

# *Fractions*

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# Fractions

## Content Descriptions

Year	Strand & SubStrand	Year Level
1	Fractions and Decimals	Recognise and describe one-half as one of two equal parts of a whole. (ACMNA016)
2	Fractions and Decimals	Recognise and interpret common uses of halves, quarters and eighths of shapes and collections (ACMNA033)
3	Fractions and Decimals	Model and represent unit fractions including $\frac{1}{2}$ , $\frac{1}{4}$ , $\frac{1}{3}$ , $\frac{1}{5}$ and their multiples to a complete whole (ACMNA058)

# Fractions

## Content Descriptions

Year	Strand & SubStrand	Year Level
4	Fractions and Decimals	Investigate equivalent fractions by exploring fractions used in contexts (ACMNA077)
4	Fractions and Decimals	Count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line (ACMNA078)
4	Fractions and Decimals	Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation (ACMNA079)

# Fractions

## Content Descriptions

Year	Strand & SubStrand	Year Level
5	Fractions and Decimals	Compare and order common unit fractions and locate and represent them on a number line (ACMNA102)
5	Fractions and Decimals	Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator (ACMNA103)
5	Patterns and Algebra	Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107)
5	Chance	List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions (ACMSP116)
5	Chance	Recognise that probabilities range from 0 to 1 (ACMSP117)

# Fractions

## Content Descriptions

Year	Strand & SubStrand	Year Level
6	Fractions and Decimals	Compare fractions with related denominators and locate and represent them on a number line (ACMNA125)
6	Fractions and Decimals	Solve problems involving addition and subtraction of fractions with the same or related denominators (ACMNA126)
6	Fractions and Decimals	Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies (ACMNA127)
6	Patterns and Algebra	Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133)

# Fractions

## Content Descriptions

Year	Strand & SubStrand	Year Level
7	Real Numbers	Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)
7	Real Numbers	Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)
7	Real Numbers	Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
7	Real Numbers	Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)

# Fractions

## Content Descriptions

Year	Strand & SubStrand	Year Level
8	Number and Place Value	Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies(ACMNA183)
10	Patterns and Algebra	Apply the four operations to simple algebraic fractions with numerical denominators(ACMNA232)
10	Linear and non-linear relationships	Solve linear equations involving simple algebraic fractions (ACMNA240)

# Fractions

## Fraction language

Traditionally, the term ‘fraction’ was used to describe a part of a whole. (That is only true for fractions between zero and one.)

The word comes from the Latin *frango* – I break.

In our module, we take a fraction to mean a nonnegative rational number, that is, a number of the form  $\frac{m}{n}$  where  $n$  is a positive integer and  $m$  is a positive integer or 0.



# Fractions

## Fraction language

For the fraction  $\frac{5}{7}$ ,

the top number is called the ***numerator***,  
the line is called the ***vinculum*** and  
the bottom number is called the ***denominator***.

# Fractions

## Fraction language

A mixed number consists of a whole number plus a fraction, for example  $3 \frac{1}{7}$ .

Every improper fraction can be written as a mixed number and vice versa.

# Fractions

## Fraction language

We call a fraction a **proper fraction** if the numerator is less than the denominator.

For example,  $\frac{3}{4}$        $\frac{199}{203}$

If the numerator is greater than or equal to the denominator, the fraction is said to be an **improper fraction**.

For example,  $\frac{4}{3}$        $\frac{8}{8}$        $\frac{199}{198}$

# Fractions

## What is 1

- It is the “basic” unit for numbers
  - It is the standard measure
  - It is the “separator” between whole numbers and fractions
  - It is the multiplicative identity:  $n \times 1 = n$
- Anything multiplied by 1 gives the number you started with

# Fractions

## The importance of zero

- It is the “place keeper” symbol for our number system
- It is the Origin point from which all other numbers spread
- It is the “separator” between positive and negative numbers

- It is the additive identity:  $n + 0 = n$

Zero added to any other number gives the number you started with

# Fractions

## 0 and 1 together

Boundaries for all “Proper” fractions

i.e. between 0 and 1 are all the fractions

AND

Between 0 and 1 there are an infinite number of fractions

# Fractions

## A number between

No matter how small you go, there will always be another fraction between the fraction you have and 0.



# Fractions

## Fractions - some misconceptions

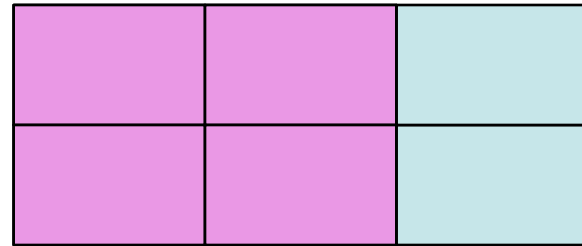
A child says  $\frac{2}{3} + \frac{2}{3} = \frac{4}{6}$



Add them together



and draws this picture

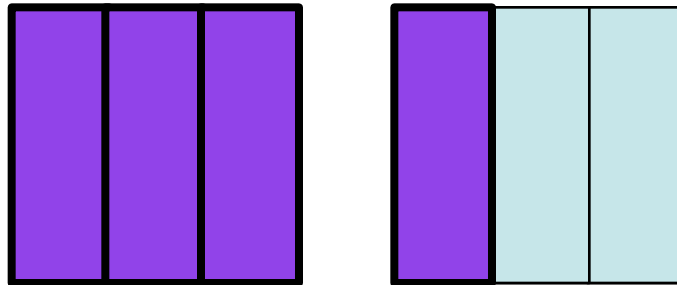




# Fractions

## Fractions - some misconceptions

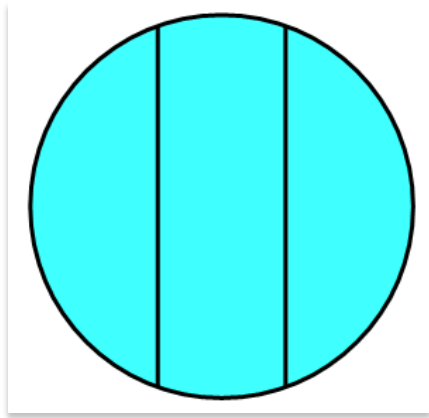
What fraction does this picture represent?



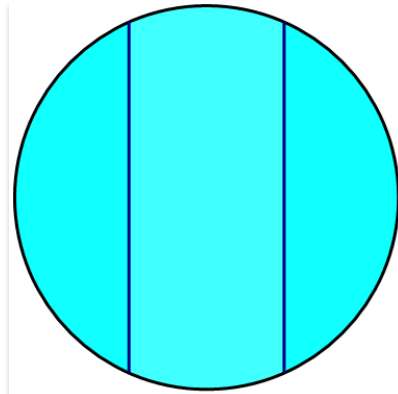
# Fractions

## Fractions - some misconceptions

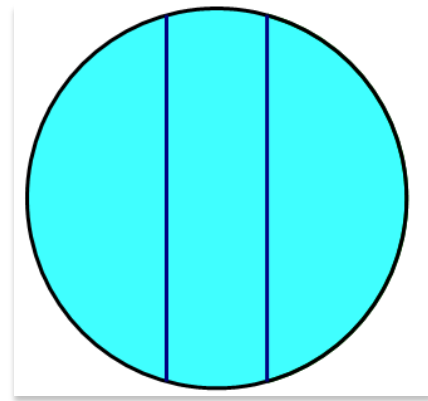
Which circle is divided into thirds by area?



Equal  
Widths



Centre is  
Half of the  
circle



Each is  $\frac{1}{3}$   
total area

# Fractions

## Fractions - some misconceptions

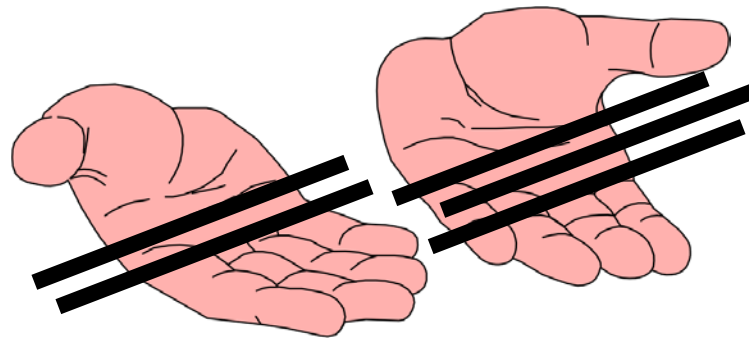


Sometimes you can have a bigger half.

# Fractions

## Fractions - early strategies

Stick in hands



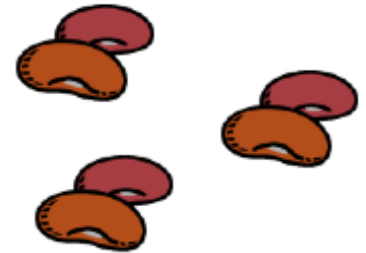
Small children should be exposed to puzzles and to cutting things up to develop a sense of 'part/whole' relationships

# Fractions

## Fractions - early strategies

Magic beans

(Lima beans sprayed gold on one side)



- Take a handful.
- Throw them.
- Talk about the number of gold out of the total number of beans.
- Link to 'numerator' and 'denominator'.

# Fractions

## Representing fractions

There are two main ways to represent fractions.

- The number line

The number line is better to use for addition, subtraction and order.

- Area diagrams

The area model is better to use for multiplication.

# Fractions

## Fractions - the number line

Introducing the number line

- Mark in zero and one other reference point
- Convention of negative numbers to the left, zero in the middle and positive numbers to the right
- Move towards children drawing their own

# Fractions

## Fractions - the number line

Use

- Masking tape on the floor
- String across the room
- Chalk in the playground
- Magnetized numbers on a blackboard or whiteboard
- Cash-register rolls
- Number ladder



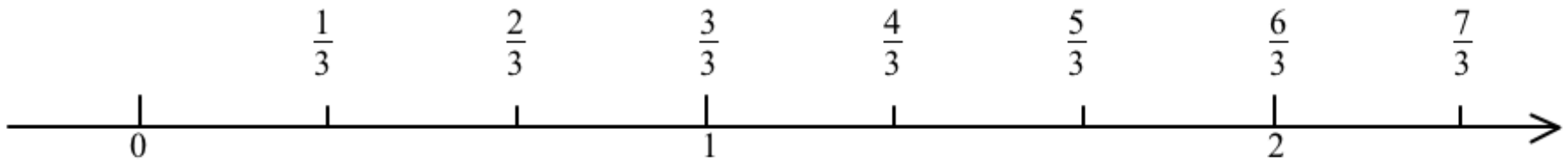
# Fractions

## Fractions - the number line

- Step through the introduction of the number line very slowly
- Do not assume this has been done before
- Remind the children all the time, where is the one?

# Fractions

## Fractions - the number line



To represent thirds on a number line

- Draw a line segment
- Mark in whole numbers 0, 1, 2 etc
- Divide the segment into three equal lengths
- Label each marker one third, two thirds etc

# Fractions

## Watch for confusion...

- Defining fractions on number line
- A fraction is both
  - A point **on** the number line
  - AND
  - The distance from 0, a length

# Fractions

## Fractions - folding paper

Folding paper helps develop the vocabulary needed

Connects to the number line

Start by folding paper strips.

Streamers are cheap and easy to use.

Begin with

halving quartering eighthing

Then move on to

thirding sixthing twelfthing

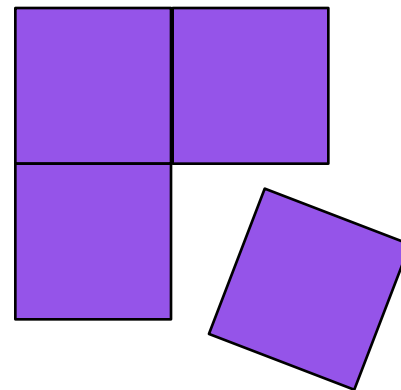
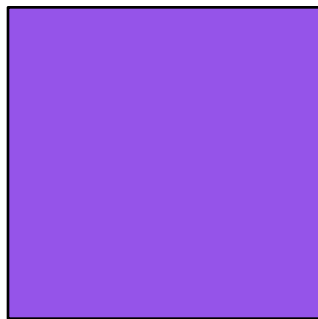
fifthing tentthing

# Fractions

## Folding paper

Make posters using kindergarten squares

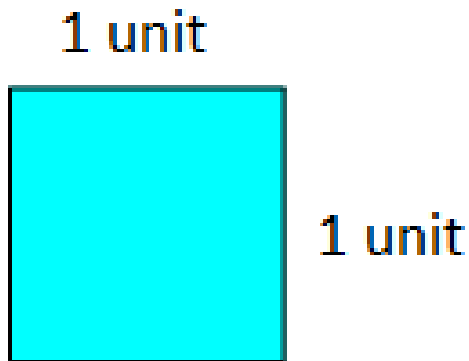
- Show understanding of cutting the whole
- Begin to introduce the idea of equivalence



# Fractions

## Area model

The unit square is a square with each side of length 1 unit.



Side length = 1

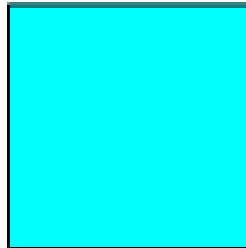
Area = 1

# Fractions

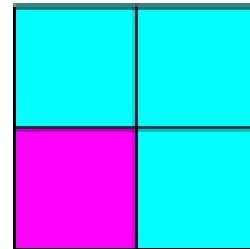
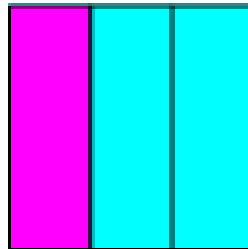
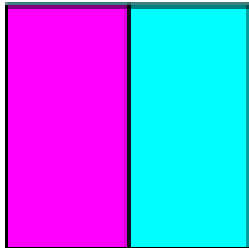
## Area model

Remind the children all the time, what is the one?

1 unit



1 unit



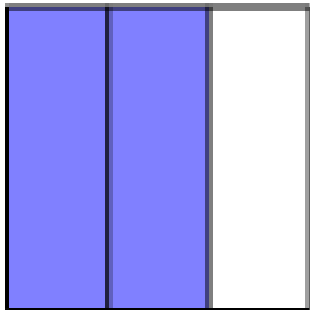
# Fractions

## Area model

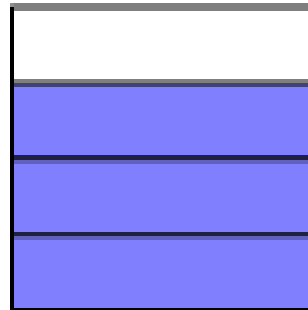
Shade parts of the unit square

- Denominator gives number of parts
- Numerator tells us “How many parts to take”

$$\frac{2}{3}$$



$$\frac{3}{4}$$





# Fractions

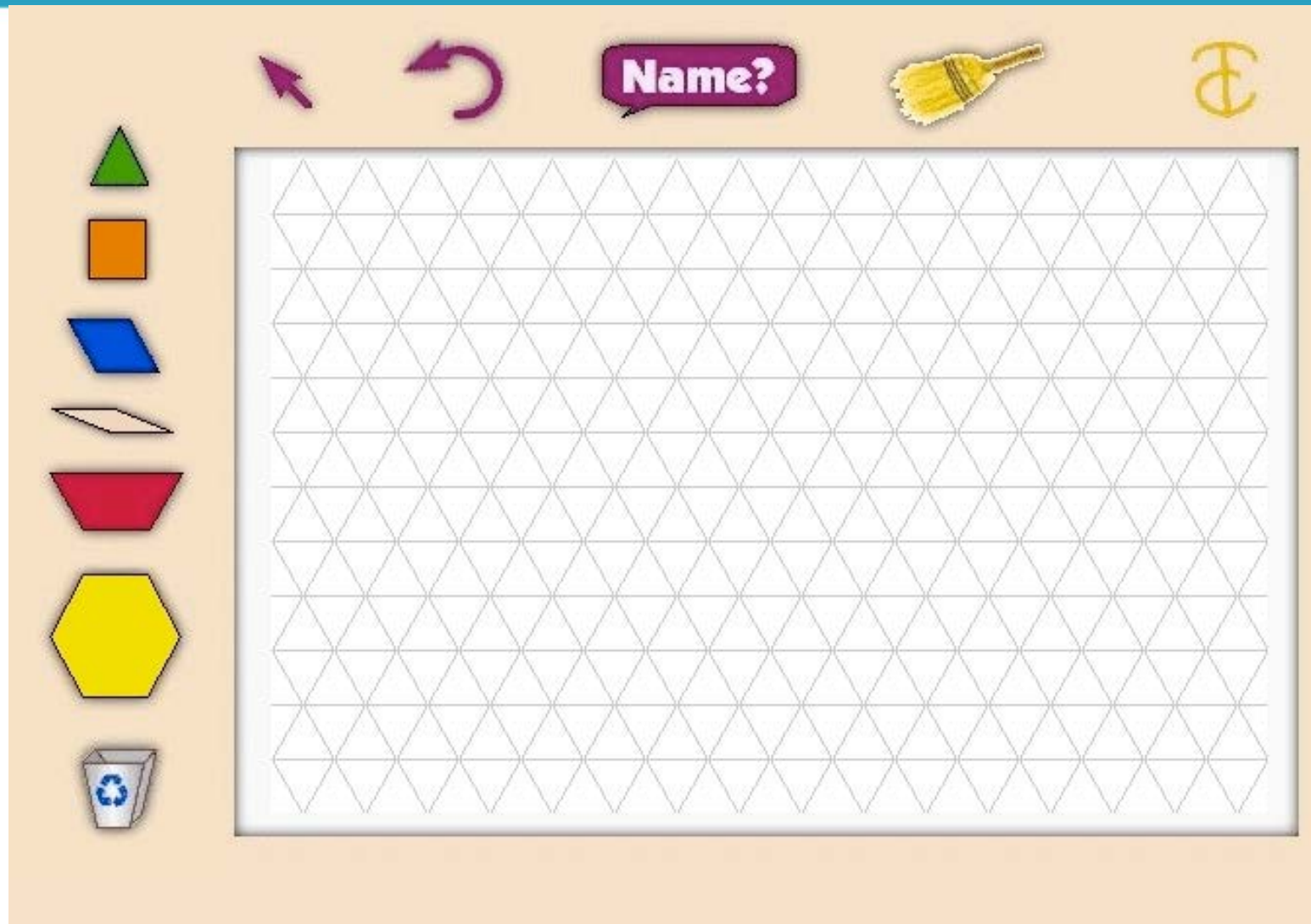
## Using pattern blocks (area model)

A different way of using manipulatives

- Pattern blocks can be used to represent fractions
- The issue here is to make sure we always know what the one is
- Use after unit square and number line are firmly understood
- There is a nice java applet on the web
- [http://www.arcytech.org/java/patterns/patterns\\_j.shtml](http://www.arcytech.org/java/patterns/patterns_j.shtml)

# Fractions

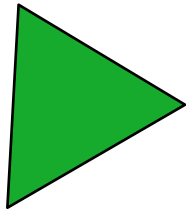
## Using pattern blocks



# Fractions

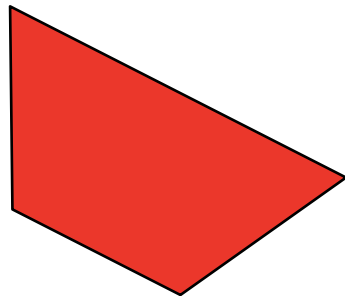
## Using pattern blocks

If



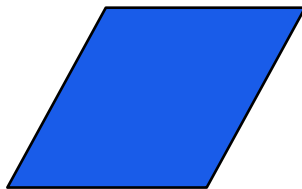
is 1,

then



is 3

and

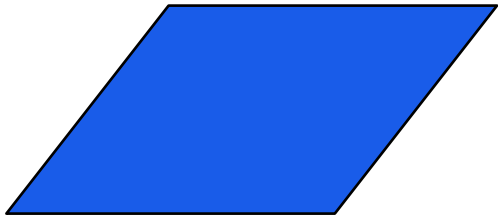


is 2.

# Fractions

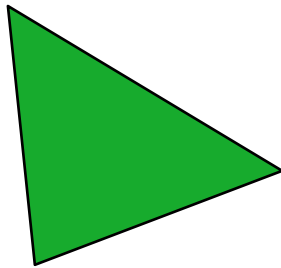
## Using pattern blocks

If



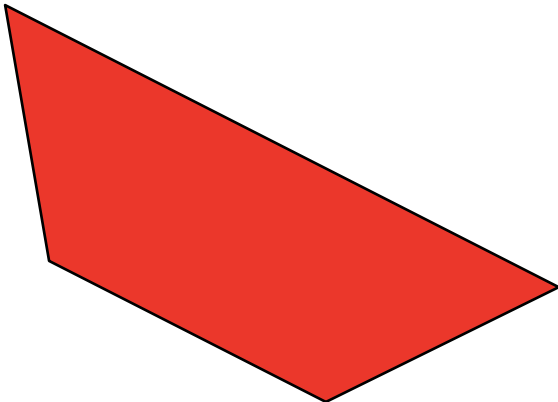
is 1,

then



is  $\frac{1}{2}$

and

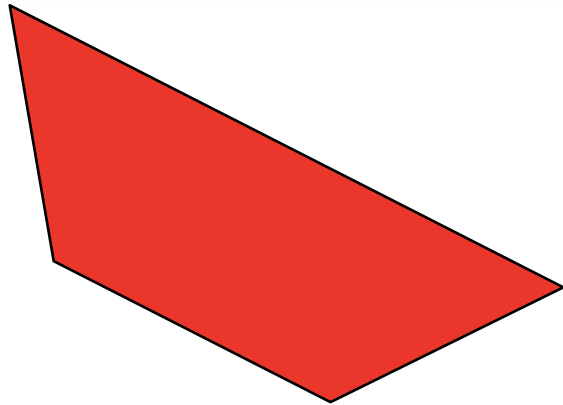


is  $1\frac{1}{2}$  .

# Fractions

## Using pattern blocks

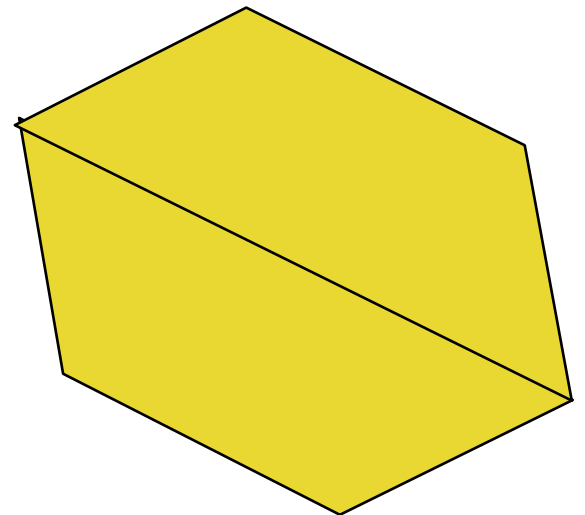
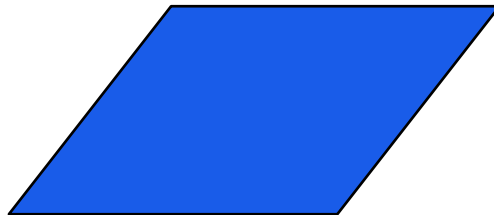
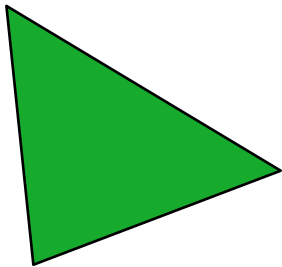
If



is 1

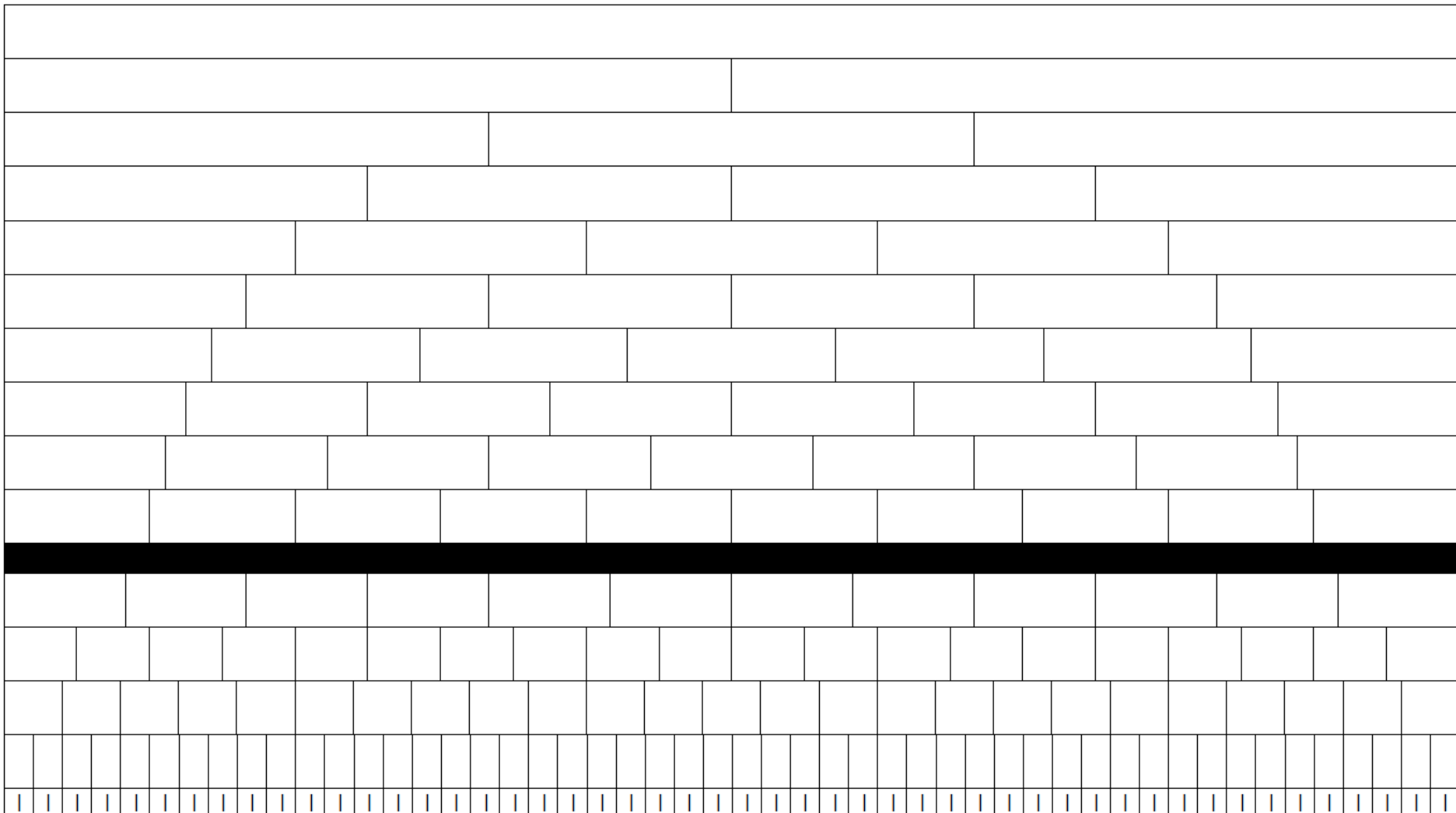


Then what value do each of these have?



# Fractions

## Fraction Walls

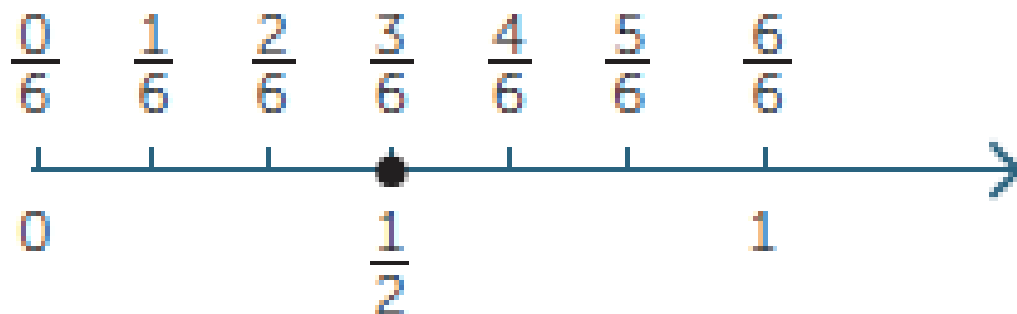


# Fractions

## Equivalent fractions

We say that two fractions are equivalent if they label the same place on the number line.

For example, we can represent the fractions  $\frac{1}{2}$  and  $\frac{3}{6}$  on the same number line.



# Fractions

## Equivalent fractions

Starting with a fraction, the fractions obtained by multiplying its numerator and denominator by the same whole number are equivalent.

Starting with a fraction, the fractions obtained by dividing its numerator and denominator by the same whole number are equivalent.



# Fractions

## Simplest form

A fraction is said to be in simplest form if the only common factor of the numerator and the denominator is 1.

To reduce a fraction to an equivalent fraction in simplest form we use the method of cancelling.

For example, when reducing  $\frac{6}{8}$  to its simplest form we divide the top (numerator) and bottom (denominator) by the highest common factor of 6 and 8, which is 2 and we get  $\frac{3}{4}$ .

# Fractions

## Representing fractions - dominoes

Use a domino standing on its end as a fraction for ordering activities.



# Fractions

## Ordering fractions

Before students start to do arithmetic with fractions they should have a strategies for choosing the larger (or smaller) of two fractions and ordering fractions from smallest to largest (or vice-versa).

# Fractions

## Addition of fractions

Addition of fractions is straightforward if the denominators are the same.

$$\frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

When the denominators are different, we use equivalent fractions to express the fractions using a common denominator and then proceed exactly as

before.

$$\begin{aligned}\frac{3}{4} + \frac{1}{6} &= \frac{9}{12} + \frac{2}{12} \\ &= \frac{11}{12}\end{aligned}$$

# Fractions

## Subtraction of fractions

Subtracting fractions uses similar ideas to addition of fractions. If the denominators of the two fractions are equal, subtraction is straightforward.

When the denominators are different, we use equivalent fractions to express the fractions using a common denominator and then proceed exactly as before.

# Fractions

## Multiplication of fractions

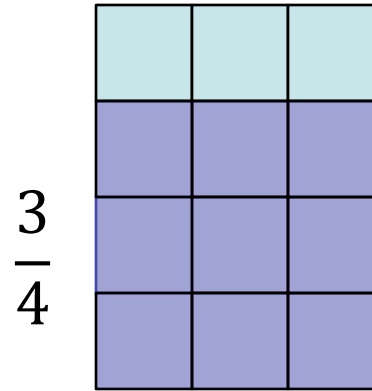
In mathematics, when we are asked for example to find two thirds of 18 oranges, we take it to mean that we divide the 18 oranges into three equal parts and then take two of these parts.



“Of” is the same as ‘x’.

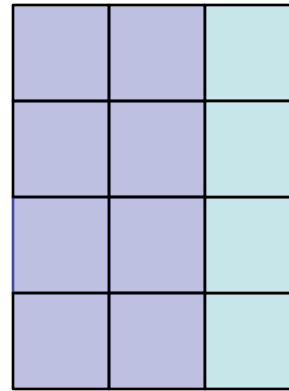
# Fractions

## Multiplication of fractions



# Fractions

## Multiplication of fractions



$$\frac{2}{3}$$

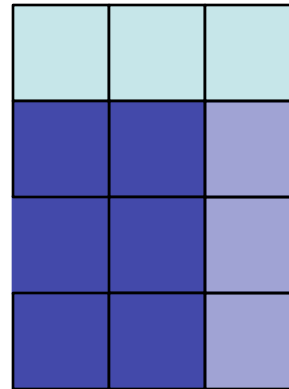


# Fractions

## Multiplication of fractions

$$\frac{2}{3} \text{ of } \frac{3}{4}$$

$$\frac{3}{4}$$



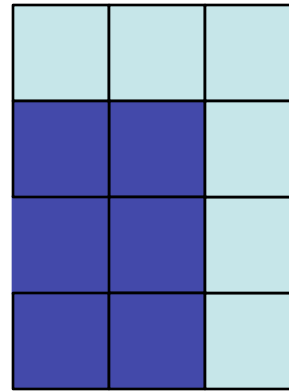
$$\frac{2}{3}$$

# Fractions

## Multiplication of fractions

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{3}{4}$$



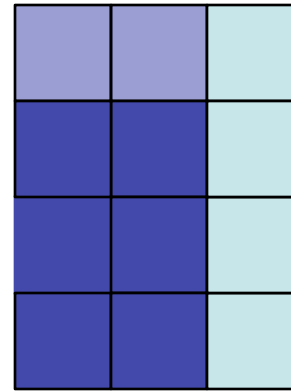
$$\frac{2}{3}$$

# Fractions

## Multiplication of fractions

$\frac{3}{4}$  of  $\frac{2}{3}$

$\frac{3}{4}$

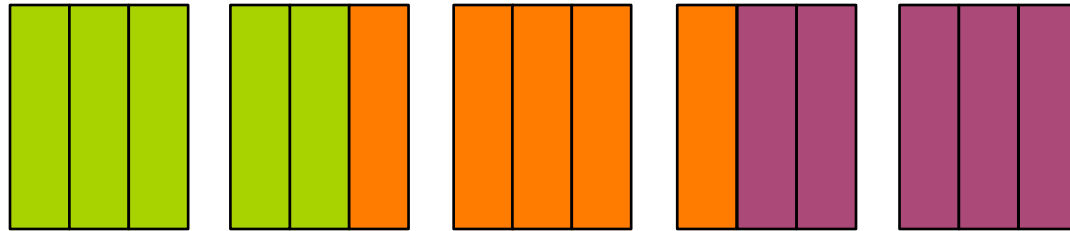


$\frac{2}{3}$

# Fractions

## Division of fractions

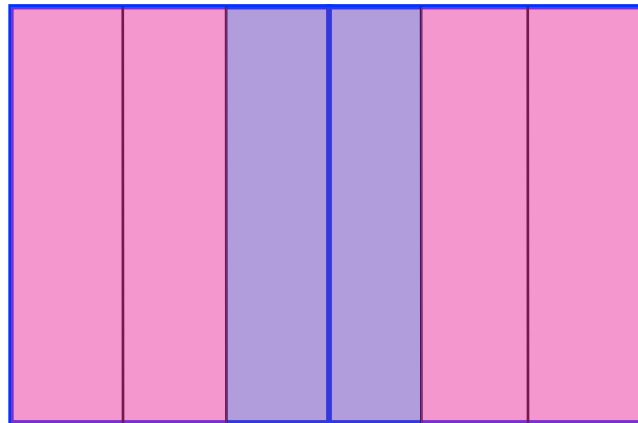
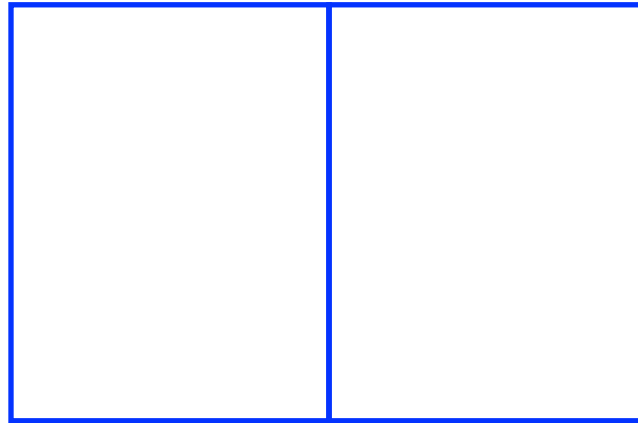
$$5 \div 3$$



# Fractions

## Division of fractions

$$2 \div 3$$



$\frac{5}{3}$

# Fractions

## Fractions and Decimals

Convert these fractions to decimals

$$\frac{1}{2}$$

$$\frac{1}{25}$$

$$\frac{1}{7}$$

$$\frac{489}{1000}$$

$$\frac{38}{40}$$

$$\frac{47}{64}$$

$$\frac{3}{5}$$

$$\frac{1}{99}$$

# Fractions

## Fractions and Decimals

Convert these fractions to decimals - which will produce a terminating decimal?

$$\frac{1}{2}$$

$$\frac{1}{25}$$

$$\frac{1}{7}$$

$$\frac{489}{1000}$$

$$\frac{38}{40}$$

$$\frac{47}{64}$$

$$\frac{3}{5}$$

$$\frac{1}{99}$$

# Fractions

## Fractions and Decimals

Look at the denominator - 'rip it apart' into its prime factorisation

$$\frac{1}{2}$$

$$\frac{1}{25}$$

$$\frac{1}{7}$$

$$\frac{489}{1000}$$

$$\frac{38}{40}$$

$$\frac{47}{64}$$

$$\frac{3}{5}$$

$$\frac{1}{99}$$



# Fractions

## Fractions and Decimals

Look at the denominator - what do you notice about the prime factorisation of the denominator?

$$\frac{1}{2}$$

$$\frac{1}{25}$$

$$\frac{1}{7}$$

$$\frac{489}{1000}$$

$$\frac{38}{40}$$

$$\frac{47}{64}$$

$$\frac{3}{5}$$

$$\frac{1}{99}$$

# Fractions

## Fractions and Decimals

Fractions with denominators that are multiples of 2s and 5s convert to terminating decimals.

$$\frac{1}{2} = 0.5 \quad \frac{1}{25} = 0.04 \quad \frac{1}{7} = 0.\overline{142857} \quad \frac{489}{1000} = 0.489$$

$$\frac{38}{40} = 0.95 \quad \frac{47}{64} = 0.734375 \quad \frac{3}{5} = 0.6 \quad \frac{1}{99} = 0.010101\ldots$$

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# Fractions

## Fractions and Decimals

Look at the denominator - what do you notice about the prime factorisation of the denominator?

$$\frac{1}{2} = 0.5 \quad \frac{1}{25} = 0.04 \quad \frac{1}{7} = 0.\overline{142857} \quad \frac{489}{1000} = 0.489$$

$$\frac{38}{40} = 0.95 \quad \frac{47}{64} = 0.734375 \quad \frac{3}{5} = 0.6 \quad \frac{1}{99} = 0.010101\dots$$

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# Fractions

## Some research – Liping Ma

$$1\frac{3}{4} \div \frac{1}{2}$$

“Knowing and Teaching Elementary  
Mathematics” Liping Ma

# Fractions

## Some research – Liping Ma

$$1\frac{3}{4} \div \frac{1}{2} = 1\frac{3}{4} \div (1 \div 2)$$

$$= 1\frac{3}{4} \div 1 \times 2$$

$$= 1\frac{3}{4} \times 2 \div 1$$

$$= 1\frac{3}{4} \times (2 \div 1)$$

$$= 1\frac{3}{4} \times 2$$

Making sense of the algorithm

“Knowing and Teaching Elementary Mathematics” Liping Ma

# Fractions

## Some research – Liping Ma

$$1\frac{3}{4} \div \frac{1}{2} = (1\frac{3}{4} \times \frac{2}{1}) \div (\frac{1}{2} \times \frac{2}{1})$$

$$= (1\frac{3}{4} \times \frac{2}{1}) \div 1$$

$$= 1\frac{3}{4} \times \frac{2}{1}$$

$$= 3\frac{1}{2}$$

Maintaining the value of the quotient

“Knowing and Teaching Elementary Mathematics” Liping Ma

# Fractions

## Some research – Liping Ma

$$1\frac{3}{4} \div \frac{1}{2} = 1.75 \div 0.5 = 3.5$$

Dividing by fractions using decimals

“Knowing and Teaching Elementary Mathematics” Liping Ma

# Fractions

## Some research – Liping Ma

$$\begin{aligned}1\frac{3}{4} \div \frac{1}{2} &= (1 + \frac{3}{4}) \div \frac{1}{2} \\&= (1 + \frac{3}{4}) \times \frac{2}{1} \\&= (1 \times 2) + (\frac{3}{4} \times 2) \\&= 2 + 1\frac{1}{2} \\&= 3\frac{1}{2}\end{aligned}$$

$$\begin{aligned}1\frac{3}{4} \div \frac{1}{2} &= (1 + \frac{3}{4}) \div \frac{1}{2} \\&= (1 \div \frac{1}{2}) + (\frac{3}{4} \div \frac{1}{2}) \\&= 2 + 1\frac{1}{2} \\&= 3\frac{1}{2}\end{aligned}$$

Applying the distributive law

“Knowing and Teaching Elementary Mathematics” Liping Ma



# Fractions

## Some research – Liping Ma

$$\begin{aligned}1\frac{3}{4} \div \frac{1}{2} &= \frac{7}{4} \div \frac{1}{2} \\&= \frac{7 \div 1}{4 \div 2} \\&= \frac{7}{2} \\&= 3\frac{1}{2}\end{aligned}$$

You don't have to multiply  
"Knowing and Teaching Elementary  
Mathematics" Liping Ma

# Fractions

## Always, sometimes, never questions

A whole number  
can not be  
written as a fraction

When comparing  
fractions, the fraction  
with the bigger number  
in the denominator is  
always smaller

There is no such thing  
as a smallest fraction

Every fraction can be  
written as a decimal

# Fractions

## Ratio

Ratios provide a way of comparing two or more related quantities.

Ratios are closely connected to fractions, but in many problems they are more convenient to use than fractions.

# Fractions

## Ratio

The ratio, by weight, of butter to flour in a biscuit dough is  $2 : 7$ .  
What fraction of the biscuit dough is flour?

**Solution**

There are  $2 + 7 = 9$  equal parts, 7 of which are flour.

Hence fraction of flour in the dough =  $\frac{7}{9}$ .

# Fractions

## Ratio

For example, in the photograph of a train (taken from the page of a textbook), the scale is 1 : 200.



A scale drawing has *exactly* the same shape as the original object, but a different size. All the lengths of the original object are reduced or magnified in the drawing in exactly the same ratio. This ratio is called the **scale** of the drawing.

Scale = length on the drawing : length on the actual object

The scale of a drawing can be given as the ratio of two numbers. For example, in the photograph of a train below, the scale is 1 : 200.



Scale = 1 : 200

This means that a length of 1 cm on the photograph corresponds to a length of 200 cm, or 2 m, on the actual carriage. Thus, the scale can also be written as:

Scale = 1 cm : 2 m

# Fractions

## Ratio

Students in years 6 to 8 should be involved in very practical exercises to do with ratio:

- cordial mixing
- recipes
- scale drawing

# Fractions

## History

In general, ancient civilisations avoided fractions by giving special names to parts of various measures.

We still do this today. For example, instead of saying  $1 \frac{13}{60}$  hours we say 1 hour 13 minutes.

# Fractions

## History


Egyptian fraction notation was developed in the Middle Kingdom of Egypt (2080–1640 BC), altering the Old Kingdom's Eye of Horus numeration system.

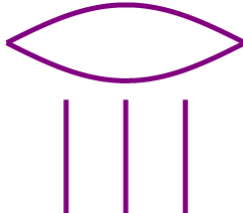


# Fractions

## History

With few exceptions they wrote all their fractions as unit fractions, that is, with numerator 1.

To write the unit fractions used in their Egyptian fraction notation the Egyptians placed the hieroglyph  above the numeral for the denominator.

For example  $\frac{1}{3} =$  

# Fractions

## History

There were separate symbols for some common non-unit fractions such as  $\frac{2}{3}$  and  $\frac{3}{4}$  but most fractions, as we know them, were expressed as a sum of unit fractions.

For example  $\frac{2}{7}$  can be written as

$$\frac{1}{7} + \frac{1}{14} + \frac{1}{21} + \frac{1}{42}$$

# Fractions

## History

Up until the Hellenistic period the Greeks preferred to think of fractions in terms of ratios and proportions. They did not tend to think of them as we do, and certainly not as points on the number line.

# Fractions

## History

Hindu mathematicians are believed to be the first to indicate fractions with numbers rather than words. Brahmagupta (c. 628) and Bhaskara (c. 1150) were early Hindu mathematicians who wrote fractions as we do today, but without the bar (vinculum). They wrote one number above the other to indicate a fraction.

# Fractions

## History

The next step in the evolution of fraction notation was the addition of the horizontal fraction bar. This is generally credited to the Arabs who used the Hindu notation, then improved on it by inserting this bar in between the numerator and denominator, which was later named the vinculum. Later on, Fibonacci (c.1175-1250), was the first European mathematician to use the vinculum as it is used today.

# Fractions

## In everyday life

Liquids	
METRIC	CUP
30ml	100°
60ml	1/4 cup
80ml	1/3 cup
125ml	1/2 cup
180ml	3/4 cup
250ml	1 cup
310ml	1 1/4 cups
375ml	1 1/2 cups
430ml	1 3/4 cups
500ml	2 cups
625ml	2 1/2 cups
750ml	3 cups
1L	4 cups
1.25L	5 cups
1.5L	6 cups
2L	8 cups
2.5L	10 cups

Cup and Spoon Sizes	
CUP	Metric
¼	60ml
1/3	80ml
½	125ml
1 cup	250ml
SPOON	
1/4 teaspoon	1.25ml
1/2 teaspoon	2.5ml
1 teaspoon	5ml
1 tablespoon	(4 teaspoons) 20ml

MUFFIN PANS		
Mini	30ml	1 1/2 Tbs
Regular 1	80ml	1/3 cup
Regular 2	125ml	1/2 cup
Texas	180ml	3/4 cup
	Source: <a href="http://www.taste.com.au">www.taste.com.au</a>	

# Fractions

## In everyday life

### Combination Spanner Set Sizes:

- Imperial:  $\frac{1}{4}$ ,  $\frac{5}{16}$ ,  $\frac{3}{8}$ ,  $\frac{7}{16}$ ,  $\frac{1}{2}$ ,  $\frac{9}{16}$ ,  $\frac{5}{8}$ ,  $\frac{11}{16}$ ,  $\frac{3}{4}$ ,  $\frac{13}{16}$ ,  $\frac{7}{8}$ ,  $\frac{15}{16}$ , 1"
- Metric: 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 22, 24mm

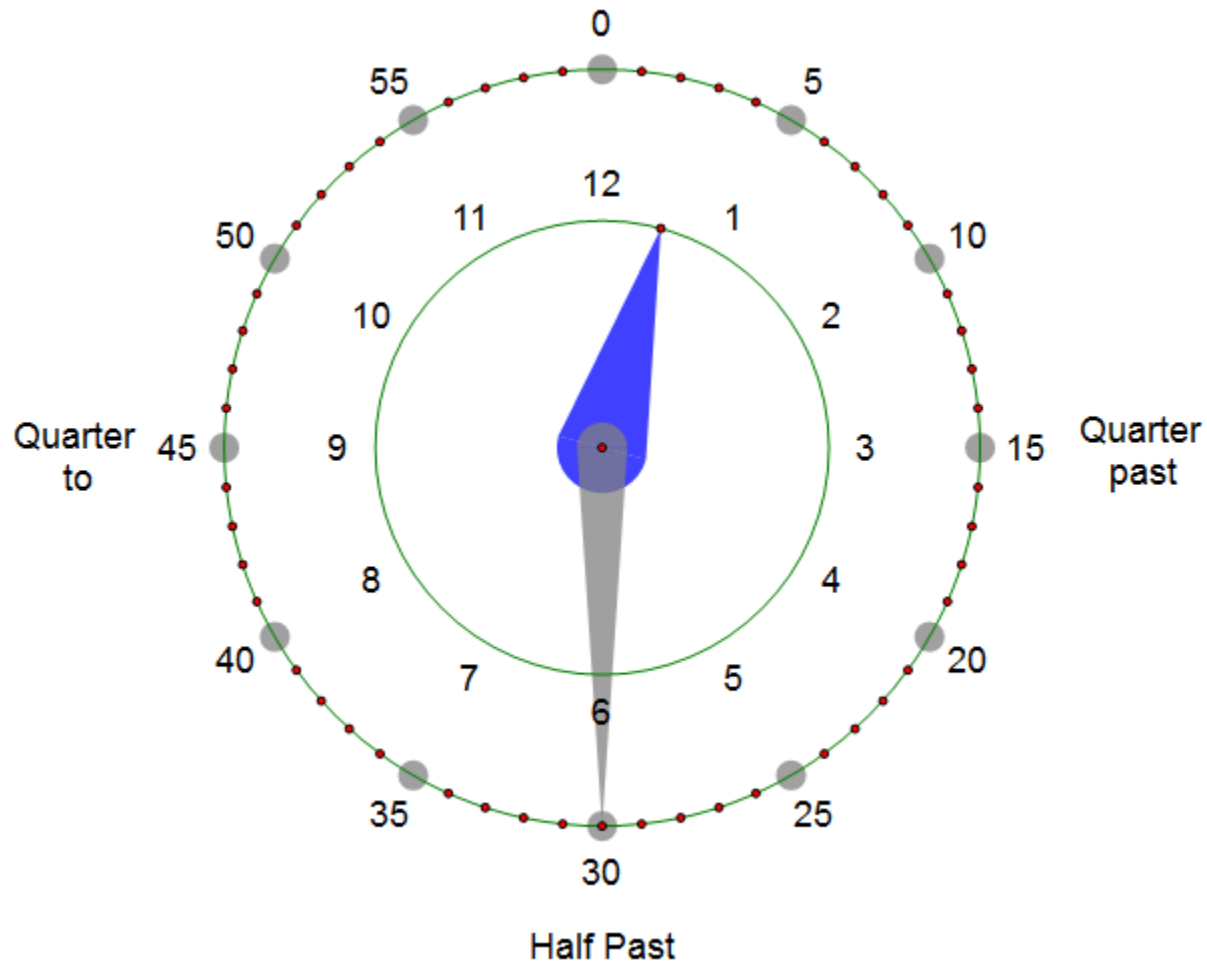
### Flare Nut Spanner Set Sizes:

- Imperial:  $\frac{3}{8}$  x  $\frac{7}{16}$ ,  $\frac{1}{2}$  x  $\frac{9}{16}$ ,  $\frac{5}{8}$  x  $\frac{11}{16}$ "
- Metric: 10 x 11, 12 x 13, 15 x 17mm



# Fractions

## In everyday life

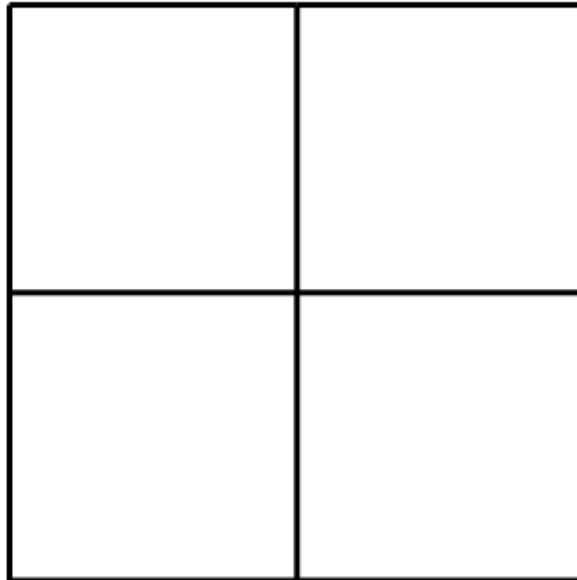




# Fractions

## Addition: Using a Grid

$$\frac{1}{2} + \frac{1}{5}$$

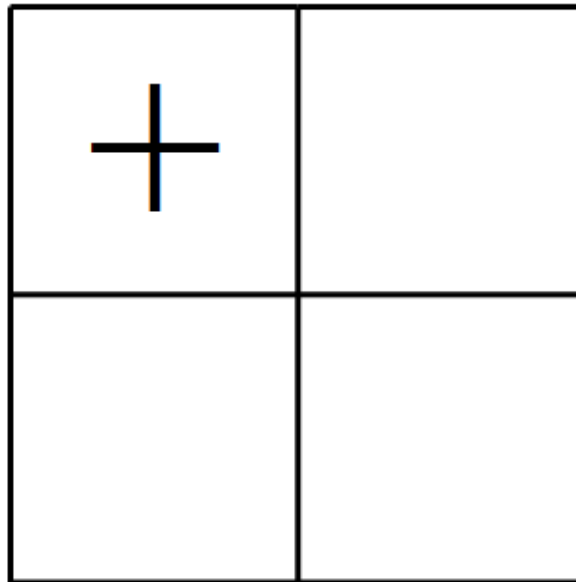


# Fractions

## Addition: Using a Grid

$$\frac{1}{2} + \frac{1}{5}$$

$$1 \frac{1}{2}$$



$$\frac{1}{5}$$

# Fractions

## Addition: Using a Grid

$$\frac{1}{2} + \frac{1}{5}$$

$$1 \frac{1}{2}$$

+	<small>2 x 1</small> 2
<small>1 x 5</small> 5	<small>2 x 5</small> 10

$$\frac{1}{5}$$

# Fractions

## Addition: Using a Grid

$$\frac{1}{2} + \frac{1}{5}$$

$$1 \frac{1}{2}$$

+	<small>2 x 1</small> 2
<small>1 x 5</small> 5	<small>2 x 5</small> 10

$$\frac{1}{5}$$

7

# Fractions

## Addition: Using a Grid

$$\frac{1}{2} + \frac{1}{5}$$

$$1 \frac{1}{2}$$

+	<small>2 x 1</small> 2
<small>1 x 5</small> 5	<small>2 x 5</small> 10

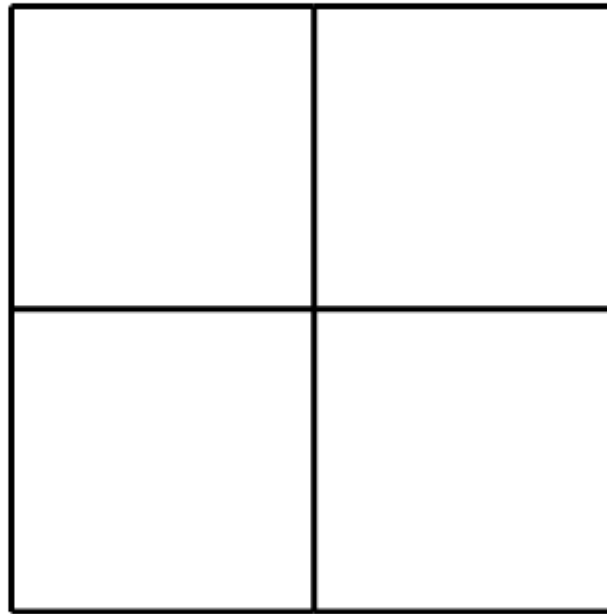
$$\frac{1}{5}$$

$$7 \frac{1}{10}$$

# Fractions

## Subtraction: Using a Grid

$$\frac{7}{5} - \frac{1}{4}$$

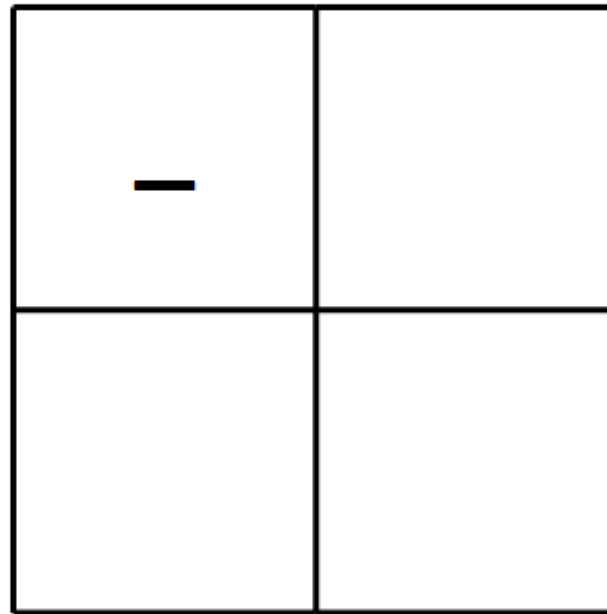


# Fractions

## Subtraction: Using a Grid

$$\frac{7}{5} - \frac{1}{4}$$

$$7 \frac{1}{5}$$



$$\frac{1}{4}$$

# Fractions

## Subtraction: Using a Grid

$$\frac{7}{5} - \frac{1}{4}$$

$$7 \frac{1}{5}$$

—	<small>5 x 1</small> 5
<small>7 x 4</small> 28	<small>5 x 4</small> 20

$$1 \frac{1}{4}$$



# Fractions

## Subtraction: Using a Grid

$$\frac{7}{5} - \frac{1}{4}$$

$$7 \frac{1}{5}$$

—	<small>5 x 1</small> 5
<small>7 x 4</small> 28	<small>5 x 4</small> 20

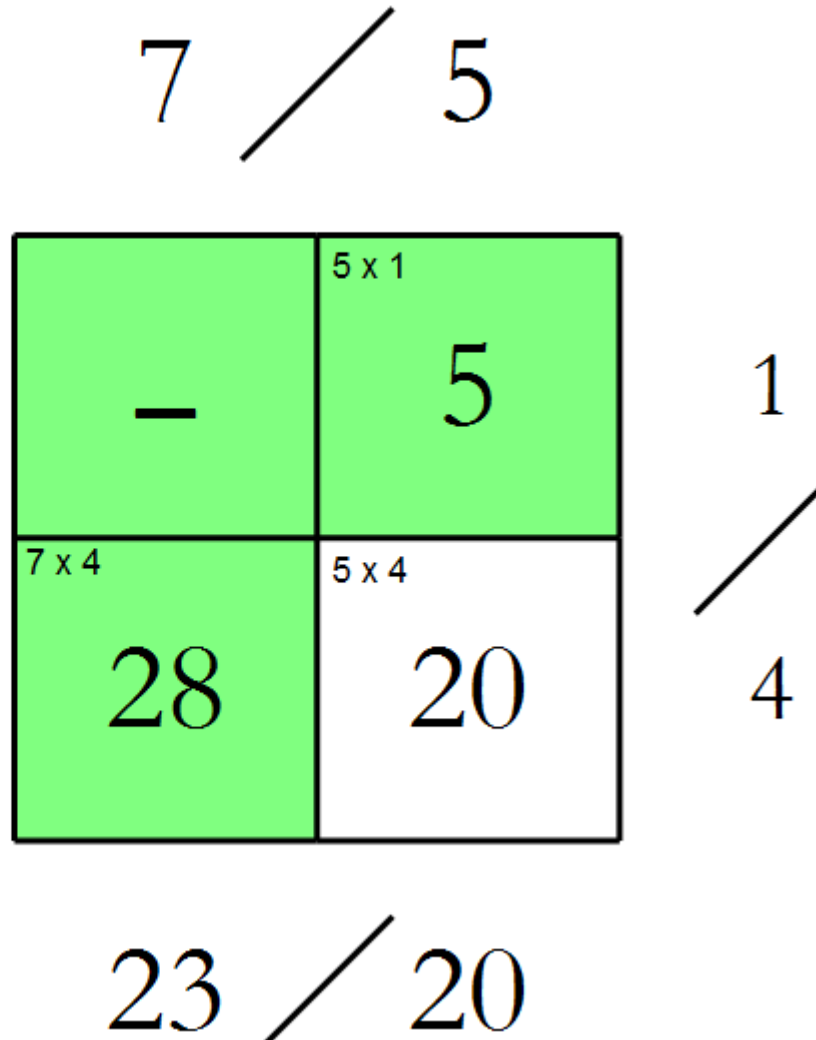
$$1 \frac{1}{4}$$

$$23$$

# Fractions

## Subtraction: Using a Grid

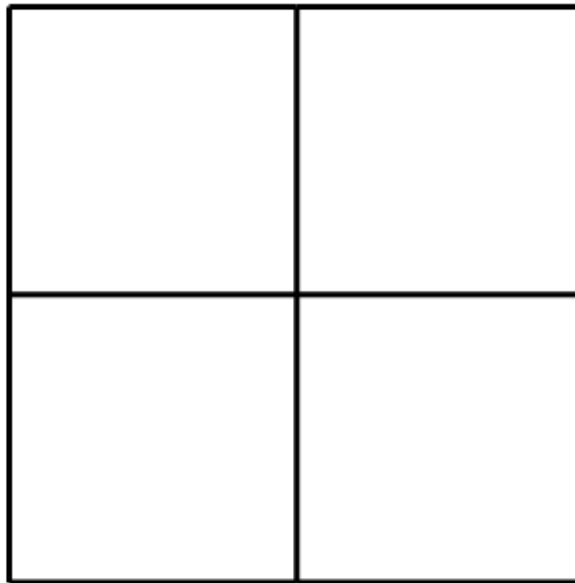
$$\frac{7}{5} - \frac{1}{4}$$



# Fractions

## Multiplication: Using a Grid

$$\frac{8}{10} \times \frac{7}{9}$$

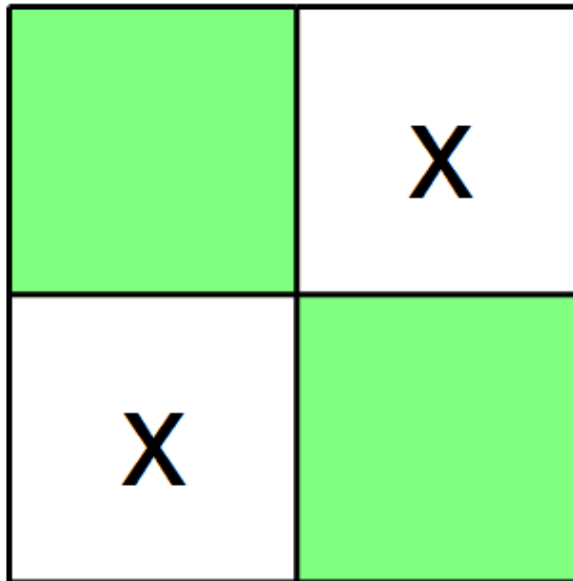


# Fractions

## Multiplication: Using a Grid

$$\frac{8}{10} \times \frac{7}{9}$$

$$8 \text{ } \diagup \text{ } 10$$



$$7 \text{ } \diagup \text{ } 9$$

# Fractions

## Multiplication: Using a Grid

$$\frac{8}{10} \times \frac{7}{9}$$

$$8 \text{ } \diagup \text{ } 10$$

8 x 7 56	X
X	10 x 9 90

$$\begin{array}{r} 7 \\ \diagup \\ 9 \end{array}$$

# Fractions

## Multiplication: Using a Grid

$$\frac{8}{10} \times \frac{7}{9}$$

8 x 7 56	X
X	10 x 9 90

$$8 \text{ } \diagup \text{ } 10$$

$$\begin{array}{r} 7 \\ \diagdown \\ 9 \end{array}$$

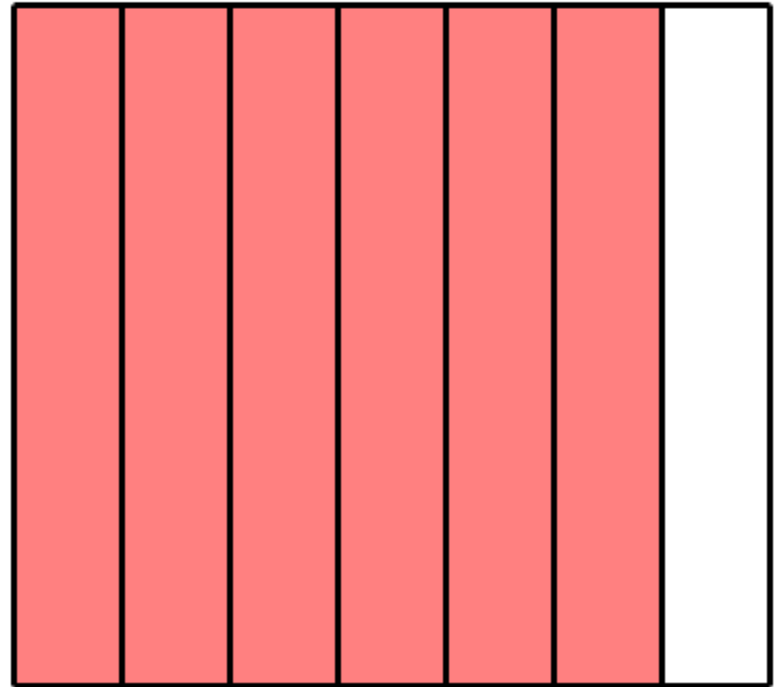
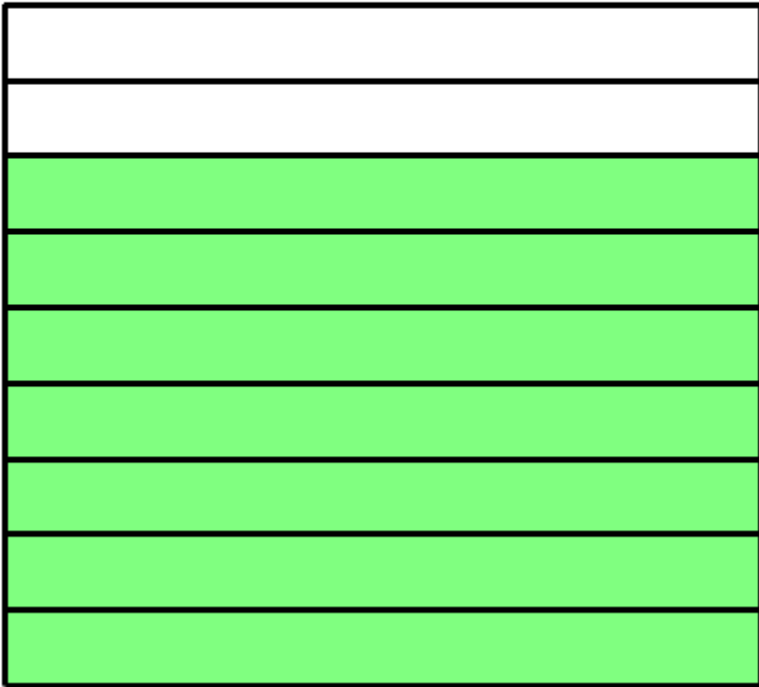
$$56 \text{ } \diagup \text{ } 90$$

# Fractions

$$\frac{7}{9}$$

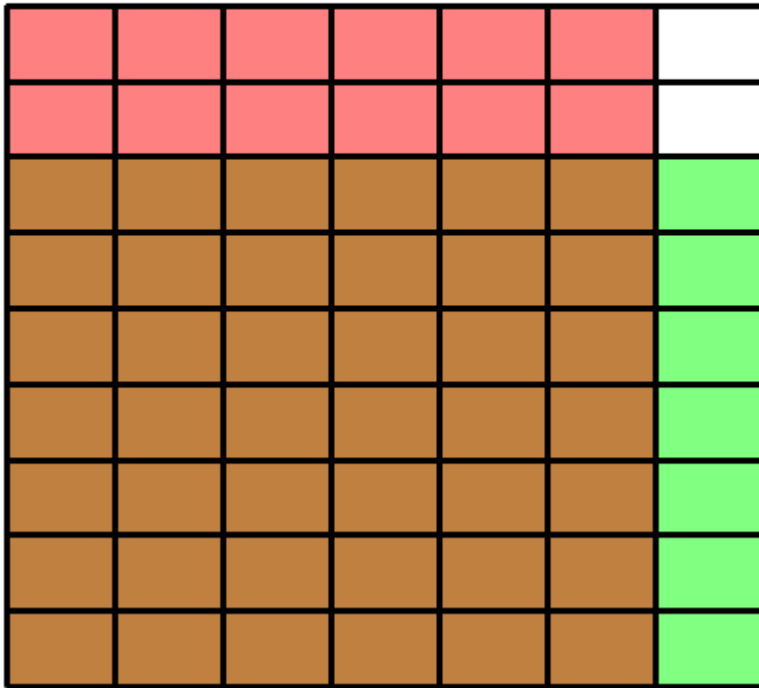
x

$$\frac{6}{7}$$



# Fractions

$$\frac{7}{9} \times \frac{6}{7}$$



The number of double shaded “squares” is the numerator, eg:  $7 \times 6 = 42$

The total number of “squares” is the denominator: eg  $9 \times 7 = 63$

$$\text{So } \frac{7}{9} \times \frac{6}{7} = \frac{42}{63}$$



# Fractions

## Division of fractions

Stay change flip process

The first term "stays" the same

$$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3}$$

The last term "flips"

The operation changes from  
division to multiplication

# Fractions

## As “suspended” divisions

Connected to exact values.

In +, - and x we almost always carry out the calculation immediately. Not so with division.

Relates to  $\sqrt{\quad}$ , logs, exponentials

# Fractions

## Order of operations and algebra

Take the fraction addition:  $\frac{2}{5} + \frac{3}{4}$

Instead of calculating each step, leave the operations visible

$$\text{So } \frac{2}{5} + \frac{3}{4} = \frac{2 \times 4 + 3 \times 5}{5 \times 4}$$

# Fractions

## Order of operations and algebra

Now consider :  $\frac{3}{7} + \frac{2}{5}$

What is the missing term?

$$\frac{3 \times \square + 2 \times 7}{7 \times 5}$$

# Fractions

## Algebraic fractions

Now consider :  $\frac{x}{7} + \frac{2}{5}$

What does this become?

# Fractions

## Algebraic fractions

And if:  $\frac{x}{7} + \frac{2}{5} = \frac{58}{70}$

What is the value of x?

# AMSI - Schools

## The Team

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