Circles, Pythagoras and Trigonometry

AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Circles

Van Hiele levels of Geometric Thought

Research by Pierre and Dina van Hiele in the 1950's.

- There is some natural development of spatial thinking but deliberate instruction is required.
- It is based on the firm belief that it is inappropriate to teach children Euclidean geometry following the same logical construction of axioms, definitions, theorems and proofs that Euclid used to construct the system. Children don't think on a formal deductive level, and therefore can only memorise geometric facts and 'rules', but not understand the relationships between the ideas, if taught using this approach.

The van Hiele theory puts forward a hierarchy of levels of thinking spanning the ages of about five years through to academic adults.

Circles

Level 1: Visual

- 'nonverbal thinking‘
- Shapes are judged by their appearance and generally viewed as 'a whole', rather than by distinguishing parts.
- Although children begin using basic shape names, they usually offer no explanation or associate the shapes with familiar objects. For example, a child might say, "It's a square because it looks like one", or "I know it's a rectangle because it looks like a box".

  This could be likened to young children's ability to recognise some words by sight, before they understand the individual letter sounds and how they blend together to form words.

Circles

Level 2: Descriptive

- Children can identify and describe the component parts and properties of shapes. For example, an equilateral triangle can be distinguished from other triangles because of its three equal sides, equal angles and symmetries.
- Children need to develop appropriate language to go with the new specific concepts. However, at this stage the properties are not 'logically ordered', which means that the children do not perceive the essential relationships between the properties. So, with the equilateral triangle for example, they do not understand that if a triangle has three equal sides it must have three equal angles.

Circles

Level 3: Informal Deduction

- the properties of shapes are logically ordered. Students are able to see that one property precedes or follows from another, and can therefore deduce one property from another. They are able to apply what they already know to explain the relationships between shapes, and to formulate definitions. For example, they could explain why all squares are rectangles. Although informal deduction such as this forms the basis of formal deduction, the role of axioms, definitions, theorems and their converses, is not understood.

Circles

Where does it fit?

Foundation - Sorts, describes and names squares, circles, triangles, rectangles, spheres and cubes.

(ACMMG009)

Nothing in between.

Level 8 - Investigates the relationship between features of circles such as circumference, area, radius and diameter. Uses formulas to solve problems involving circumference and area. ***(ACMMG197) TIMESMG17***

Circles

Australian Curriculum

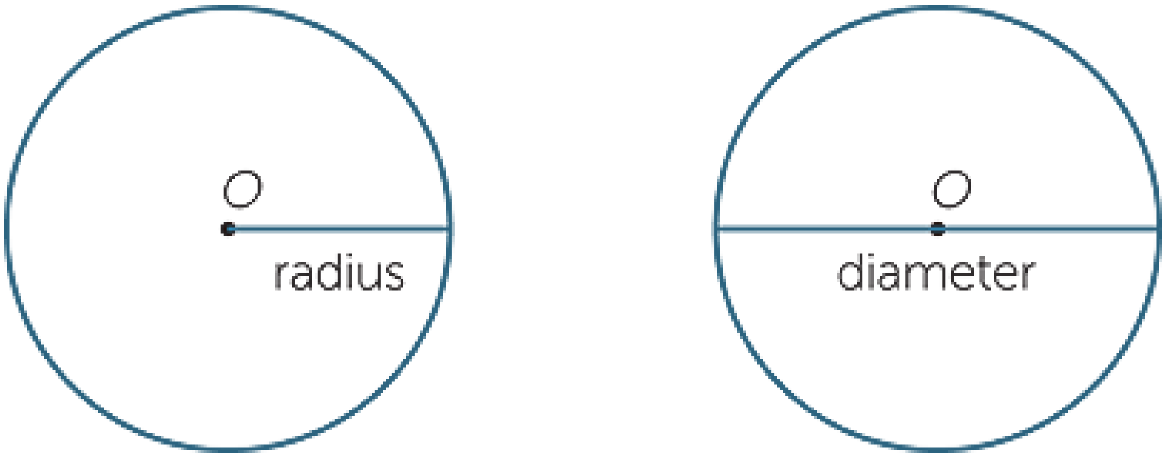
Nothing other than circle recognition until L8.

8	Using Units of Measurement	Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area. (ACMMG216)
9	Using Units of Measurement	Calculate the areas of composite shapes. (ACMMG216)
9	Using Units of Measurement	Calculate the surface area and volume of cylinders and solve related problems (ACMMG1)
10	Using Units of Measurement	Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids. (ACMMG242)
10A	Using Units of Measurement	Solve problems involving surface area and volume of right pyramids, right cones, spheres and related composite solids. (ACMMG271)

Circles

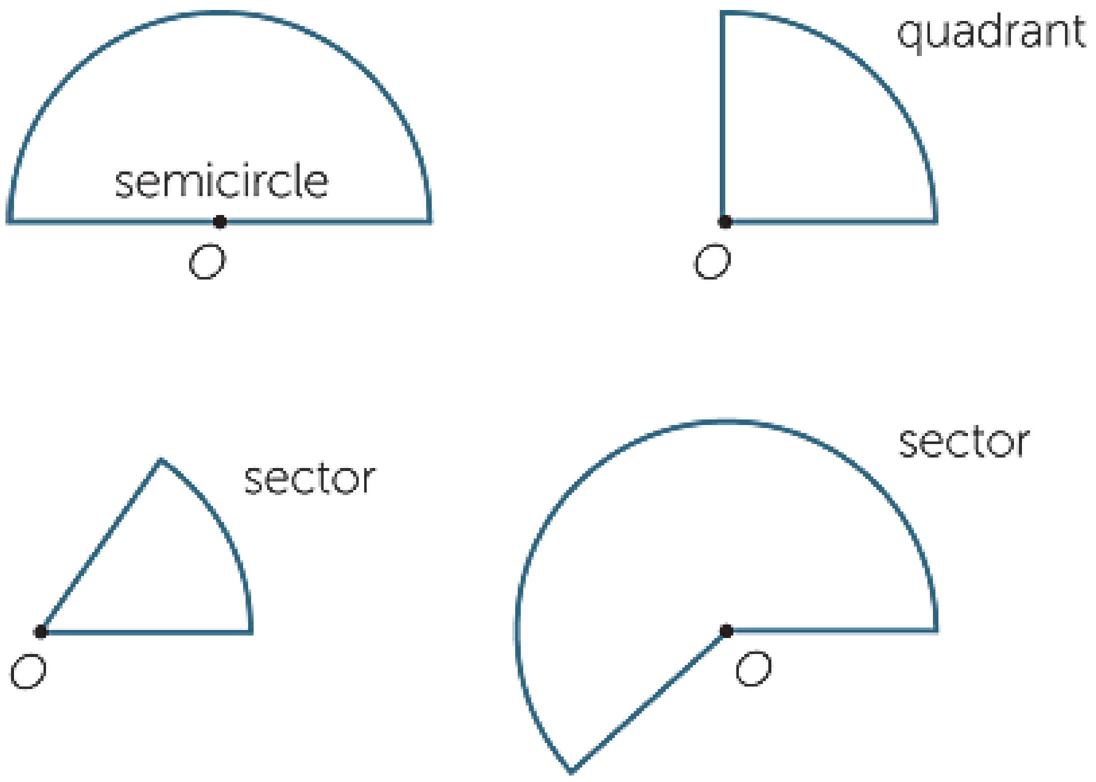
Definitions

A **circle** is the path traced out by a point, moving in a plane, that is always a fixed distance (the radius) from a fixed point (the centre).



Circles

Definitions



Circles

Activity 1:Consider a can of three tennis balls.

Which is greater, the height or circumference of the container?

Assume the balls touch both the top and bottom of the container.

Height = $3 \times diameter$

Circumference = $\pi \times diameter$

Circles

Definitions

The **circumference** of a circle is the distance around the circle.

Circles

Student Activity 2

Discover the formula for circumference.

Step 1 - Find an approximate formula.

Draw a circle and measure the diameter using string.

To the nearest whole number find how many times the diameter goes into the circumference by stepping the diameter around the circumference.

Establish that c is approximately $3d$

Circles

Student Activity 2

Step 2 – Improve the formula.

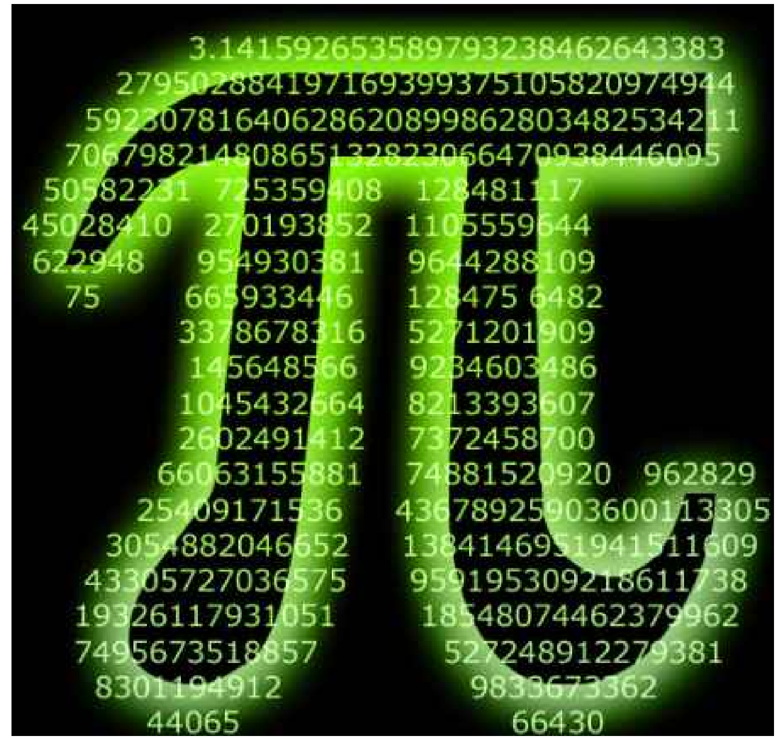
Take 5 round objects and measure the diameter and circumference using string.

Complete this table:

	Diameter, d	Circumference, d	$\frac{c}{d}$
1			
2			
3			
4			
5			

Circles

Circumference



C = 2πr or πd

π = 3.14159265358979323846264338327…

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History of Pi

- Babylonian tablet shows pi = 3.125. (1900–1680 BC)
- 1 Kings 7:23 (Christian Bible) pi = 3 can be deduced.
- The *Rhind Papyrus* (ca.1650 BC). The Egyptians calculated the area of a circle by a formula that gave the approximate value of 3.1605 for pi.
- Archimedes (287–212 BC), showed that pi is between $3\frac{1}{7}$ and $3\frac{10}{71}$.
- Mathematicians began using the Greek letter π in the 1700s.

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"Fun" Pi facts

The number π is an irrational number. (*Cannot be written exactly as a fraction.*)

Its approximate value, correct to 7 decimal places, is 3.1415927, but the decimal expansion of π continues forever with no apparent pattern.

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \frac{\pi^2}{6}$$

This number is one of the most remarkable of all numbers in mathematics and reappears somewhat mysteriously in many places.

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“Fun” Pi facts

Pi continues indefinitely in a random pattern.

For most people their figures of their birthdate can be found in Pi.

e.g. If you were born on 25/06/1985 then

25061985 appears starting at the 68 268 digit.

http://www.mypiday.com/

Circles

“Fun” Pi facts

Pi Day is an annual celebration of π. Pi Day is observed on March 14 (which is 3/14 in the *month/day format*). In 2015 this became 3/14/15, the first 4 decimal places. (Though as mathematicians we know π rounded to 4 d.p. from 3.14159 should become 3.1416. Maybe next year?)

Pi Approximation Day is observed on July 22 (22/7 in the *day/month* date format), since the fraction $\frac{22}{7}$ is a common approximation of π.

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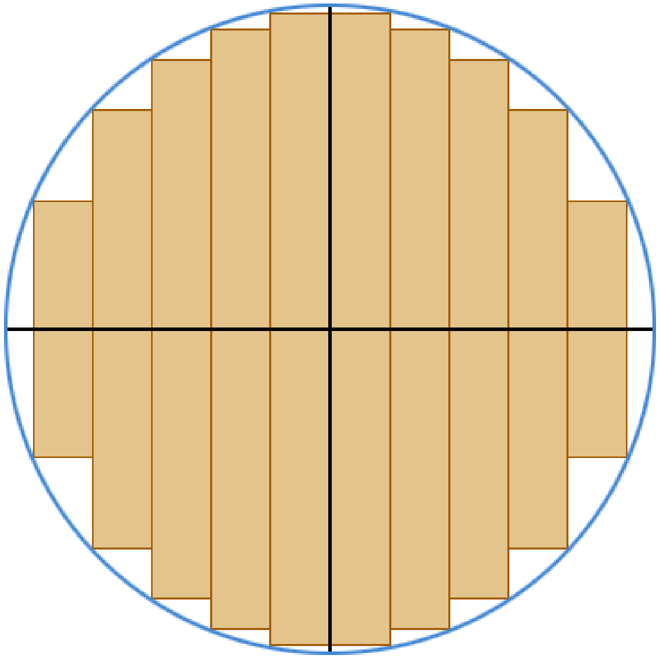
Area

$A = \pi r^2$



Model this with students?

Counting squares,

measuring the $L \times W$ of the rectangles.

How can we improve the accuracy of the model?

Formulas for Circles

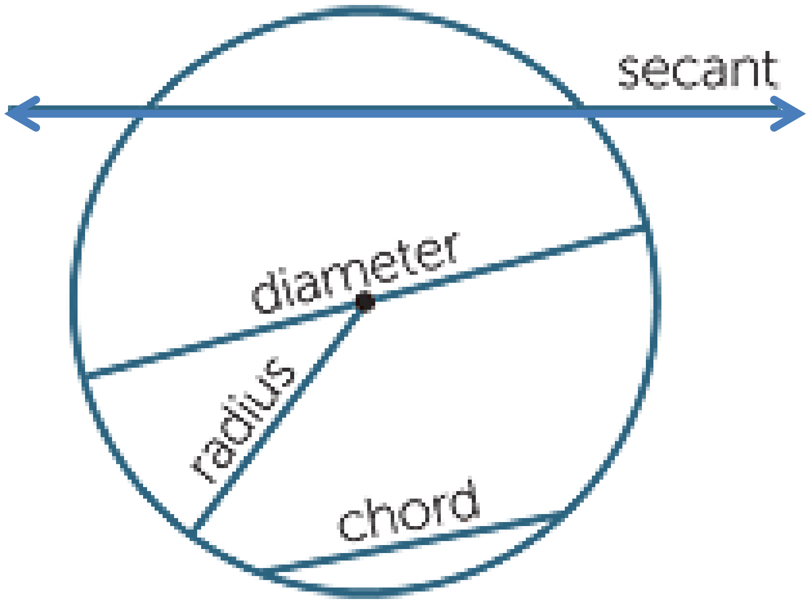
Is this just a coincidence?

circumference	One dimension	r^1
area	Two dimensions	r^2
volume	Three dimensions	?

What do we predict the power of the *r* will be in the formula for volume of a sphere?

Circles

More definitions



Any interval joining a point on the circle to the centre is called a radius.

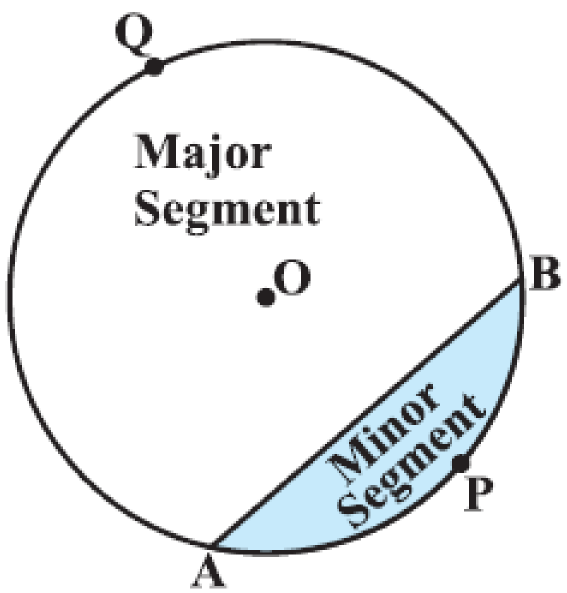
An interval joining two points on the circle is called a chord.

A chord that passes through the centre is called a diameter.

A line that cuts a circle at two distinct points is called a secant.

Circles

More definitions



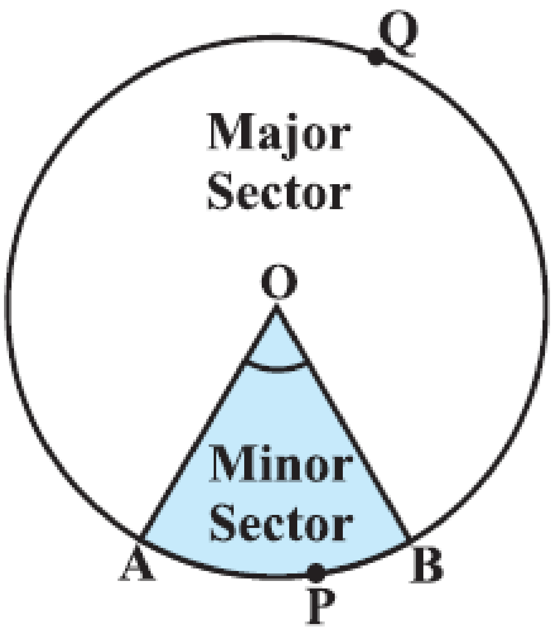
The area created by a chord and the arc is called a **segment**.

http://www.ekshiksha.org.in/eContentShow.do?documentId=160

Circles

More definitions

The area created by two radii and the arc is called a **sector**.



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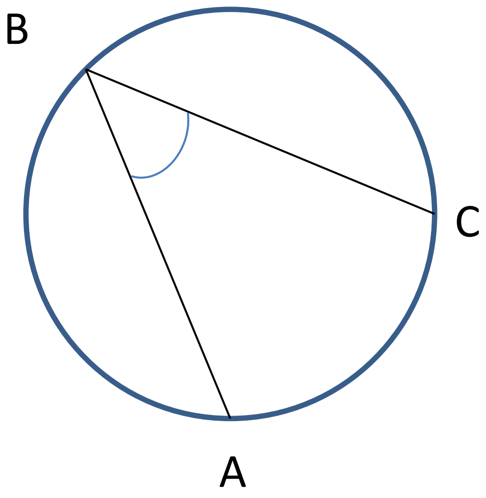
Circles

Inscribed angle

An inscribed angle is made from three points on a circle.



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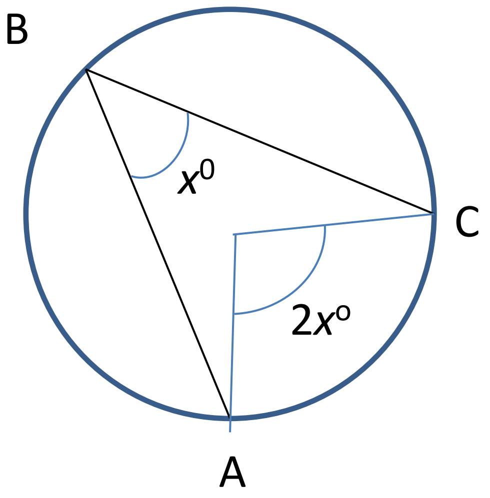
Inscribed angle Theorems

An inscribed angle x^0 is half the central angle $2x^0$



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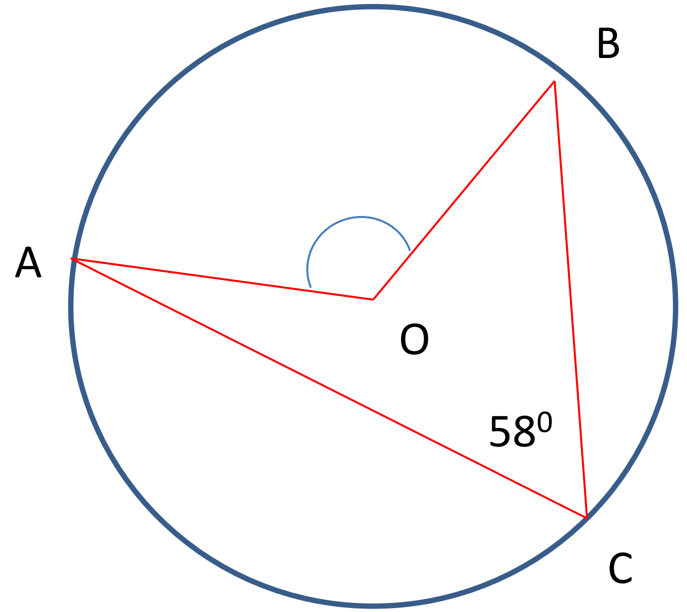
Example

What is the size of $\angle BOA$?

$$2 \times 58 = 116^o$$



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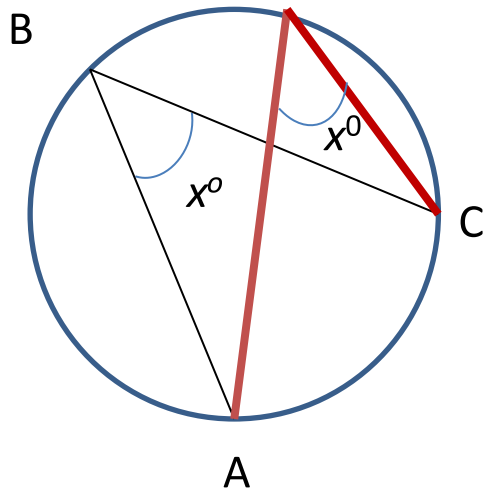
Angles Subtended by Same Arc

Angle x is always equal.



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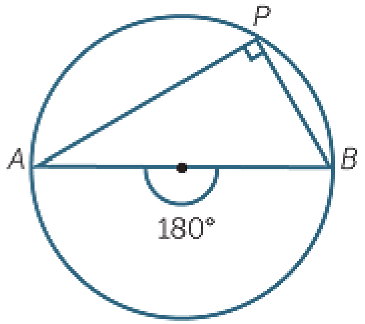
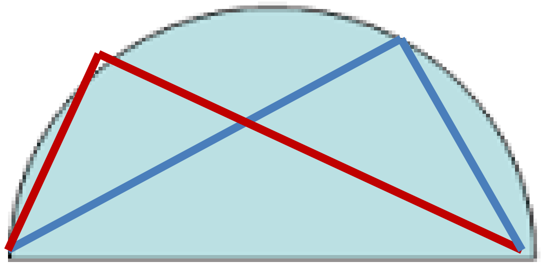
Angles in a semi-circle

An angle inscribed in a semi-circle is always a right angle.
(Inscribed means contained within in another figure touching at as many places as possible)



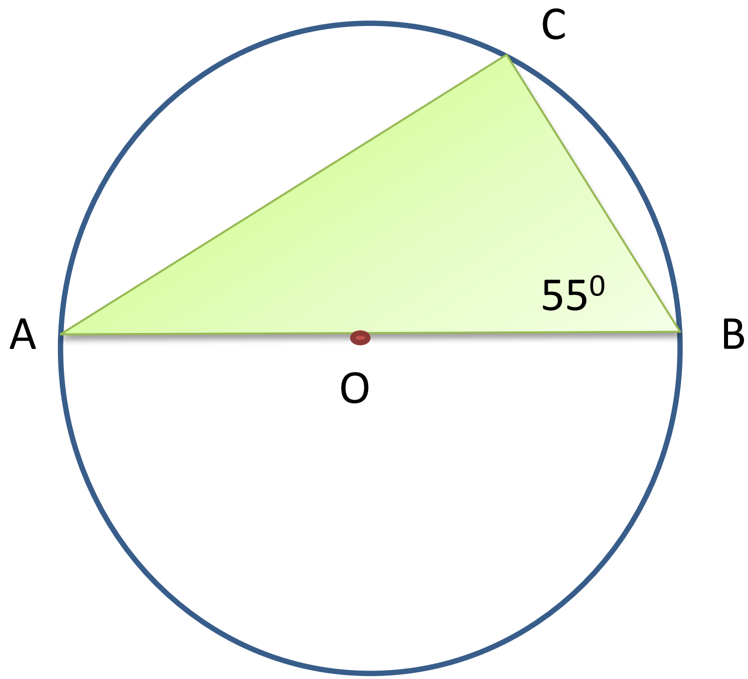
Why? This is an application of the theorem that the inscribed angle is half the central angle.



Circles

Example

What is the size of $\angle BAC$?



The top angle, C, is 90^0

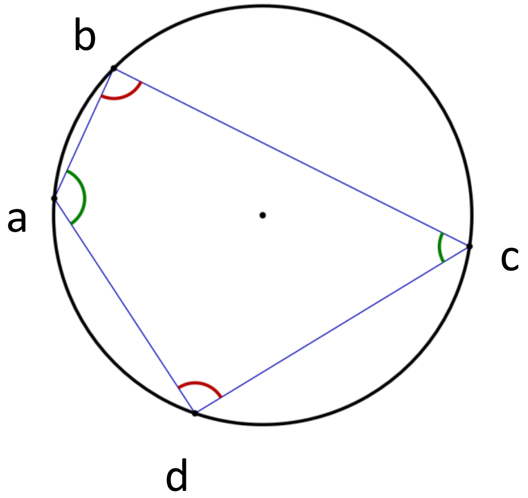
$180 – (55 + 90) = 35^O$

Circles

Cyclic Quadrilateral

A quadrilateral inscribed in a circle is called a cyclic quadrilateral.

A cyclic quadrilateral's opposite angles add to 180^0



$a + c = 180^0$

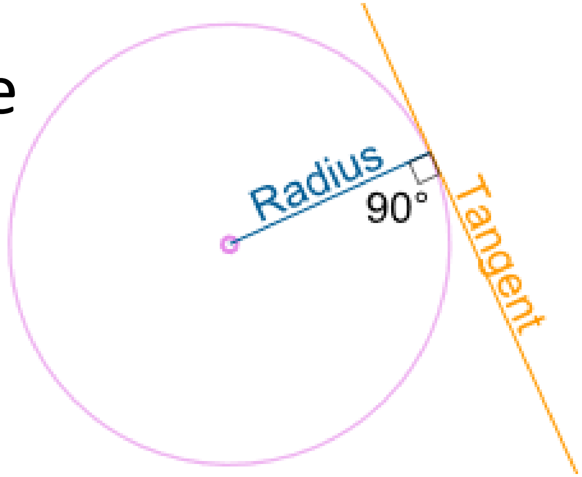
$b + d = 180^0$

Circles

Tangent Angle

A tangent is defined as a line that touches a circle at only one point.

A tangent always forms a right angle with the circle's radius.



Circles

Proofs

Thinking back to the van Hiele levels of Geometric thought.

It is inappropriate to teach children Euclidean geometry following the same logical construction of axioms, definitions, theorems and proofs that Euclid used to construct the system. Children don't think on a formal deductive level, and therefore can only memorise geometric facts and 'rules', but not understand the relationships between the ideas, if taught using this approach.

Circles

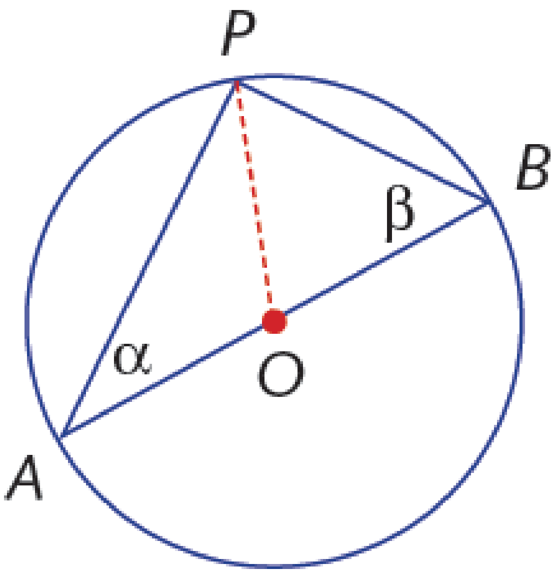
Proofs

Using geometrical proofs only comes in at Year 10 and the only applicable one to Circles is:

10A	Geometric Reasoning	Prove and apply angle and chord properties of circles (ACMMG272)

Circles

Proofs – Thales Theorem



The lines OA, OB and OP are all radii.

Hence, have two isosceles triangles, $\triangle AOP$ and $\triangle BOP$.

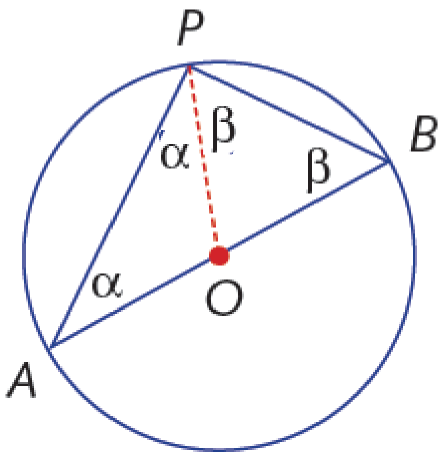
Let $\angle BAP = \alpha$ and $\angle ABP = \beta$.

Circles

Proofs – Thales Theorem

Then $\angle OPA = \alpha$ (base angles of isosceles $\triangle OPA$)

and $\angle OPB = \beta$ (base angles of isosceles $\triangle OPB$).



Adding up the interior angles of the triangle $\triangle ABP$,

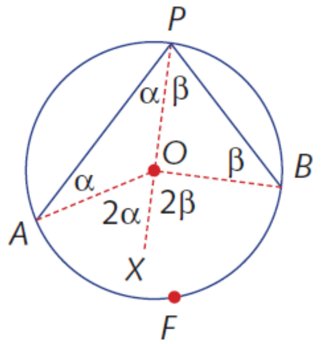
$$\alpha + \alpha + \beta + \beta = 180°$$

$$\alpha + \beta = 90°$$

so $\angle APB = 90°$, which is the required result.

Circles

Proofs



$\overline{AO}$, $\overline{PO}$ and $\overline{BO}$ are all radii.

$\therefore$ both $\triangle AOP$ and $\triangle BOP$ are isosceles triangles with two equal angels.

Let the equal angles be α and β respectively.

Remember the exterior angle of a triangle is equal to the sum of the two opposite interior angles.

$\therefore \angle AOX = 2\alpha$ and $\angle BOX = 2\beta$.

Hence, $\angle AOB = 2\alpha + 2\beta = 2(\alpha + \beta)$ and $\angle APB = \alpha + \beta$.

The conclusion is that $\angle APB$ is half the $\angle AOB$, **an angle subtended at the centre of the circle by the arc *AFB***.

Circles

Proofs – Cyclic quadrilaterals

The key to finding the relationship is to draw the radii AO and BO.

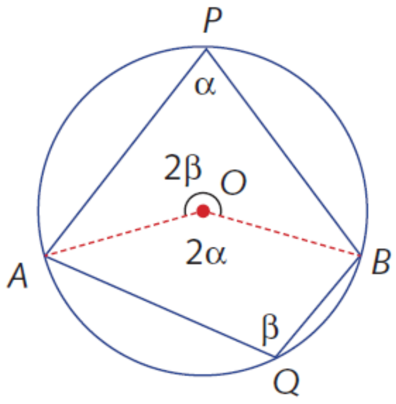
First, $\angle P$ is half the angle $\angle AOB$ at the centre on the same arc AQB; we have marked these angles α and 2α.

Second, $\angle Q$ is half the reflex angle $\angle AOB$ at the centre on the same arc APB; we have marked these angles β and 2β.

$$2\alpha + 2\beta = 360° \qquad \text{(angles in a revolution at } O\text{)}$$

so $$\alpha + \beta = 180°$$

Hence, the opposite angles $\angle P$ and $\angle Q$ are supplementary.



Circles

Great Circles

Not in the F – 10 course.

Is in the 2016 Year 12 Further Maths Geometry and Measurement Module.

It can be a "fun" extension and some familiarity is probably a good thing.

Section Heading

Spherical Geometry

The Geometry and Measurement in Further Maths Y12

Spherical geometry, including:

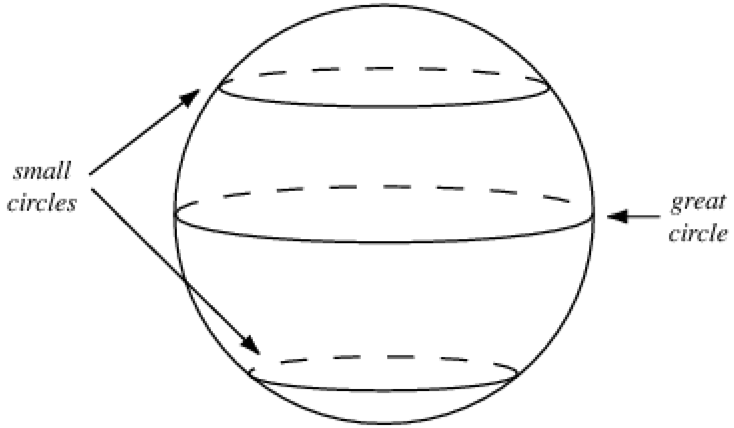
- circle mensuration; arc length using the rule $s = r \times \frac{\pi}{180} \times \theta°$ with practical applications
- arc length of a sector of a circle, and the areas of sectors and segments with practical applications
- use of trigonometry and Pythagoras' theorem in two and three dimensions to solve problems involving the solution of right-angled triangles within a sphere
- use of a sphere of radius 6400 km as a model of the earth, and meridians and parallels and their use in locating points on the surface of the earth in terms of latitude and longitude (specified in decimal degrees) using the Greenwich meridian and the equator as reference
- use of meridians to determine the shortest distance from any point on the earth to a pole or the equator
- use of a great circle to determine the shortest distance between two points on the surface of the earth that have the same longitude
- use of 15° of longitude as equating to a 1 hour time difference to identify time zones, and determining travel times of journeys that cross two or more time zones from departure and arrival times.

Circles

Great Circles

Great circles are circles on the surface of a sphere whose plane passes through the center of the sphere.

For example the equator is a **great circle** on the sphere of the globe.



http://mathworld.wolfram.com/GreatCircle.html

Circles

Great Circles

Why do planes fly along arcs rather than straight lines? Especially on long flights.

http://www.ms.unimelb.edu.au/~statphys24/website/img/Flight-Mapbig.jpg

Circles

Great Circles

Planes will generally fly the shortest distance between 2 points to save fuel.

The shortest distance between 2 points on a sphere is the line along the great circle joining those point and the center of the Earth.

Circles

Great Circles

What Is a Great Circle?

Have you ever heard of a great circle? How about a small circle? Do the lines of longitude on Earth form great circles? How about the lines of latitude? How do these ideas explain the routes that airplanes fly? Keep on reading to find out!

By Jason Marshall, PhD, The Math Dude

October 24, 2014

Episode #218

Have you ever been on a flight, grabbed the magazine out of the seat-back pocket in front of you, and stumbled upon the page that shows the routes that airplanes take when flying hither and thither?

Were you surprised by what you saw? I know I was the first time I saw it...because the lines showing the flight paths between various cities around the globe are not straight!

Instead, these flight paths follow what appear to be crazy looping arcs. As it turns out, these arcs aren't crazy at all—they're called great circles, and they really are pretty great. What are they? And why do airplanes fly along them? Stay tuned because those are exactly the questions we'll be answering today.

http://www.quickanddirtytips.com/education/math/what-is-a-great-circle

Pythagoras

When to introduce?

Level 9 Investigates Pythagoras' Theorem and its application to solving simple problems involving right angled triangles that generate results that can be integral, fractional or irrational numbers *(ACMMG222)* *TIMESMG15*

Level 10A Applies Pythagoras' Theorem and trigonometry to solving three-dimensional problems in right-angled triangles *(ACMMG276)* *TIMESMG24*

Pythagoras

Australian Curriculum

9	Pythagoras and Trigonometry	Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles (ACMMG222)
10	Pythagoras and Trigonometry	Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245)
10A	Pythagoras and Trigonometry	Pythagoras' theorem and trigonometry to solving three-dimensional problems in right-angled triangles (ACMMG276)

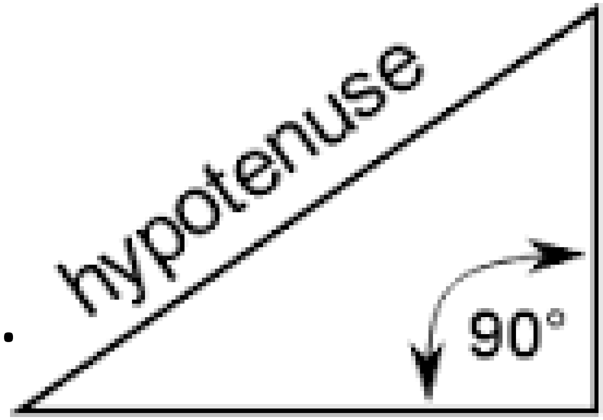
Pythagoras

Some definitions

A right angled triangle contains a 90º angle.

(The right angle is always the largest angle in a right-angled triangle. Why?)

The side opposite the largest angle is the longest side.
This side is called the Hypotenuse.



Pythagoras

Hypotenuse

The word is connected with a Greek word meaning “to stretch”. The ancient Egyptians discovered that if you take a piece of rope, mark off 3 units, then 4 units and then 5 units, this can be stretched to form a triangle that contains a right angle.

This was very useful to the Egyptian builders.

Video from SCOOTLE

http://splash.abc.net.au/home#!/media/1469315/

Pythagoras

History – Who was Pythagoras?

Pythagoras(569-500 B.C.E.) was born on the island of Samos in Greece. Not much more is known of his early years.

Pythagoras gained his famous status by founding a group, the Brotherhood of Pythagoreans, who were devoted to the study of mathematics. The group was almost cult-like in that it had symbols, rituals and prayers.

(Youtube Donald Duck in Mathemagic land)

Pythagoras

History

Legend has it that upon completion of his famous theorem, Pythagoras sacrificed 100 oxen.

Although he is credited with the discovery of the famous theorem, it may well have been a member of his group. The group wanted to keep their findings secret and consequently kept them from the public.

Unfortunately, this vow of secrecy prevented an important mathematical idea from being made public.

Pythagoras

Irrational numbers

The Pythagoreans had discovered irrational numbers! e.g. $\sqrt{2}$

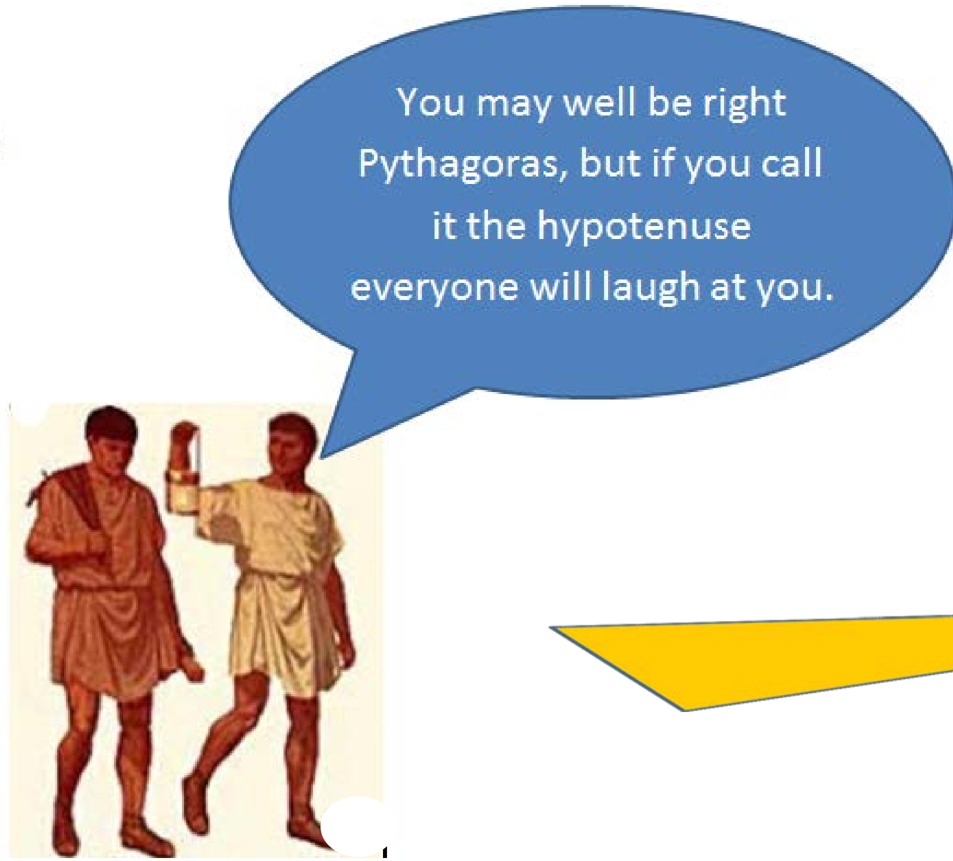
The fact these numbers cannot be expressed exactly as a fraction deeply disturbed the Pythagoreans, who believed that "All is number."

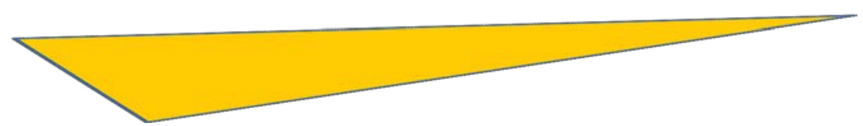
They called these numbers "alogon," which means "unutterable." So shocked were the Pythagoreans by these numbers, they put to death a member who dared to mention their existence to the public.

It would be 200 years later that the Greek mathematician Exodus developed a way to deal with these unutterable numbers.

Pythagoras

Name only the Hypotenuse not opposite and adjacent.

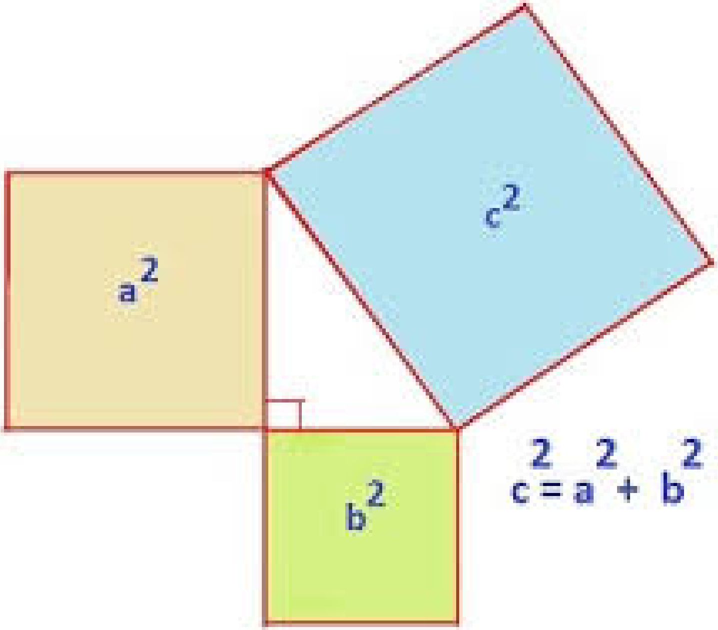


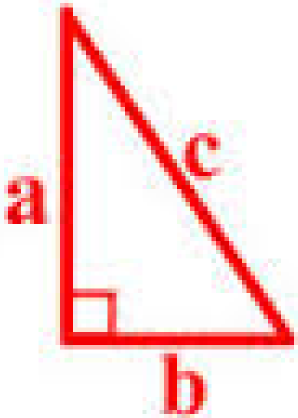


Pythagoras

Formula

$$a^2+b^2=c^2$$





$$a^2+b^2=c^2$$

Pythagoras

Pythagorean triples

There are some whole number values that satisfy this equation. These are know as Pythagorean triples.

e.g. 3,4,5 $3^2 + 4^2 = 5^2$

5,12,13 $5^2 + 12^2 = 13^2$

37,684,685 $37^2 + 684^2 = 685^2$

Pythagoras

“Two” question types

Finding the hypotenuse.

$$a^2 + b^2 = c^2$$

Finding one of the shorter sides.

$$a^2 = c^2 - b^2$$

Do we teach the two formulas or get the students to rearrange?

Pythagoras

Plenty of apps – some useful

This free one is called Pythagoras and shows graphically a proof of the Theorem.

Trigonometry

When?

<u>Level 9</u>

Applies trigonometry to solve right-angled triangle problems. Understands the terminology "adjacent" and "opposite" sides and hypotenuse in right angled triangles. Uses trigonometric ratios to find unknown sides and angles in right-angled triangles. ***(ACMMG224) TIMESMG23***

<u>Level 10</u>

Solves right-angled triangle problems including those involving direction and angles of elevation and depression. ***(ACMMG245) TIMESMG23***

Trigonometry

When?

Level 10A

Establishes the sine, cosine and area rules for any triangle and solves related problems. *(ACMMG273)* *TIMESMG24*

Uses the unit circle to define trigonometric functions, and graphs them with and without the use of digital technologies. *(ACMMG274)* *TIMESMG25*

Solves simple trigonometric equations. *(ACMMG275)*

Units for measuring angles

Angles are measured using many different units.

Student calculators use degrees, radians or gradians.

A common difficulty students have finding the correct answer is when their calculator is set for the incorrect unit.

Teachers need to be aware of how to change the settings for this for the common student calculators.

Trigonometry

Degrees- why 360^o?

Not universal agreement on this. Some say it has to do with Claudius Ptolemy (100-170 AD) who used 360 degrees when he set up his table of sine values.

Some say that as we have 365 days in a year, close to 360 and this is too big a coincidence to ignore.

Trigonometry

Degrees- why 360°?

Probably the most accepted answer is:

Babylon used a sexagesimal (base 60) system.

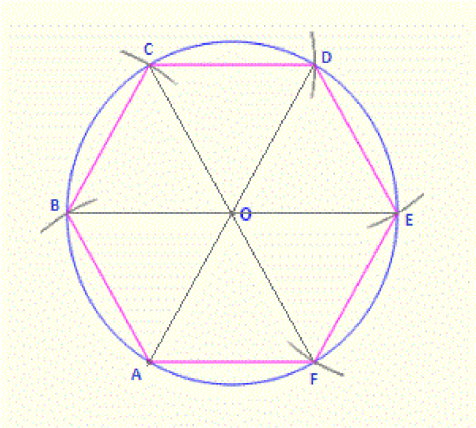
They also found that the perimeter of a hexagon was exactly 6 times the radius of the circumscribed circle.

Combining these two thoughts

it seemed natural to them to divide

the circle into 6 × 60 = 360 divisions

called degrees.

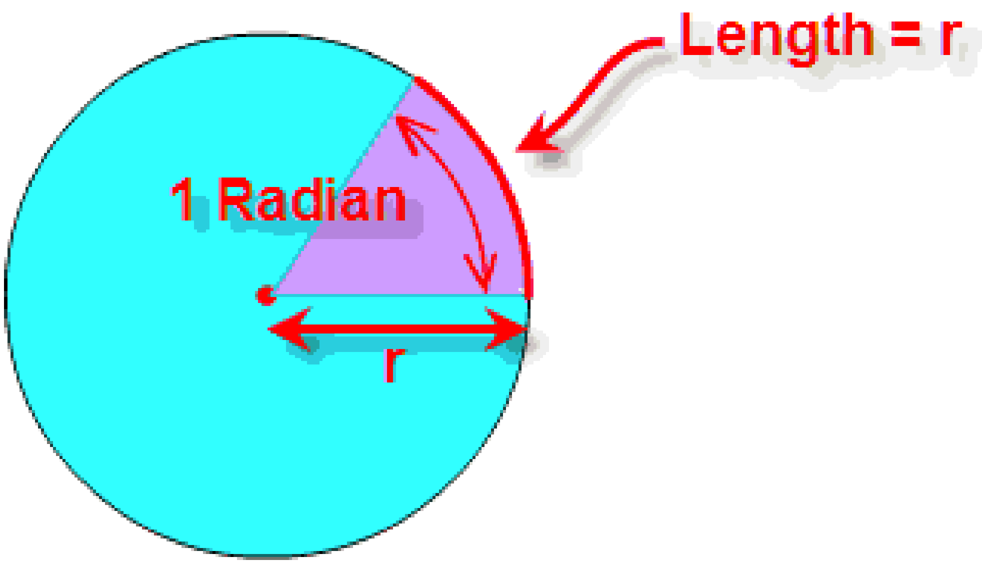


http://www.mathopenref.com/constinhexagon.html

Trigonometry

Radians

The radian is the angle subtended by an arc of a circle that has the same length as the circle's radius.

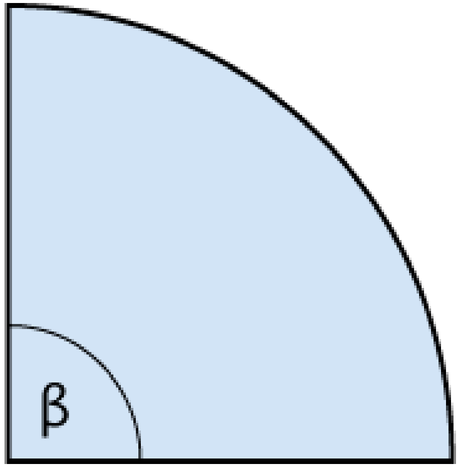


Radians are used extensively in Senior Maths.

Trigonometry

Gradians

The *grad*, also called *grade*, *gradian*, or *gon*, is 1/400 of a turn, so a right angle is 100 grads.



β = 100 gon = 90°

http://bl.gg/

How does the Calculator find values?

What happens when I type the sin (or cos or tan for that matter) of an angle into my calculator?

Is the calculator just reading off of a list created from people who used rulers to physically measure the distance on a graph or is there a mathematical function that defines it?

Trigonometry

No, it does not read off a list.

One method is to use the Taylor series for sine:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots,$$

where x is in radians, and the ! Symbol is factorial e.g. 4! = 4 × 3 × 2 × 1

The more terms you take the more decimal places you can find.

e.g. sin 23^0 is found by:

1. Convert 23 to radians by multiplying by $\frac{\pi}{180}$ ≈ 0.4014257

2. Substitute into the above formula.

$$\text{Sin } 0.4014257 = 0.4014257 - \frac{0.4014257^3}{3!} + \frac{0.4014257^5}{5!} - \frac{0.4014257^7}{7!}$$

$$= 0.390731102008345$$

Trigonometry

SOH-CAH-TOA ?

Absolutely, but don't start here or the students will memorise the process and not get any understanding.

$$\text{Sin}\Theta = \frac{Opposite}{Hypotenuse}$$

$$\text{Cos}\Theta = \frac{Adjacent}{Hypotenuse}$$

$$\text{Tan}\Theta = \frac{Opposite}{Adjacent}$$

Trigonometry

Introducing the idea

Set students the quick challenge of making some estimations.

e.g. How high is the gutter?

How tall is that tree?

Can 1 million dollars in $1 coins fit under the table?

How high is the roof?

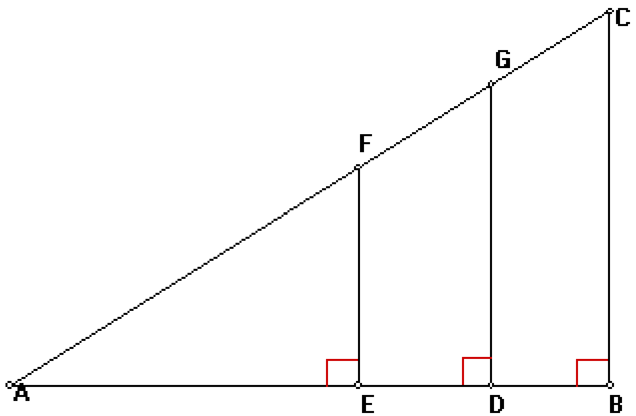
The discussion that follows is about strategies not necessarily the correct answer.

Trigonometry

Trig ratios – using tan

Estimating the height of objects by measuring the distance to them and using the tan button.

Talk about similar triangles, but you don't necessarily call them that, all having the same ratio of length and height.



What does each triangle have in common? - the angle A

Uses similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles ***(ACMMG223)*** ***TIMESMG22*** ***TIMESMG23***

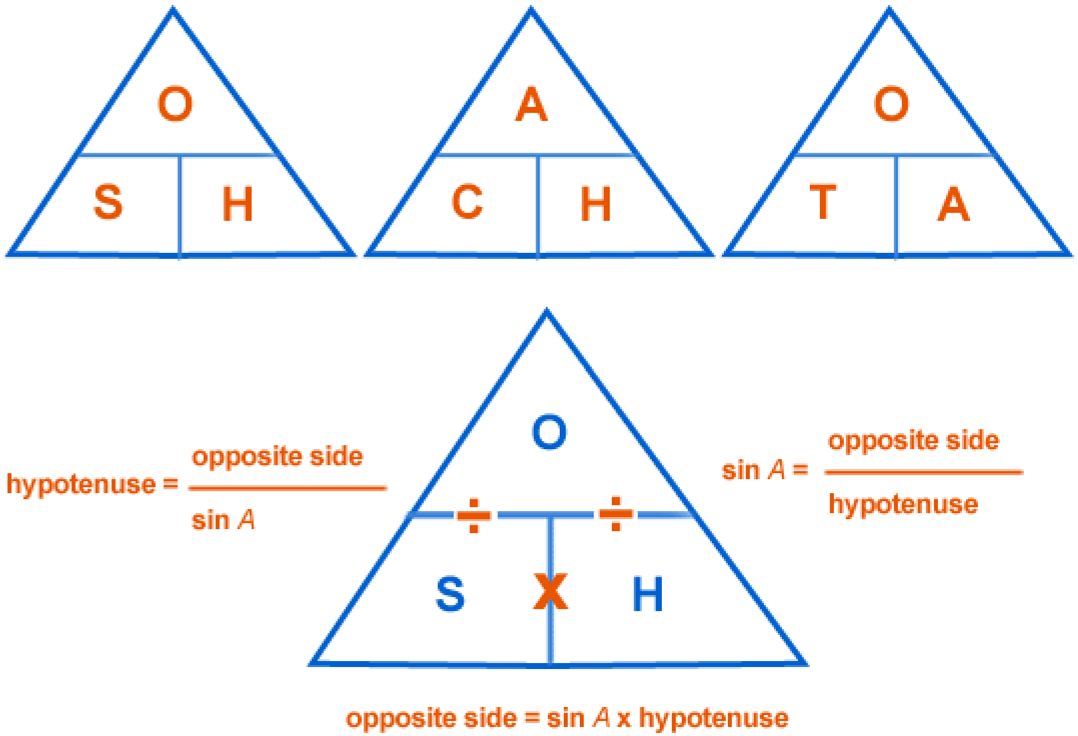
Trigonometry

History

Trigonometry (from Greek trigonon, "triangle" and metron, "measure“.

Trigonometry

An alternative to SOH-CAH-TOA



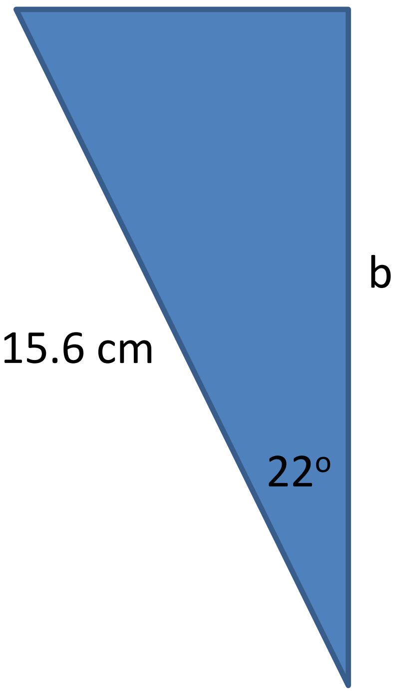
https://www.tutorbee.com.au/blog/trigonometric-equations-and-an-easy-way-to-remember-trig-formulas

Trigonometry

Only 3 types of questions

Type 1: The unknown is the numerator.

Find *b to 2 d.p.*



Step 1. Label the sides

Step 2. We Hyp. & Adj.

using SOH-CAH-TOA, the cos ratio.

Step 3. Substitute values.

$\cos 22^0 = \frac{b}{15.6}$ (Note: *b* is the numerator)

Step 4. $b = 15.6cos22^o$ = 14.46m

Trigonometry

Only 3 types of questions

Type 2: The unknown is the denominator.

Find *y*.

Step 1. Label the sides

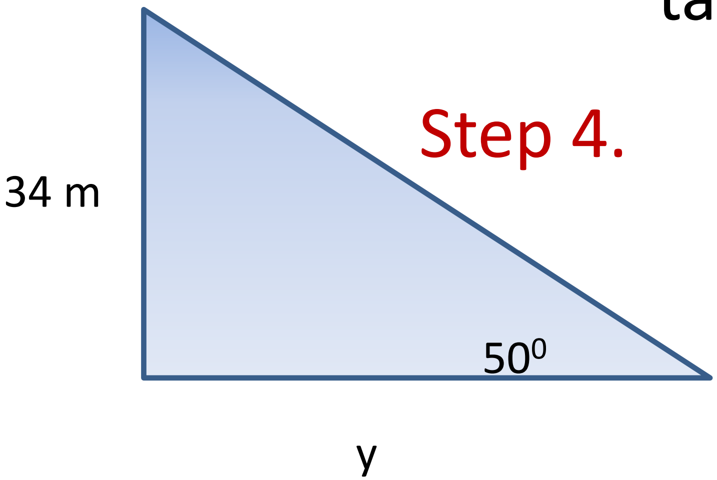
Step 2. We have Opp. & Adj.

Using SOH-CAH-TOA, the tan ratio.

Step 3. Substitute values

tan 50° = $\frac{34}{y}$ (*y* is the denominator)

Step 4. $y = \frac{34}{tan 50^0}$ = 28.53 m



Trigonometry

Only 3 types of questions

Type 3: Finding the angle.

Find ɵ to 2 d.p.

Step 1. Label the sides

Step 2. Identify Opp & Adj using SOH-CAH-TOA, the tan ratio.

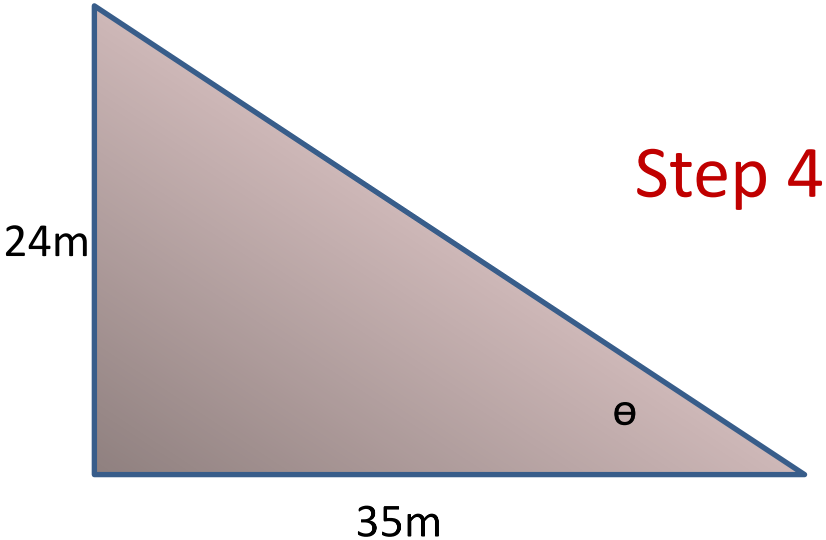
Step 3. Substitute values

$$\tan \theta = \left(\frac{24}{35}\right)$$

Step 4.

$$\theta = \tan^{-1}\left(\frac{24}{35}\right)$$

$$\theta = 34.44^0$$



Trigonometry

Do Not use of Minutes & seconds

There is no mention of degrees, minutes and seconds in the syllabus at any level from F to 10.

Not used in Victorian or NSW Year 12 Exams.

Still in some old textbooks.

Trigonometry

Standard notation

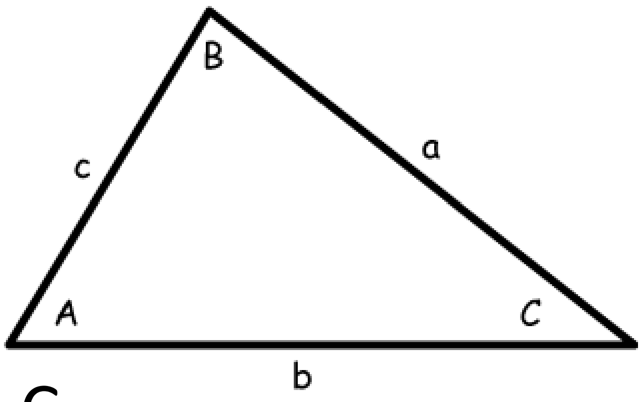
For the triangle *ABC* or Δ*ABC*, the corners are labelled with capital letters.

The sides with lower case letters.

The corner and opposite side

have the same letter.

i.e. the interval *AB*, $\overline{AB}$, is opposite C.



Note: $A = \angle BAC$ or $\angle CAB$

Trigonometry

Not only right angled triangles!

Level 9 & 10 only use right angled triangles.

Level 10A introduces the Sine Rule, Cosine Rule and Area formulas.

Sine Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \Leftrightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

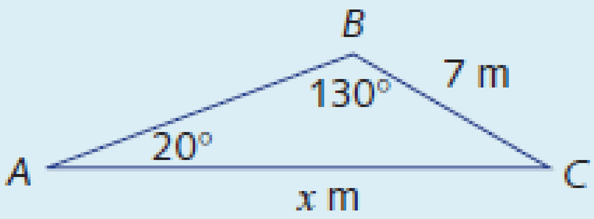
Area = $\frac{1}{2} absinC$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Trigonometry

10A – Sine rule

Find the value of x, correct to one decimal place.



Solution

Apply the sine rule to $\triangle ABC$.

$$\frac{x}{\sin 130^\circ} = \frac{7}{\sin 20^\circ}$$

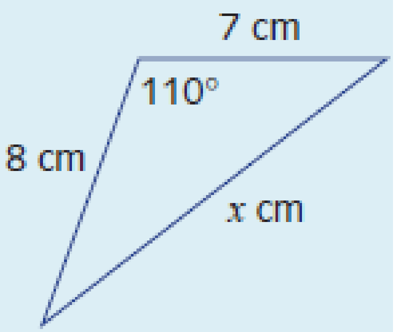
$$x = \frac{7 \sin 130^\circ}{\sin 20^\circ}$$

$$\approx 15.7 \qquad \text{(correct to one decimal place)}$$

Trigonometry

10A –Cosine rule

Find the value of x, correct to one decimal place.



Solution

Applying the cosine rule:

$$x^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \times \cos 110°$$
$$= 151.30 \ldots$$

so $\quad x \approx 12.3 \qquad$ (correct to one decimal place)

Trigonometry

10A –Area Formulas

Heron's Rule:

$$S = \frac{A + B + C}{2}$$

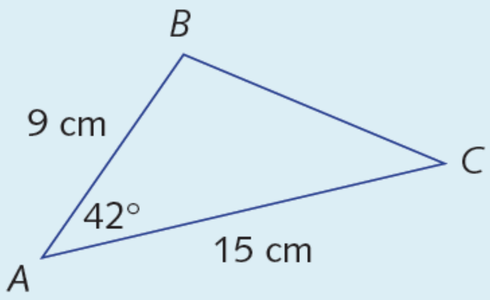
Area = $\sqrt{S(S - A)(S - B)(S - C)}$

Trigonometry

10A –Area Formulas

$$\text{Area} = \frac{1}{2}absinC$$

Calculate the area of the triangle ABC, correct to one decimal place.



Solution

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 9 \times 15 \times \sin 42°$$
$$\approx 45.2 \qquad \text{(correct to one decimal place)}$$

So the area of the triangle is 45.2 cm^2.

Trigonometry

The Unit Circle

Comes in at 10A.

10A	Pythagoras and Trigonometry	Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies (ACMMG274)
10A	Pythagoras and Trigonometry	Solve simple trigonometric equations (ACMMG275)

Trigonometry

The Unit Circle

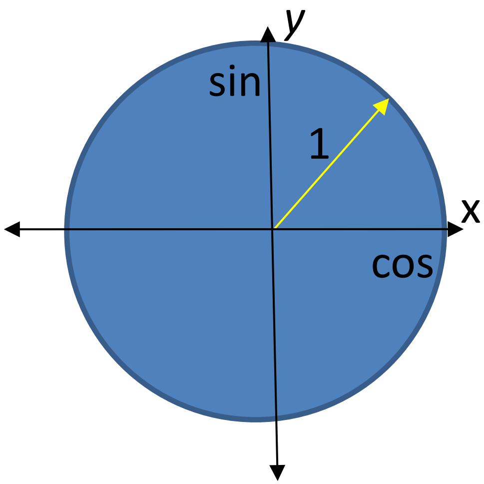
A circle of radius 1.

Pythagoras, $a^2 + b^2 = c^2$

under a radius of 1 becomes:

$x^2 + y^2 = 1$

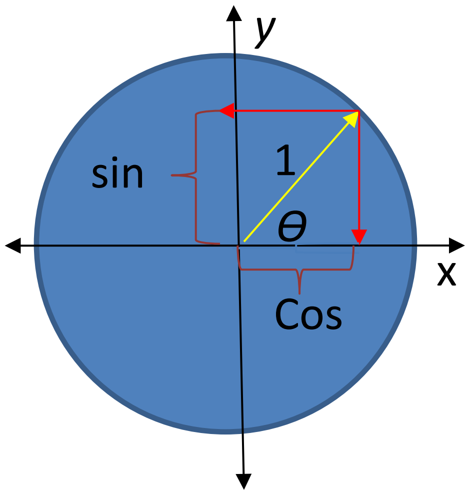
or $sin^2 + cos^2 = 1$



Trigonometry

The Unit Circle

A circle of radius 1.



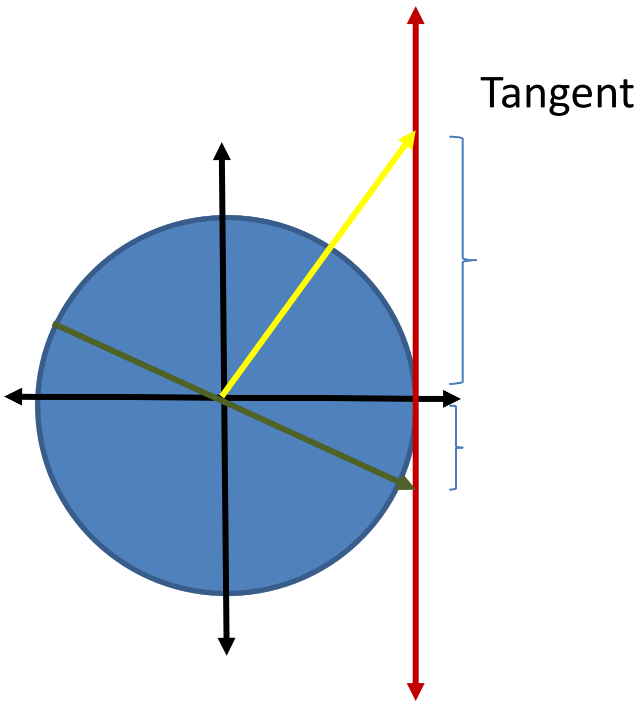
The cos ratio becomes

$$\cos \theta = \frac{Adj.}{Hyp.} = \frac{x}{1}$$

$$\sin \theta = \frac{Opp.}{Hyp.} = \frac{y}{1}$$

Trigonometry

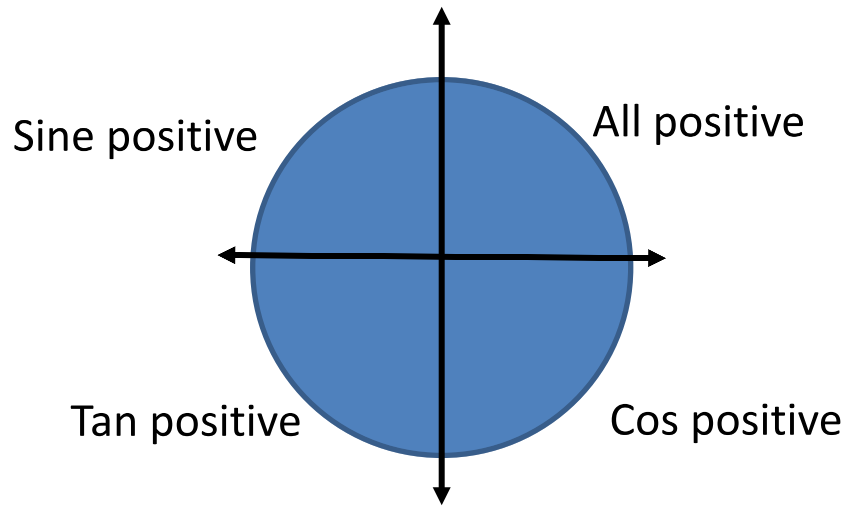
The Unit Circle



Trigonometry

The Unit Circle

Must be understood to allow students to solve trigonometric equations.



Trigonometric equations

At this level students are not expected to be able to use the special triangles algorithm to find the exact values for 0, 30, 45, 60 and 90.

You may still choose to do this, however, a unit circle approach may lead to a better understanding of where the extra values are coming from and what the graph may look like.

Trigonometry

The Unit Circle

This Youtube link shows a good trick for remembering the table of values for sin and cos.

	0º	30º	45º	60º	90º
sinθ	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	1
cosθ	1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undef.

https://www.youtube.com/watch?v=xXGfp9PKdXM

Trigonometry

Trigonometric graphs

An opportunity to model this out.

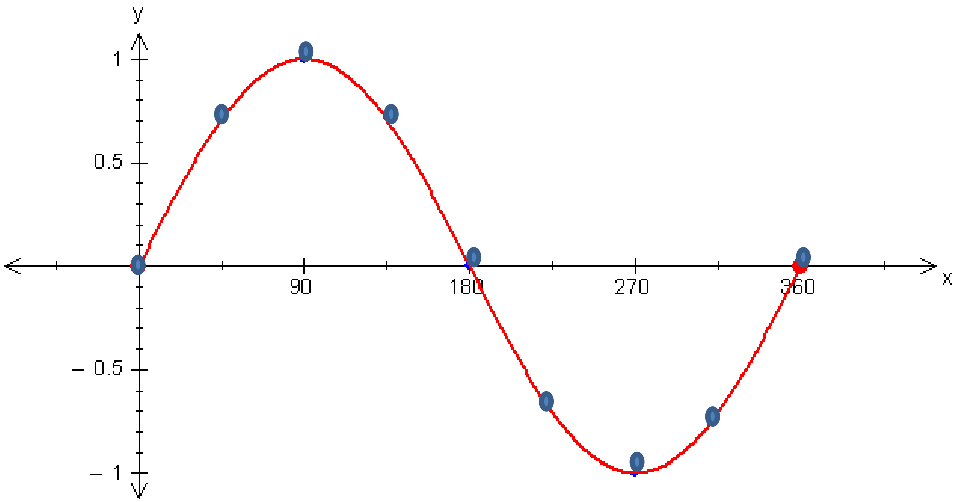
Begin with a table of values

x^o	0	45	90	135	180	225	270	315	360
Sin x	0	0.707	0	0.707	0	-0.707	-1	-0.707	0

Trigonometry

Trigonometric graphs

And then plot points.



Then progress to using a software package and the graphics calculator.

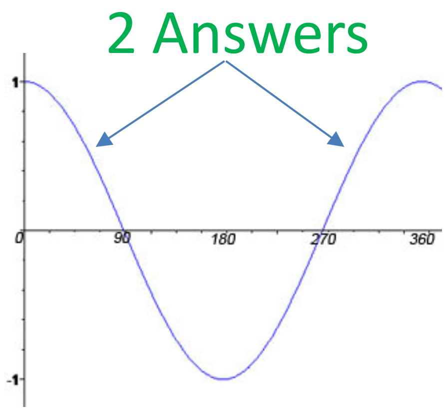
Trigonometry

Trigonometric equations

To solve $2\cos\theta - 1 = 0$ (+1 then divide by 2 on both sides)

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$



From the graph we can see there are two answers.

Trigonometric equations

Ask the question when does cos θ = 0.5?

Use the table of values to find this occurs at $\theta = 60^o$

Move around the unit circle in an anti-clockwise manner to find other values of θ for which cos θ= 0.5.

$\theta = 60^o$ or 300^o

Remember radians are not part of the 10A course.

AMSI

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