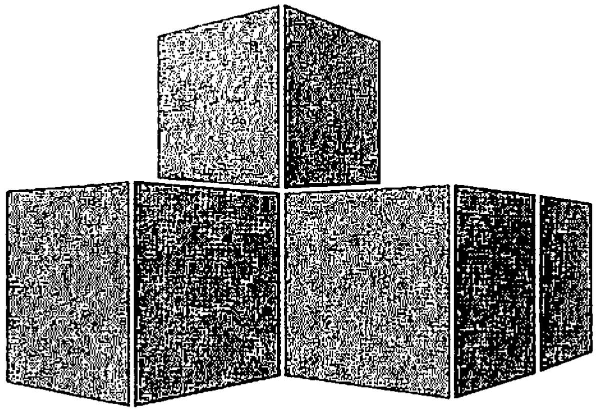

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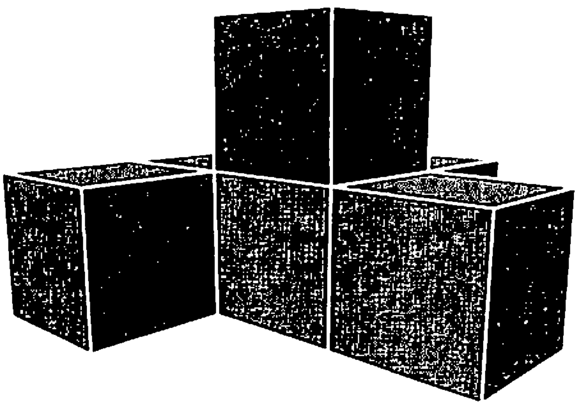

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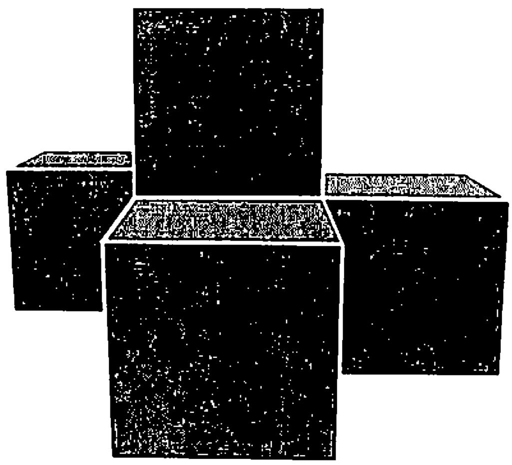


▲ Wayne's Wheel

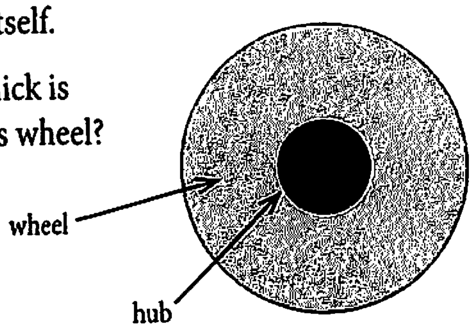
Wayne's wheel goes around four times when it rolls ten meters.

How thick is Wayne's wheel?

▼ Wayne's Wheel

The circumference of the hub of Wayne's wheel is the same as the diameter of the wheel itself.

How thick is Wayne's wheel?



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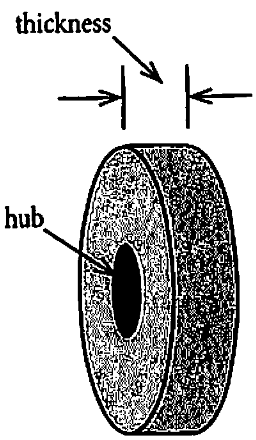
◀ Wayne's Wheel

The thickness of Wayne's wheel is the same as the radius of the hub.

How thick is Wayne's wheel?



▶ Wayne's Wheel

Two useful formulas about circles:

$$C = \pi d \text{ and } d = 2r$$

where C = circumference, d = diameter, and r = radius.

How thick is Wayne's wheel?

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○ Wayne's Wheel

Common sense, but easy to forget: When a wheel rolls around once—if it doesn't slip—the distance it travels is the *circumference* of the wheel.

How thick is Wayne's wheel?

△ Wayne's Wheel

Wayne's wheel is thicker than a bicycle wheel but thinner than one from a car.

How thick is Wayne's wheel?

from *United We Solve*

Geometry, measurement, circles.

Cluster: Round and Round

Build It #1

There are six blocks in all.

One of the blocks is yellow.

Build It #1

The green block shares one face with each of the other five blocks.

Build It #1

The two red blocks do not touch each other.

Build It #1

The two blue blocks do not touch each other.

Build It #1

Each red block shares an edge with the yellow block.

Build It #1

Each blue block shares one edge with each of the red blocks.

Stick Figures 1

There are twelve sticks in the figure. The sticks are unbroken and they don't overlap.

Make the figure!

Stick Figures 1

There are eight sticks in the square.

Make the figure!

Stick Figures 1

There are four sticks in the interior of the square.

All of the sticks are the same length.

Make the figure!

Stick Figures 1

There are six sticks in the triangle.

Make the figure!

Stick Figures 1

The triangle and the square share a side.

Make the figure!

Stick Figures 1

In the figure, both the rectangle and the triangle are regular polygons.

Make the figure!

What's the Pattern?

The sixth number is the third number times four and it is the first number times eight.

What is the seventh number in the pattern?

What's the Pattern?

The third number is the second number plus one, and the fourth number is the third number plus one.

What is the seventh number in the pattern?

What's the Pattern?

The fifth number is the third number plus the fourth number.

What is the seventh number in the pattern?

What's the Pattern?

When you add the first six numbers together, the sum is twenty.

What is the seventh number in the pattern?

What's the Pattern?

The third number raised to the third power equals the sixth number.

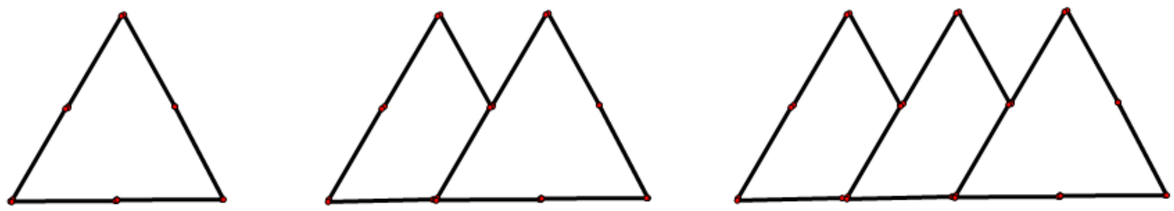
What is the seventh number in the pattern?

What's the Pattern?

The first and second numbers are the same.

What is the seventh number in the pattern?

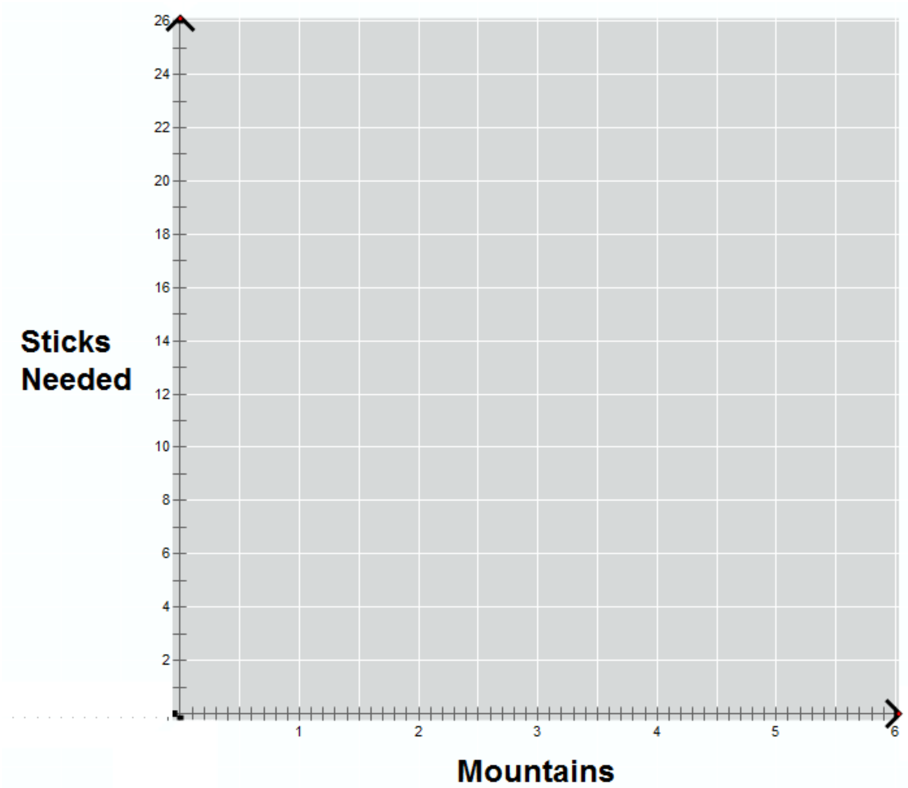
Mountains Activity



Use your sticks to work out how many you need to make each of the mountain ranges given in the table

How many Mountains	1	2	3	4	5
How many sticks					

Plot points for each of the pairs of values in the table.



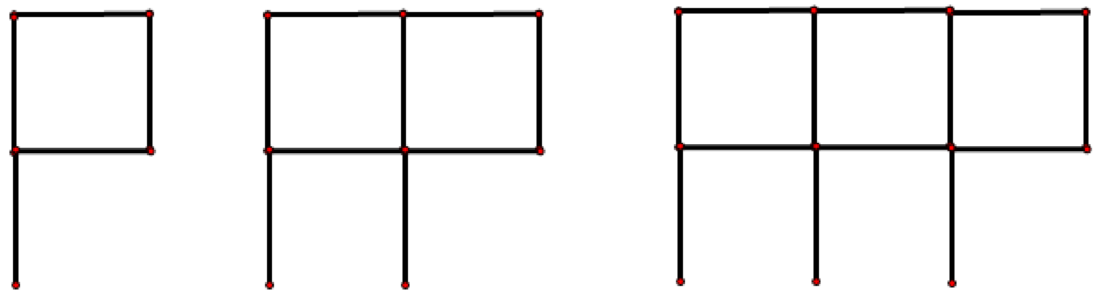
What do you notice about the pattern produced by the points?

Write down a sentence for how to create and continue the pattern (Where do you start and what do you do?)

Write this as an algebraic equation.

Can you write this expression in factorised form?

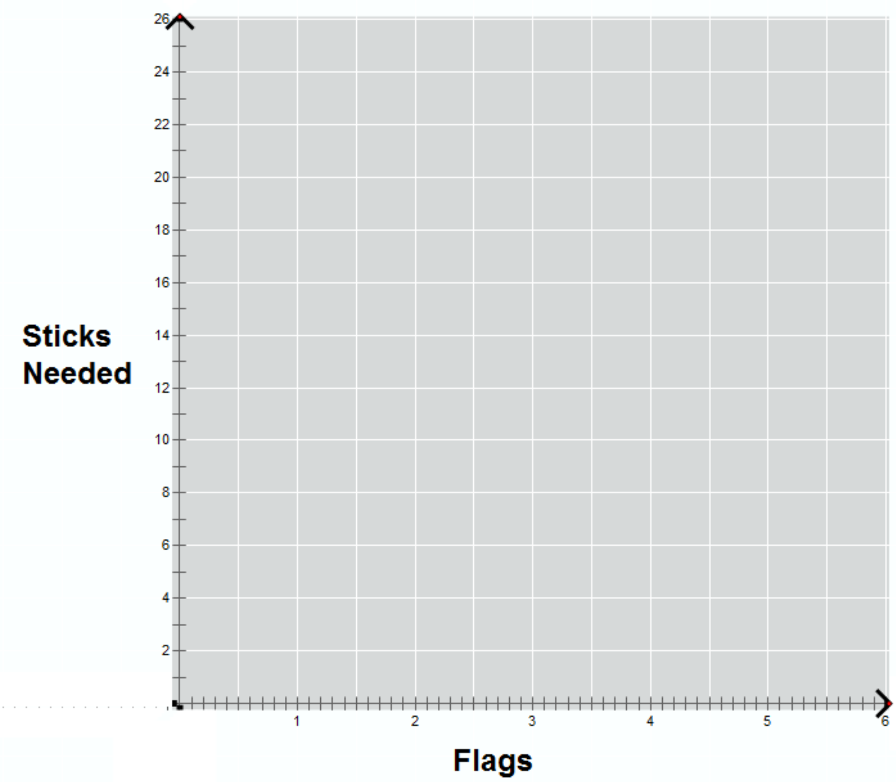
Flags Activity



Use your sticks to work out how many you need to make each of the flag sets given in the table

How many Flags	1	2	3	4	5
How many sticks					

Plot points for each of the pairs of values in the table.



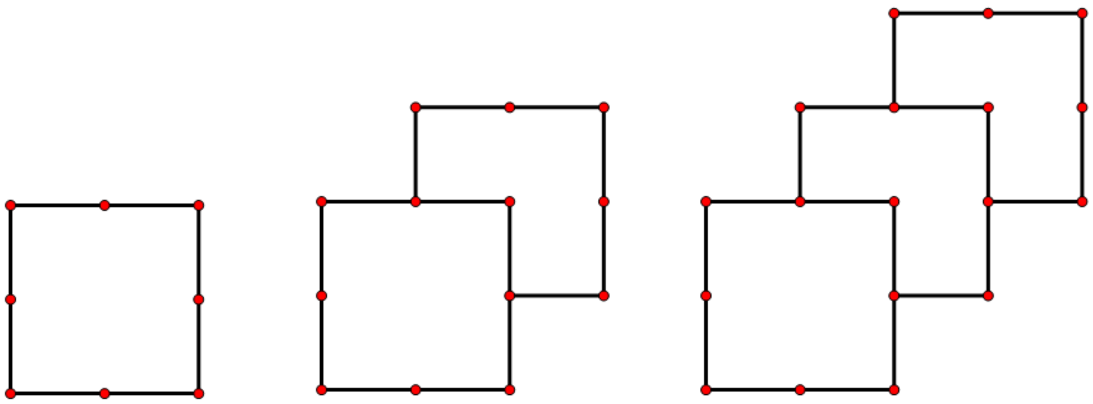
What do you notice about the pattern produced by the points?

Write down a sentence for how to create and continue the pattern (Where do you start and what do you do?)

Write this as an algebraic equation.

Can you write this expression in factorised form?

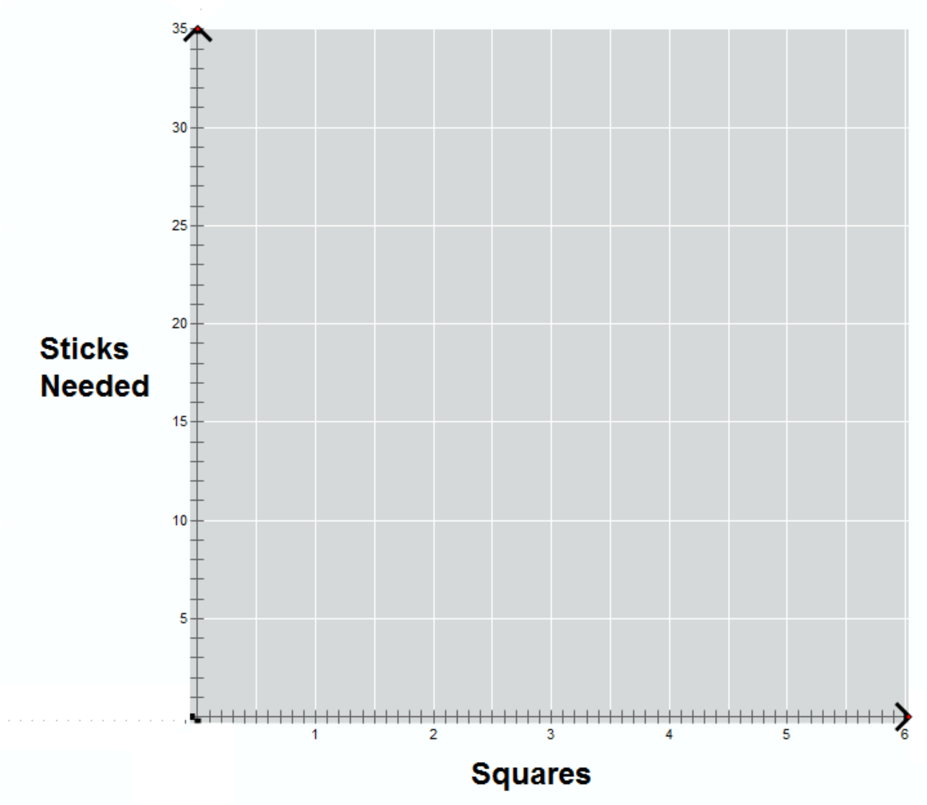
Overlapping Squares



Use your sticks to work out how many you need to make each of the squares given in the table

How many Squares	1	2	3	4	5
How many sticks					

Plot points for each of the pairs of values in the table.



What do you notice about the pattern produced by the points?

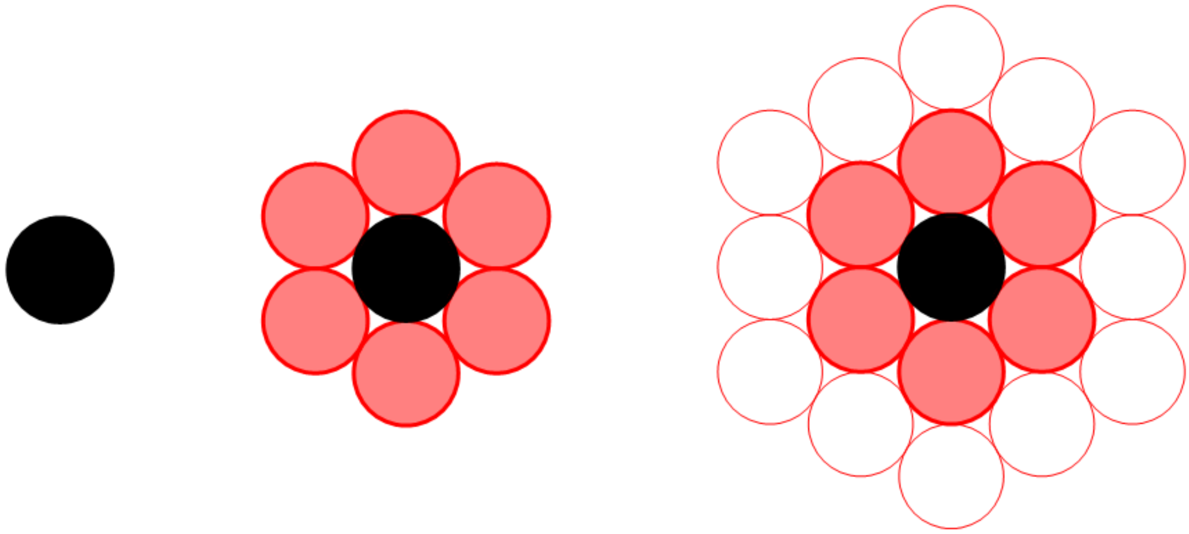
Write down a sentence for how to create and continue the pattern (Where do you start and what do you do?)

Write this as an algebraic equation.

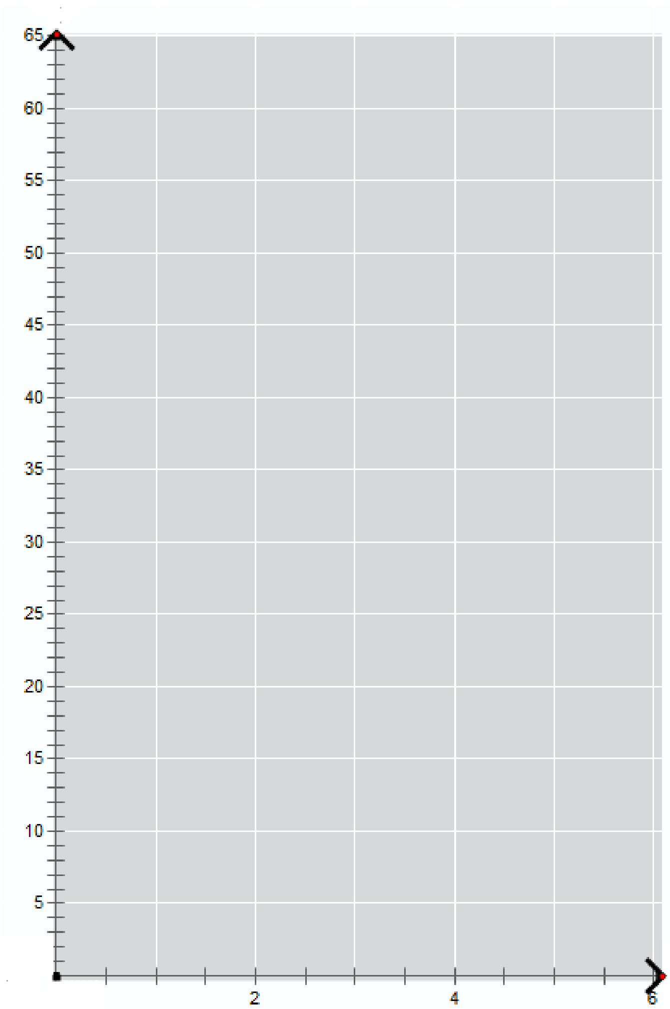
Can you write this expression in factorised form?

Packing Straws (Extending the Idea)

Straws or pipes bundle together so that end on they form the following pattern.



How many Straws	How many layers
1	
2	
3	
4	
5	



Plot points for each of the pairs of values in the table.

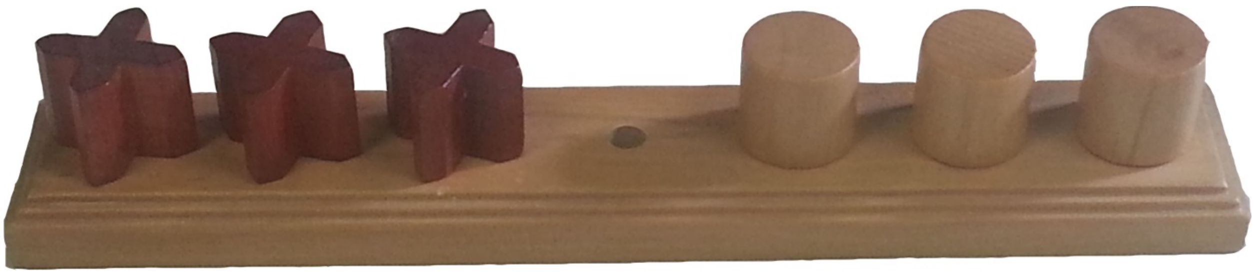
What do you notice about the pattern produced by the points?

Write down a sentence for how to create and continue the pattern (Where do you start and what do you do?)

Write this as an algebraic equation.

Can you write this expression in factorised form?

Tic Tac Toe



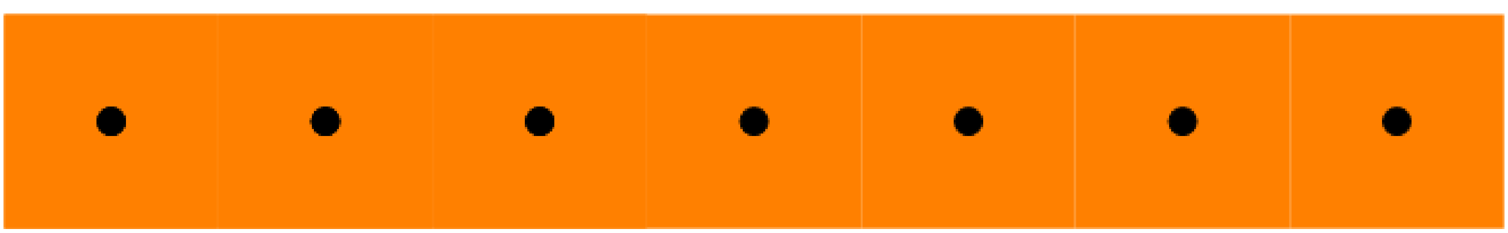
The aim is to move all of the circles from the right to the left and all of the crosses from left to right.

Pieces can only move "forward", ie to the right for crosses and to the left for circles.

A valid move is either one step forward onto a vacant spot, or jumping over ***one*** opponent piece onto a vacant spot.

How many moves does it take to complete the problem and what is the sequence?

Board



Pieces

Soupermarket Shenanigans

Part I

Stephen and Sophie both work as shelf stackers for their local Safeday Supermarket. Their manager wants them to build a display for one of the aisles using soup cans. He tells them they have to stack the cans to form a triangle.

They have 300 cans to use.

How many rows of cans will there be in the display and how many cans will be in the bottom row?

Part II

What would happen if they had 400 cans to work with?

Can you find a way to determine how many cans would be needed if the display was to have 100 rows?

Part III

Simone also works for Safeday. The manager tells her that she needs to pack bags with lollies for a lucky dip promotion. He wants all of the bags to have a different number of lollies in them, the first one with only 1 lolly, the second with 2 lollies, the third with 3 lollies and so on.

Simone has worked out that she has 5050 lollies to pack into bags. How many bags will she need?