GEOMETRY

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GEOMETRY

Space

The mathematical study of space is called *geometry*

–Geo: “earth”

–-metry: “measure”

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Space

Emphasis is on making links between two-dimensional **shapes** and three-dimensional **objects** in the physical world and their abstract geometrical representations, leading to an understanding of their properties and how they can be used to solve practical or aesthetic problems

QP1

Activity #1: Making a Cube

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F	Using Units of Measurement	Use direct and indirect comparisons to decide which is longer, heavier or holds more, and explain reasoning in everyday language (ACMMG006)
F	Shape	Sort, describe and name familiar two-dimensional shapes and three-dimensional objects in the environment (ACMMG009)
F	Location and Transformation	Describe position and movement (ACMMG010)
1	Using Units of Measurement	Measure and compare the lengths and capacities of pairs of objects using uniform informal units (ACMMG019)
1	Shape	Recognise and classify familiar two-dimensional shapes and three-dimensional objects using obvious features (ACMMG022)
1	Location and Transformation	Give and follow directions to familiar locations (ACMMG023)

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2	Using Units of Measurement	Compare and order several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units (ACMMG037)
2	Using Units of Measurement	Compare masses of objects using balance scales (ACMMG038)
2	Shape	Describe and draw two-dimensional shapes, with and without digital technologies(ACMMG042)
2	Shape	Describe the features of three-dimensional objects (ACMMG043)
2	Location and Transformation	Interpret simple maps of familiar locations and identify the relative positions of key features (ACMMG044)
2	Location and Transformation	Investigate the effect of one-step slides and flips with and without digital technologies(ACMMG045)
2	Location and Transformation	Identify and describe half and quarter turns (ACMMG046)

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3	Using Units of Measurement	Measure, order and compare objects using familiar metric units of length, mass and capacity (ACMMG061)
3	Shape	Make models of three-dimensional objects and describe key features (ACMMG063)
3	Location and Transformation	Create and interpret simple grid maps to show position and pathways (ACMMG065)
3	Location and Transformation	Identify symmetry in the environment (ACMMG066)
3	Geometric Reasoning	Identify angles as measures of turn and compare angle sizes in everyday situations(ACMMG064)

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4	Using Units of Measurement	Use scaled instruments to measure and compare lengths, masses, capacities and temperatures (ACMMG084)
4	Using Units of Measurement	Compare objects using familiar metric units of area and volume (ACMMG290)
4	Shape	Compares the areas of regular and irregular shapes by informal means (ACMMG087)
4	Shape	Compares and describes two dimensional shapes that result from combining and splitting common shapes, with and without digital technologies (ACMMG088)
4	Location and Transformation	Uses simple scales, legends and directions to interpret information contained in basic maps (ACMMG090)
4	Location and Transformation	Creates symmetrical patterns, pictures and shapes with and without digital technologies (ACMMG091)
4	Geometric Reasoning	Compare angles and classify them as equal to, greater than or less than a right angle (ACMMG089)

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5	Using Units of Measurement	Choose appropriate units of measurement for length, area, volume, capacity and mass(ACMMG108)
5	Using Units of Measurement	Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109)
5	Shape	Connect three-dimensional objects with their nets and other two-dimensional representations (ACMMG111)
5	Location and Transformation	Use a grid reference system to describe locations. Describe routes using landmarks and directional language (ACMMG113)
5	Location and Transformation	Describe translations, reflections and rotations of two-dimensional shapes. Identify line and rotational symmetries (ACMMG114)
5	Location and Transformation	Apply the enlargement transformation to familiar two dimensional shapes and explore the properties of the resulting image compared with the original (ACMMG115)
5	Geometric Reasoning	Estimate, measure and compare angles using degrees. Construct angles using a protractor (ACMMG112)

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6	Using Units of Measurement	Connect decimal representations to the metric system (ACMMG135)
6	Using Units of Measurement	Convert between common metric units of length, mass and capacity (ACMMG136)
6	Using Units of Measurement	Solve problems involving the comparison of lengths and areas using appropriate units(ACMMG137)
6	Using Units of Measurement	Connect volume and capacity and their units of measurement (ACMMG138)
6	Shape	Construct simple prisms and pyramids (ACMMG140)
6	Location and Transformation	Investigate combinations of translations, reflections and rotations, with and without the use of digital technologies (ACMMG142)
6	Location and Transformation	Introduce the Cartesian coordinate system using all four quadrants (ACMMG143)
6	Geometric Reasoning	Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles (ACMMG141)

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7	Using Units of Measurement	Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)
7	Using Units of Measurement	Calculate volumes of rectangular prisms (ACMMG160)
7	Shape	Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)
7	Location and Transformation	Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)
7	Geometric Reasoning	Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)
7	Geometric Reasoning	Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (ACMMG164)
7	Geometric Reasoning	Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (ACMMG166)
7	Geometric Reasoning	Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)

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8	Using Units of Measurement	Choose appropriate units of measurement for area and volume and convert from one unit to another (ACMMG195)
8	Using Units of Measurement	Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites (ACMMG196)
8	Using Units of Measurement	Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area (ACMMG197)
8	Using Units of Measurement	Develop the formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume(ACMMG198)
8	Geometric Reasoning	Define congruence of plane shapes using transformations (ACMMG200)
8	Geometric Reasoning	Develop the conditions for congruence of triangles (ACMMG201)
8	Geometric Reasoning	Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202)

GEOMETRY

Why teach it?

Just as arithmetic has numbers as its basic object of study, so points, lines and circles are the basic building blocks of plane geometry.

Geometry gives an opportunity for students to develop their geometric intuition, which has applications in many areas of life, and also to learn how to construct logical arguments and make deductions in a setting which is, for the most part, independent of number.

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Why teach it?

Why teach geometry?

- Applications
- Accessibility to students who prefer pictures
- Encourages flexibility
- Historical importance
- Central role in mathematics
- Logical structure
- The surprise of results
- It is in the syllabus

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Why teach it?

What are the issues when teaching geometry?

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Why teach it?

What are the issues when teaching geometry?

- Connected logical arguments
- Proof
- Statements and converses
- Necessary and sufficient conditions

Sounds like the Proficiency strands…

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Why teach it?

Pictures are helpful to people who prefer non-verbal learning.

Geometry can make other areas of mathematics more accessible. For example: algebra

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Algebra exists only for the elucidation of geometry.

William Edge

No employment can be managed without arithmetic, no mechanical invention without geometry.

Benjamin Franklin

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Why teach it?

Builders, tilers, architects, graphic designers and web designers routinely use geometric ideas in their work. Classifying such geometric objects and studying their properties are very important. Geometry also has many applications in art.

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Applications

Building

Pythagoras is used by trades people. Though this topic is taught in junior secondary. for students to begin to grasp ideas about Pythagoras, they need to have developed a solid understanding of primary space and shape ideas.

http://www.amazon.com/gp/product/images/B00004TKDP/ref=dp_otherviews_4/104-6026308-1691933?%5Fencoding=UTF8&s=hi&img=4

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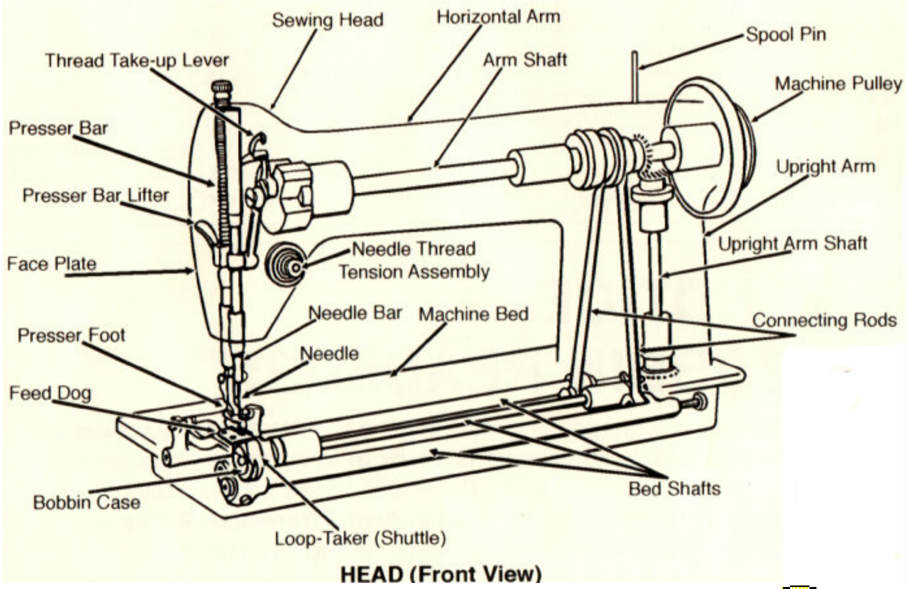
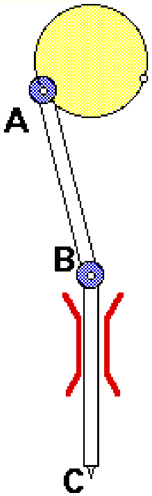
Applications

Mechanics

All mechanical inventions have moving parts that obey the laws of geometry. For example the sewing machine is an application of line segments and points, where one of the points moves around the circumference of a circle.





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Applications

Art

Throwing a pot is all about symmetry.

A potter wants to know how to find the centre of a circle - exactly!

www.joepicassos.com/ potters_wheel.html

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Applications

Scaling and similarity

Enlarging photographs

www.livingartsphotogallery.com/ pricelist.shtml

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Applications

- Land and money: the Toorak example

- How much is a strip of land one foot wide down the length of a property worth?

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History

All the classical civilisations – Egypt, Babylon, India and China had a practical geometry but none treated geometry as a deductive science.

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History

The papyri, clay tablets and other written material that have come to us invariably state problems in numbers and solve them by recipes.

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History

The Rhind papyrus which dates from about 2000 BC, some 17 centuries before Euclid gives the method of finding the area of a triangle.

http://www.touregypt.net/featurestories/numbers.htm

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History

In Egypt, the rope pullers were important members of Egyptian society.

They used ropes to form rectangular shapes and right angled triangles.

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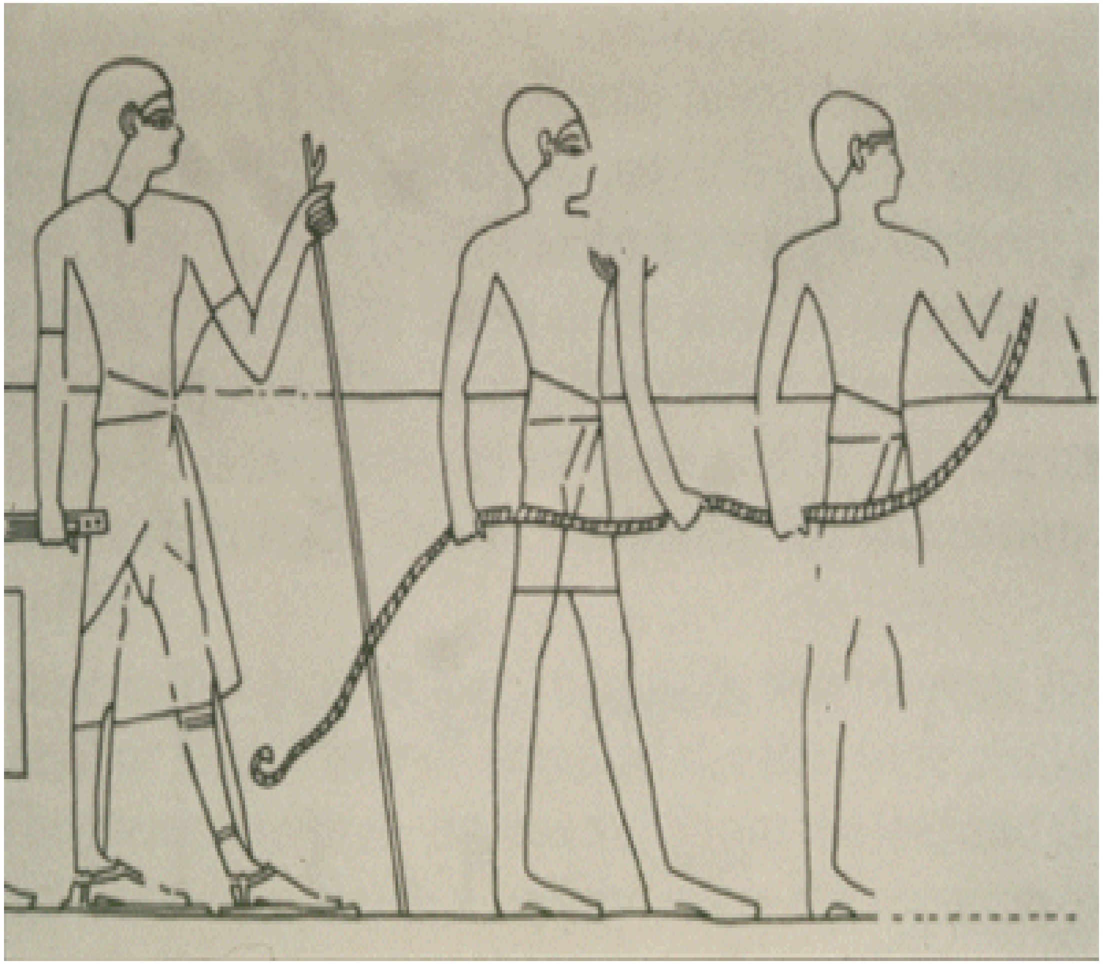
History

This picture is from the tomb of a high priest.



www.math.dartmouth.edu/. ../unit1/INTRO.html

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History

It took 1500 years before the Greeks devised a logical system that enabled them to demonstrate, on very general assumptions many of the geometric results which had been used in special cases by the earlier civilisations.

Students should not become impatient if they do not immediately understand the point of geometrical argument.

Entire civilisations missed the point altogether.

History

Euclid's *Elements* was written in about 300 BC.

The *Elements* begins with definitions and five postulates that have come to define much of the geometry we know today.

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History

Raphael's *School of Athens* (1509 - 1510)

http://www.christusrex.org/www1/stanzas/Aw-Athens.jpg

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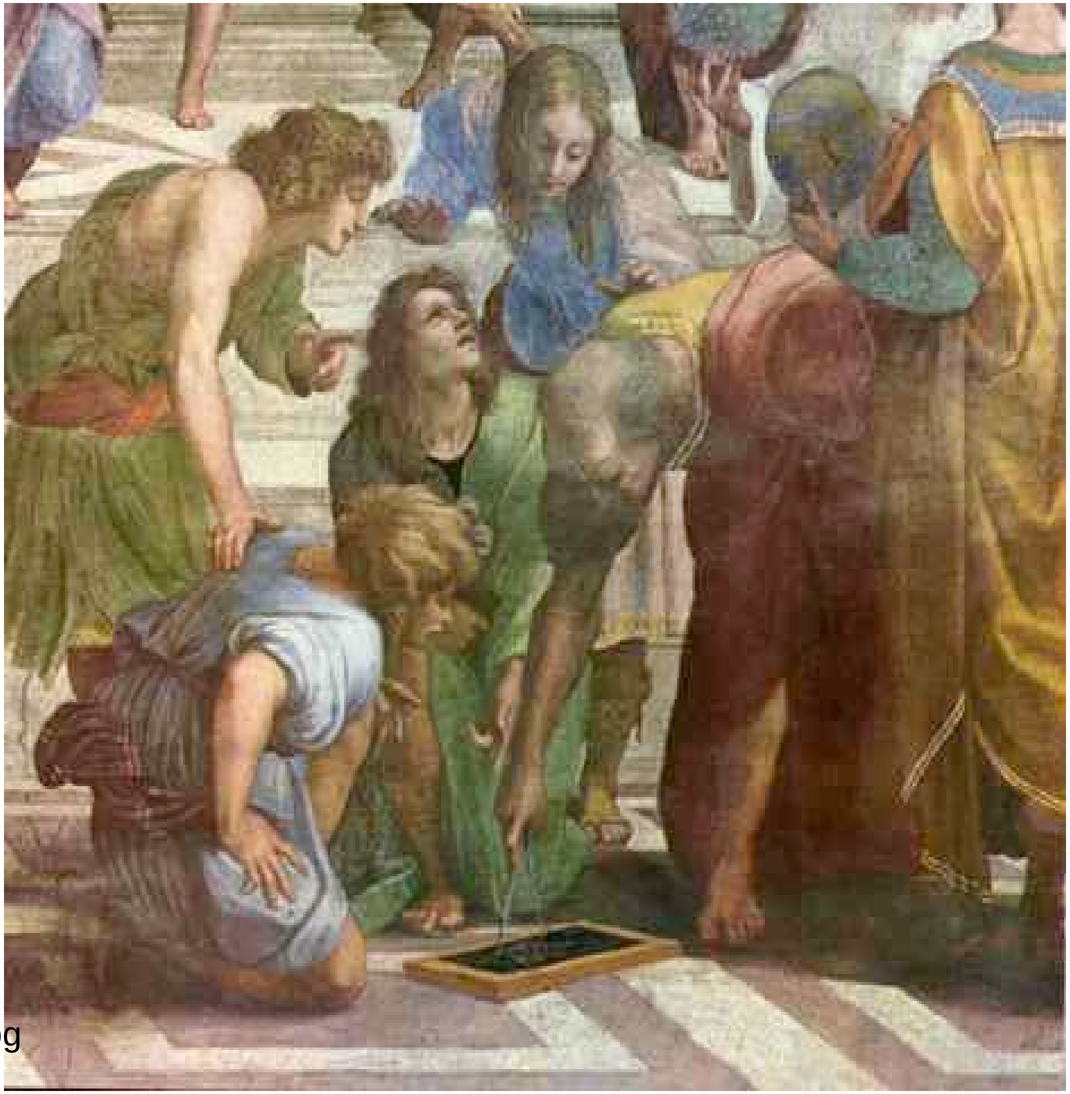
History

Detail from *School of Athens* showing Euclid (after Bramante - architect.



http://www.christusrex.org/www1/stanzas/Ad-Euclid.jpg

History

The renaissance produced a renewed interest in the work of the Greek geometers.

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History

This painting by Jacopo de Barbari was painted in 1495.

See the beautiful Rhombicuboctohedron.

http://www.math.nus.edu.sg/aslaksen/teaching/math-art-arch.shtml

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History

Dürer's *Melancholia* (1514) had mathematical and, in particular, geometric themes.

http://www.math.nus.edu.sg/aslaksen/teaching/math-art-arch.shtml

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History

Leonardo da Vinci produced the following drawings for a book by Luca Pacioli (1509).

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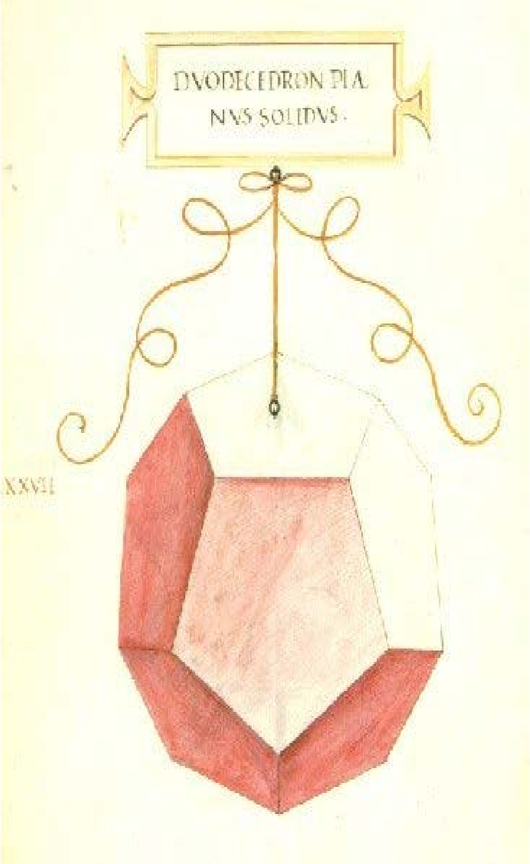
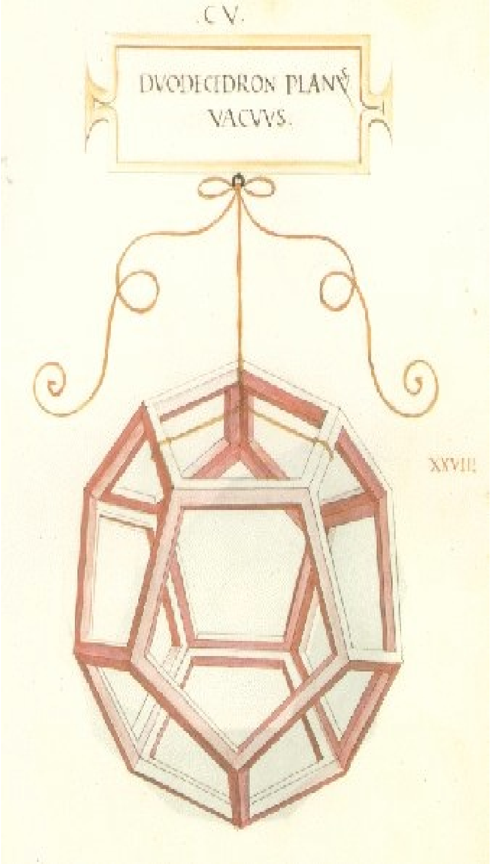
History





http://www.georgehart.com/virtual-polyhedra/leonardo.html

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History

This is a terrific website:

Art and Mathematics

http://www.math.nus.edu.sg/aslaksen/teaching/math-art-arch.shtml

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Language

Technical language is used in order to be precise and accurate in the description of spatial ideas

Technical terms should be defined carefully

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Language

Use of technical terms. Teachers should

have a clear understanding of these terms;

not underestimate students’ ability to recognise and use terms;

show discretion in using them with students;

supplement terms with informal but accurate language;

ensure geometry teaching does not degenerate into merely learning lists of technical terms

GEOMETRY

Language

Introducing...

Points

Lines

Planes

Angle

GEOMETRY

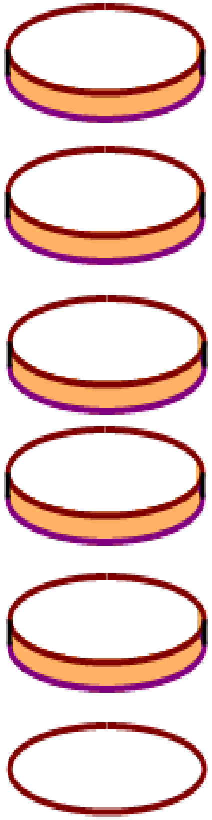
The difference between

Physical Objects

Diagrams

And

Geometric Concepts



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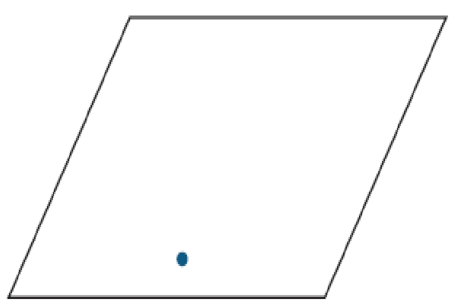
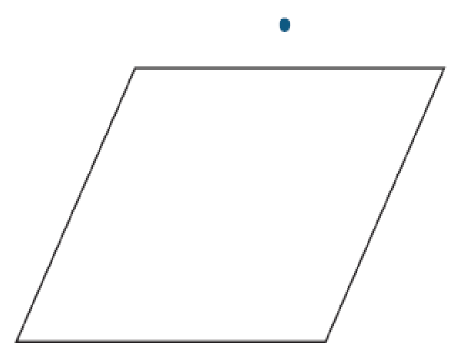
Points, lines and planes

Points

Given a point and a plane, there are two possibilities:





The point lies on the plane.

The point does not lie on the plane.

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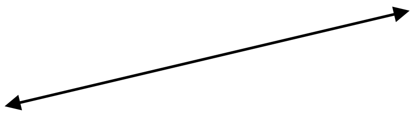
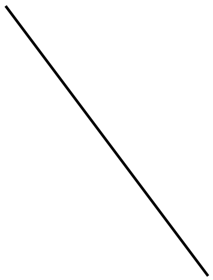
Points, lines and planes

- Lines go on forever



- When we say line in mathematics, we mean straight line not squiggles and curves
- When we draw a line like this

 it is really a line segment



GEOMETRY

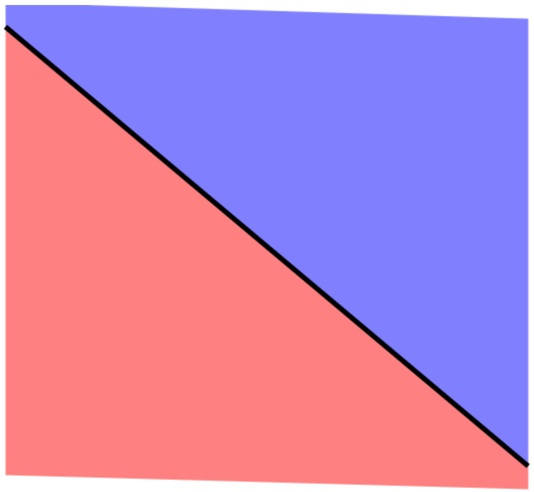
The difference between

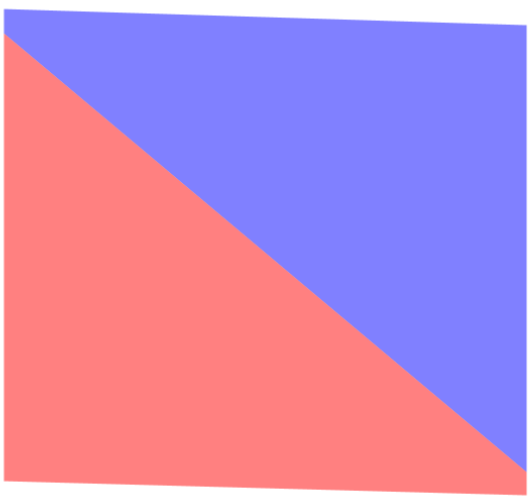
Physical Objects

Diagrams

And

Geometric Concepts





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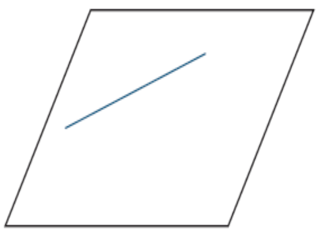
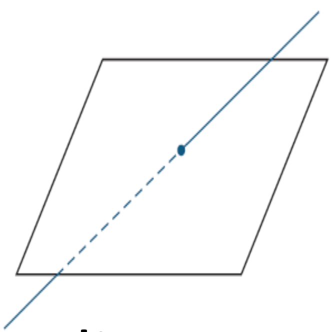
Points, lines and planes

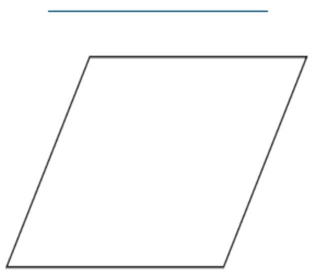
Given a line and a plane, there are three possibilities:



The line lies completely within the plane.



The line meets the plane at a single point.
We can think of the line passing through the plane.



The line never meets the plane, no matter how far each is extended.
We say they are parallel.

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Points, lines and planes

Parallel lines

- never meet
- go on forever

Students have an intuitive understanding of parallel

- Need to discuss on informal basis with students.

- Parallel implies corresponding angles equal and vice-versa.

Points, lines and planes

Perpendicular lines

- meet at 90º
- think of the letter T

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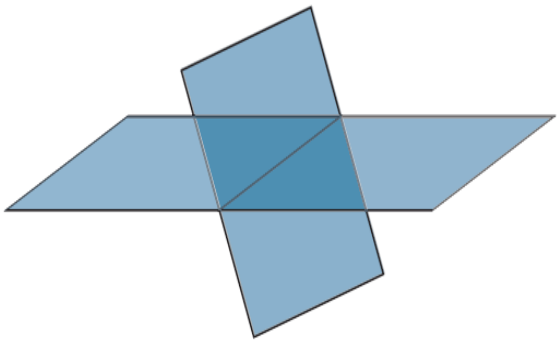
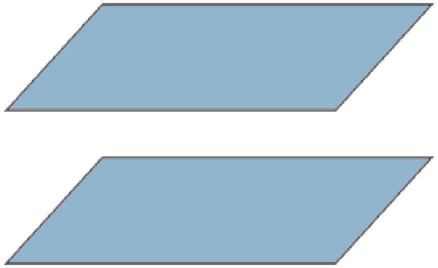
Points, lines and planes

Planes

Given two planes, there are two possibilities:





The two planes meet in a line.

The two planes never meet at all, no matter how far they are produced (extended).
We say they are parallel.

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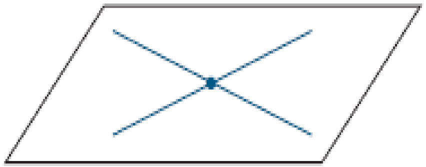
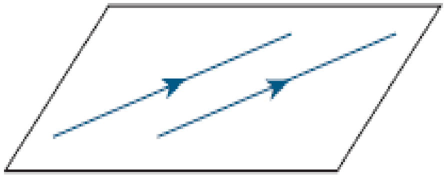
Points, lines and planes

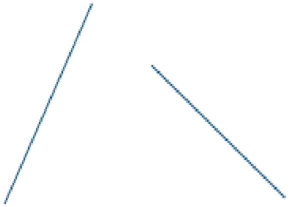
Given two lines in space, there are three possibilities:



The lines lie in a single plane and meet in a single point.



The lines lie in a single plane and are parallel.



The two lines do not lie in a single plane. They are skew lines.

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Angles

Angle is a measure of an amount of turn.

The amount of turning is called the size of the angle.

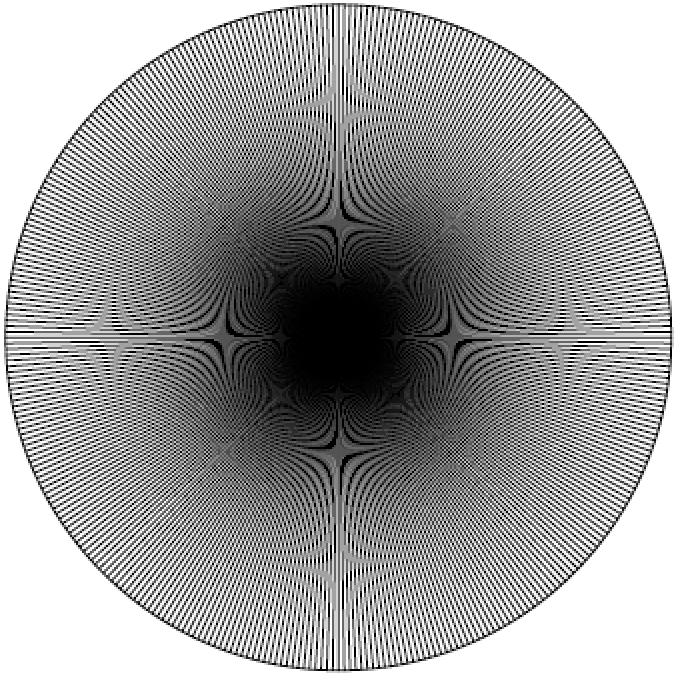
The size of the reflex angle corresponding to one full revolution was divided(by the Babylonians) into 360 equal parts, which we call degrees.

GEOMETRY

Angles

QP2

Why are there 360° in a circle?



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Angles

Go back almost 3000 years to Babylonia…

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Angles

…we call it Iraq.

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Angles

The Babylonians were astronomers.

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Angles

They wanted to know where each star would be tomorrow.

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Angles

They knew the year was about 365 days long.
If you wanted to calculate the length of the year, how would you go about it?

How long would it take you?

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Angles

The Babylonians did not use Hindu-Arabic notation and could not deal with fractions which were not of the form $\frac{1}{n}$.

Try to write $\frac{3}{7}$ in roman numerals to get an idea of the problem.

$365 = 5 \times 73$

So the factors of 365 are 1, 5, 73, and 365.

If you want to divide your cake into slices which are a whole number of degrees you don't have a lot of choice!

GEOMETRY

Angles

A simplifying compromise…

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

So the factors of 360 are
1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 10, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180 and 360.

They were willing to be out by 5 days in 365 in order to use a number which had lots of factors.

GEOMETRY

Angles

A 3000 year-old model of motion around a circle:

So, there are 360 degrees in a circle because there are 365 days in a year.

This model was useful and well-adapted to the needs of the scientists at the time.

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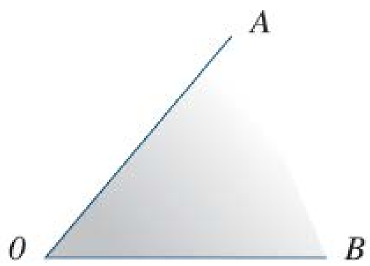
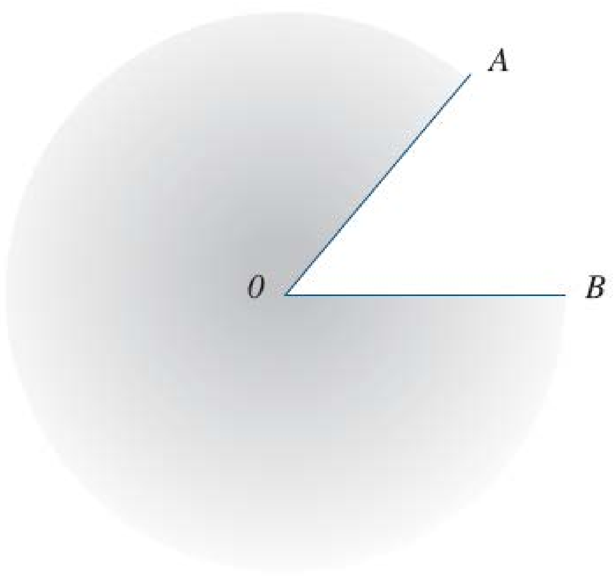
Angles

Defining an angle





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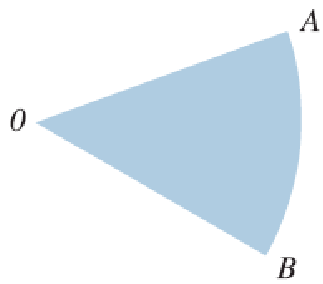
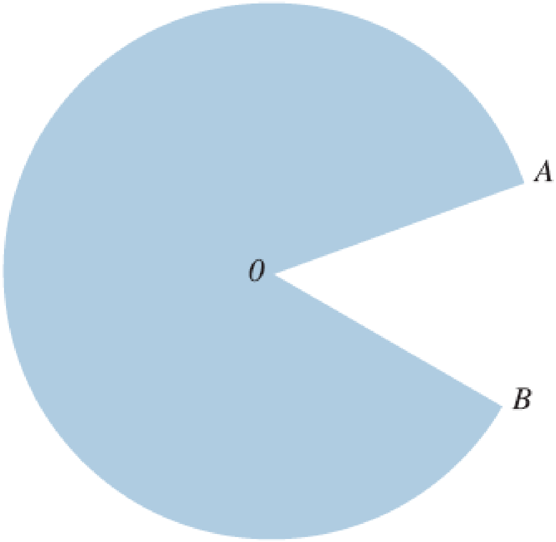
Angles

Defining an angle





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Angles

Measuring angles
Students need a great deal of practice using a pair of compasses and a protractor

★
QP2

Activity #2: Angle Measurement

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Angles

Kids in primary school learn about measuring angles before they understand irrational numbers. The most appropriate measure of angle to use at that time in their mathematical development is the degree.

Angles

When doing anything other than measuring (eg models of motion involving differentiation), radians behave much more neatly than degrees. When students learn calculus they need to be comfortable with radian measure of angles.

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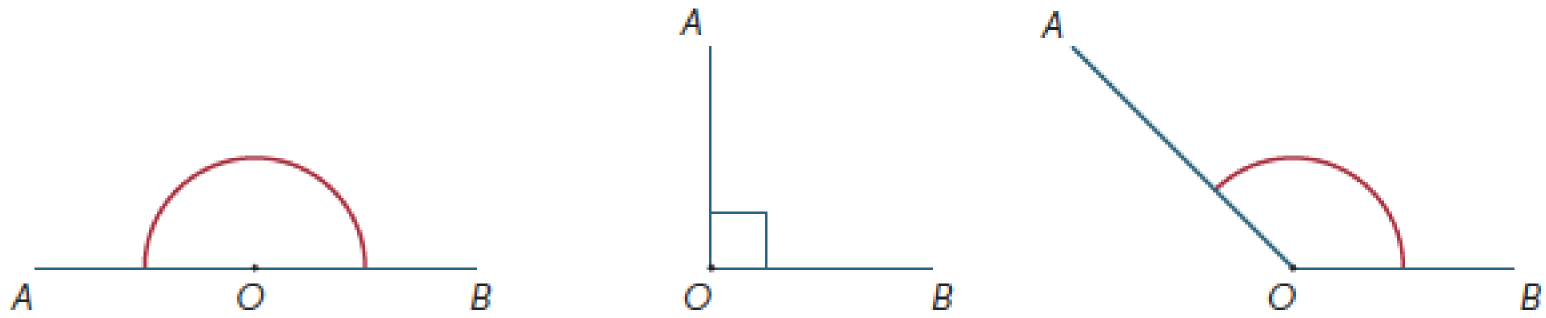
Angles

The size of a straight-angle is 180º and the size of a right-angle is 90º. Other angles can be measured (approximately) using a protractor.



Straight angle

Right angle

Obtuse angle

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Angles

- acute (less than 90^0)
- right angle (exactly 90^0)
- obtuse (between 90^0 and 180^0)
- straight (exactly 180^0)
- reflex (more than 180^0)

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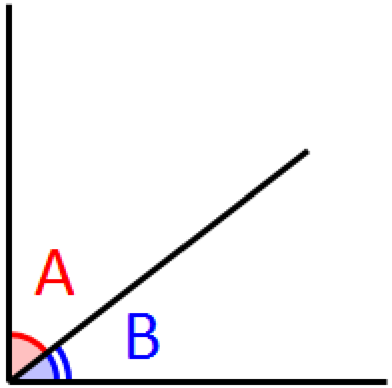
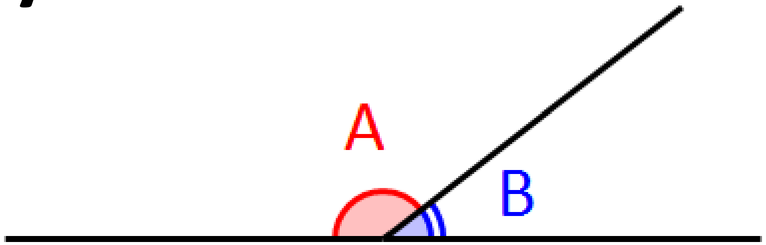
Angles

Two angles are **complementary** if their sum is 90˚



Two angles are **supplementary** if their sum is 180˚



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Pi

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Pi

The symbol π is used because it is the first letter of the Greek word $\pi\epsilon\rho\iota\mu\epsilon\tau\rho\omega\sigma$, meaning perimeter;

$$\pi = 3.14159265358979323 84 \ldots$$

Since radius = $\frac{1}{2}$ diameter,

$$\frac{\text{perimeter of circle}}{\text{radius of circle}} = 2\pi .$$

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Pi

The number we call π is irrational.

It can not be written as a fraction and its decimal expansion is infinite and does not recur.

That is why we use a symbol, π , to represent it.

There are no integers a, b such that $\frac{a}{b} = \pi$.

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Two dimensional shapes

QP3

Activity #3: Making and Classifying 2 D Shapes

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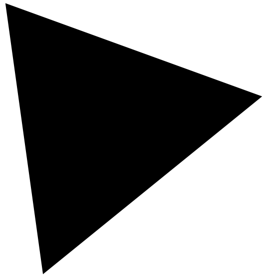
Types of triangles

Equilateral

All sides and angles equal (i.e., regular triangle)

Has 3 axes of mirror symmetry

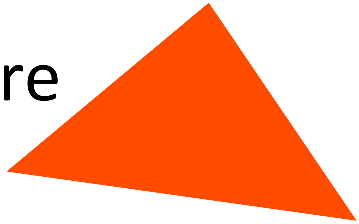


Isosceles

Iso/skeles = equal legs

Two sides equal and angles opposite those sides are also equal

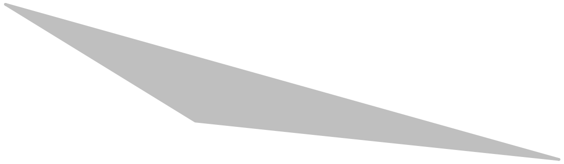
Has one axis of mirror symmetry



Scalene

No sides equal

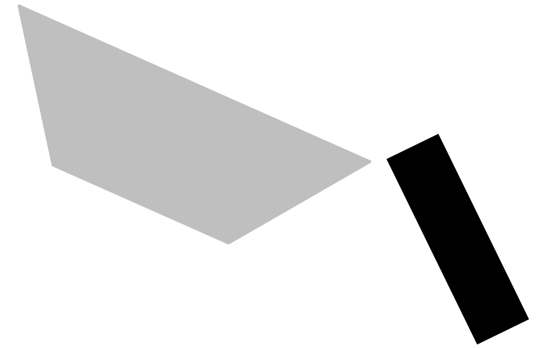
No axis of mirror symmetry



GEOMETRY

Types of quadrilaterals

“Quad” means “4”
quadrilateral is a figure with 4 sides



Parallelogram
Opposite sides parallel
(Derived property: opposite sides equal and opposite angles equal)

GEOMETRY

Types of quadrilaterals

Rectangle

Four right angles

(Derived property: diagonals are the same length – this is not true for non-rectangular parallelograms)

We only have to say “four right angles”

“Opposite sides equal” happens automatically

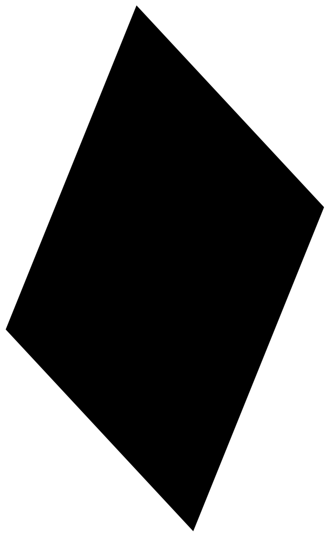
In fact, we need only say “three right angles!”

GEOMETRY

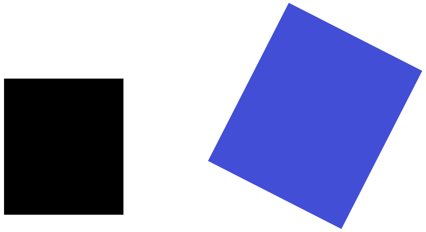
Types of quadrilaterals

Rhombus

Four equal sides



Square



Regular quadrilateral

Four sides equal and at least one angle a right angle

GEOMETRY

Types of quadrilaterals

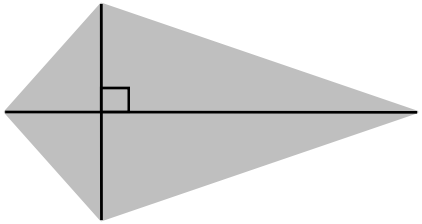
Trapezium

Only one pair of parallel sides

Kite

Two sets of two equal sides, with equal sides adjacent

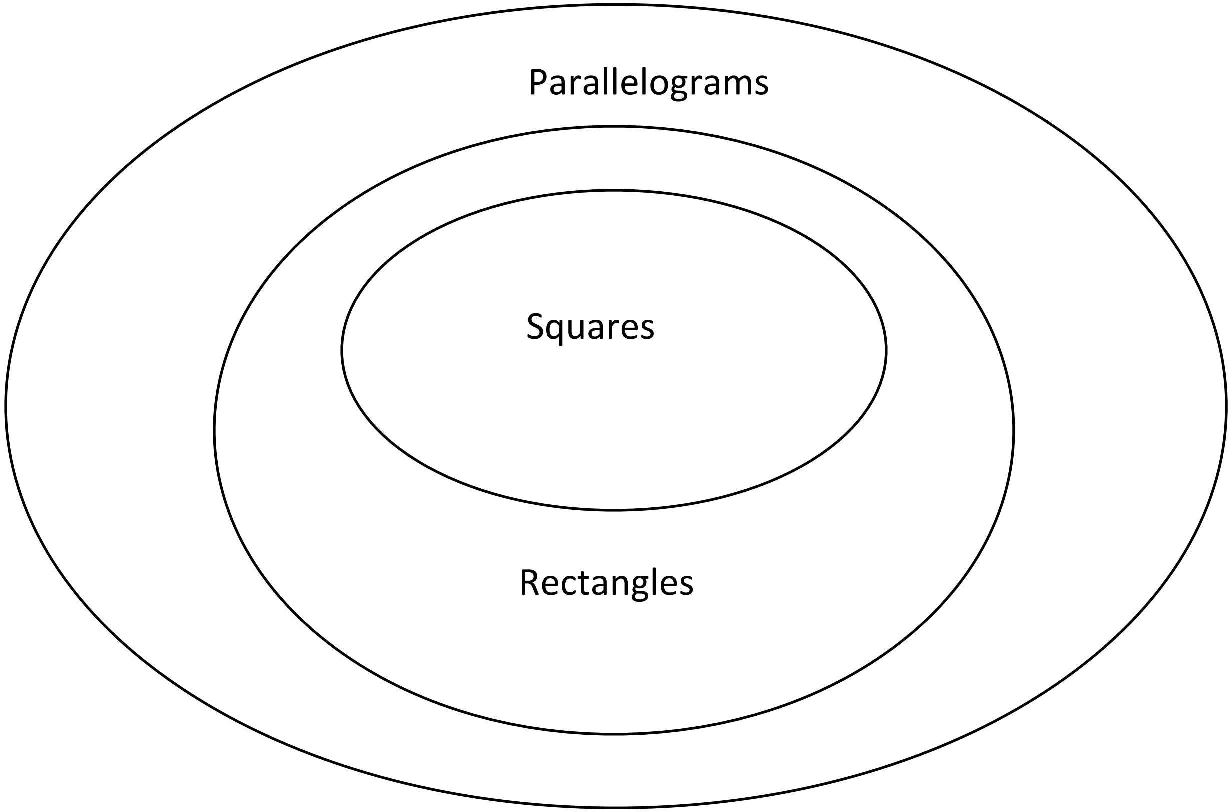
Derived property: diagonals are perpendicular



GEOMETRY

Types of quadrilaterals

Use a Venn diagram to show the relationships between the types of quadrilaterals



GEOMETRY

Polygons

A **polygon** is

a closed, plane figure (that is - 2D) consisting only of straight line segments (sides)

“Many-sided”- though poly/gon means many/angled (“gon” = knee))

Classified according to the number of sides (as before)

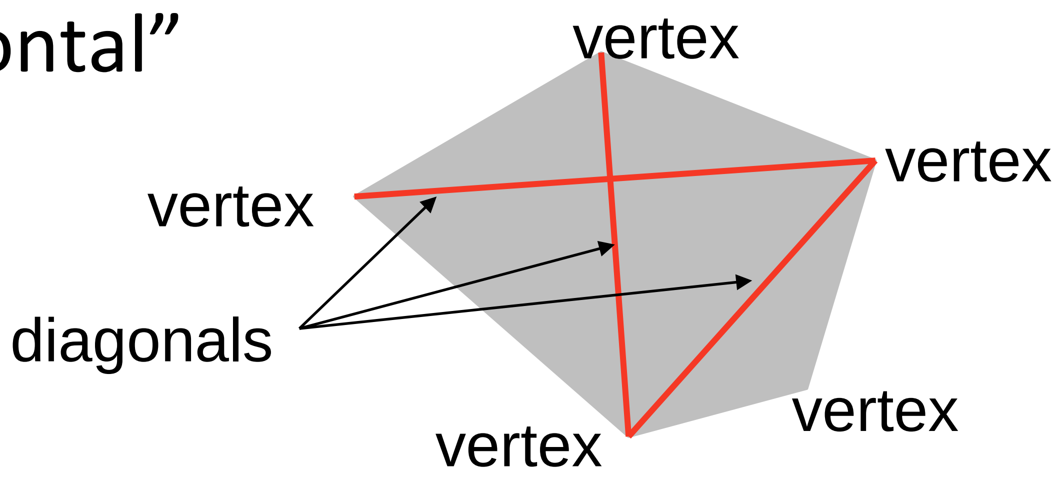
GEOMETRY

Polygon features

The corners are called vertices (1 vertex, 2 or more vertices)

A line joining one corner to a non-adjacent corner is a diagonal

Note: Don’t say “diagonal line” when you mean “not vertical nor horizontal”



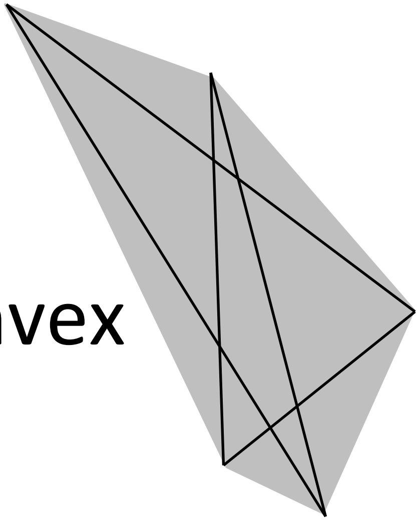
GEOMETRY

Polygon features

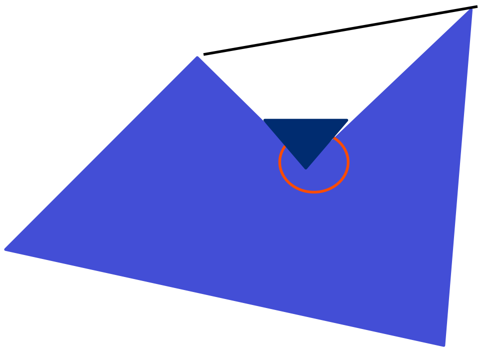
Polygons can be concave or convex

All the diagonals of a convex polygon lie inside it

A concave polygon has at least one exterior diagonal and one interior reflex angle



convex



concave

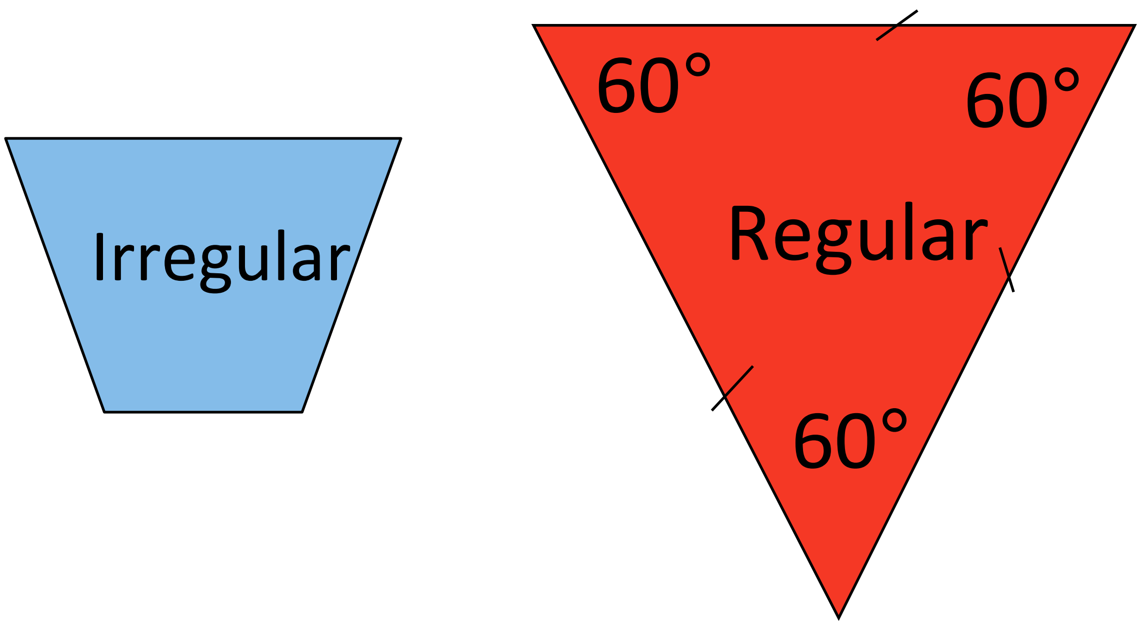
GEOMETRY

Regular polygons

A regular polygon has

all angles equal AND

all sides equal



GEOMETRY

Regular polygons

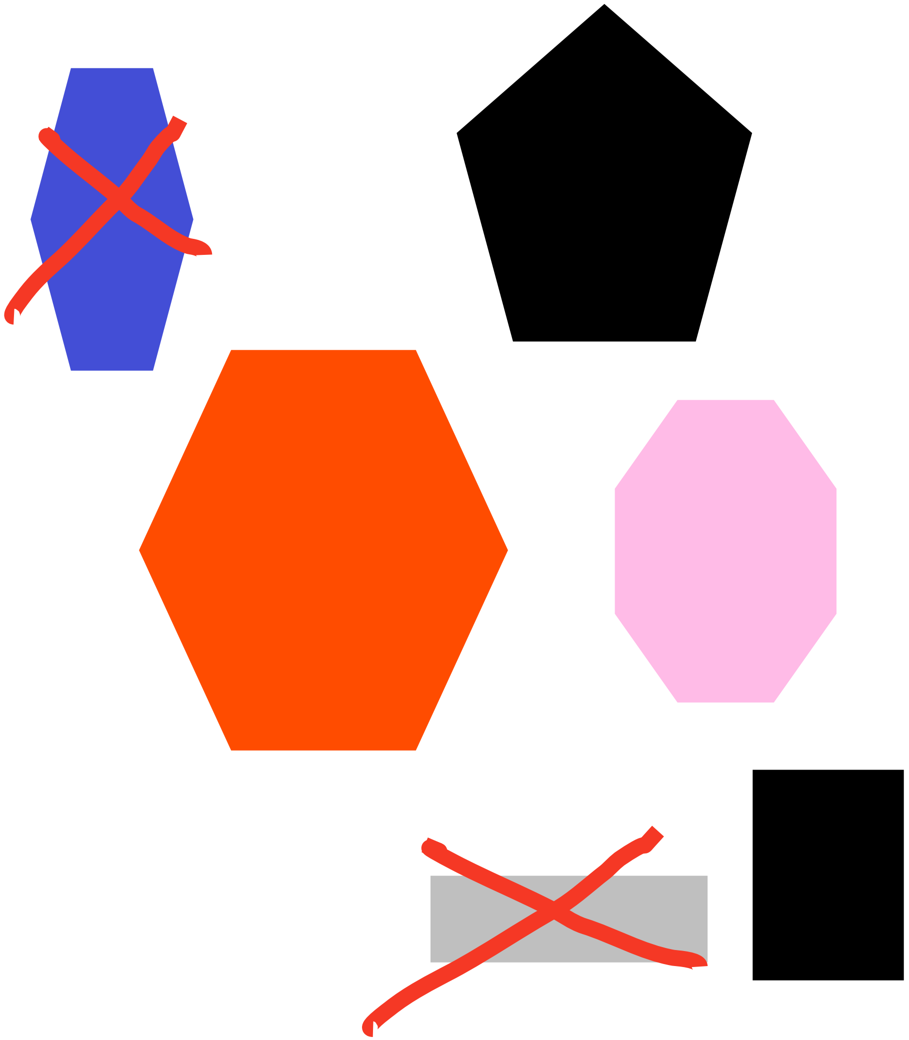
Equilateral triangle

Square

Regular pentagon

Regular hexagon

Regular octagon …

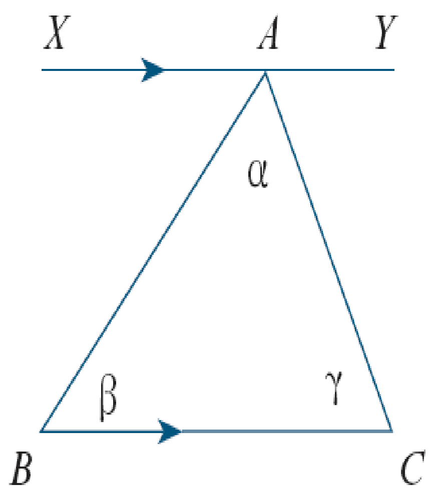


GEOMETRY

Triangles

We can prove the angle sum of a triangle is 180^o.

(The first real theorem)



We can apply this to the angle sum of a quadrilateral...

GEOMETRY

Angle sum

Number of sides	Name	Number of triangles	Angle sum
3			
4			
5			
6			
7			
8			
9			
10			
n			

GEOMETRY

Angle sum

Number of sides	Name	Number of triangles	Angle sum
3	Triangle		
4	Quadrilateral		
5	Pentagon		
6	Hexagon		
7	Heptagon		
8	Octagon		
9	Nonagon		
10	Decagon		
n	Polygon		

GEOMETRY

Angle sum

Number of sides	Name	Number of triangles	Angle sum
3	Triangle	1	
4	Quadrilateral	2	
5	Pentagon	3	
6	Hexagon	4	
7	Heptagon	5	
8	Octagon	6	
9	Nonagon	7	
10	Decagon	8	
n	Polygon	n-2	

GEOMETRY

Angle sum

Number of sides	Name	Number of triangles	Angle sum
3	Triangle	1	$1 \times 180^0 = 180^0$
4	Quadrilateral	2	$2 \times 180^0 = 360^0$
5	Pentagon	3	$3 \times 180^0 = 540^0$
6	Hexagon	4	$4 \times 180^0 = 720^0$
7	Heptagon	5	$5 \times 180^0 = 900^0$
8	Octagon	6	$6 \times 180^0 = 1080^0$
9	Nonagon	7	$7 \times 180^0 = 1260^0$
10	Decagon	8	$8 \times 180^0 = 1440^0$
n	Polygon	n-2	$(n-2) \times 180^0$

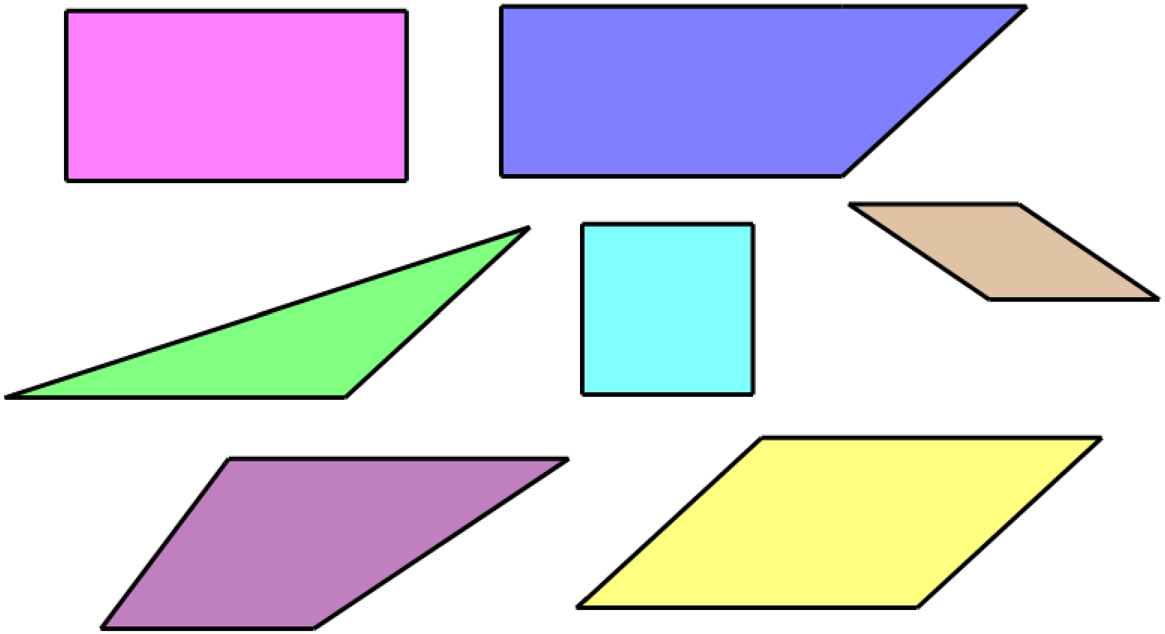
GEOMETRY

Area formulas and Reasoning

Family Tree

What comes first?
Second?
Third?
Fourth?
…

Why?



GEOMETRY

Area formulas and Reasoning

The area of a triangle is half of the area of the rectangle with the same dimensions.

Where is the rectangle this triangle “came” from?

What are it’s dimensions?

QP4

6 Card Puzzle: The Length of AB

GEOMETRY

Theory of learning

GEOMETRY

The Van Hiele Theory

Developed jointly by Pierre van Hiele and Dina van Hiele-Geldof in the 1950s

Used as a framework for teaching geometry and for considering children’s levels of understanding

GEOMETRY

The Van Hiele Theory

Level 1: Visualisation/Recognition

Level 2: Analysis

Level 3: Abstraction

Level 4: Deduction

Level 5: Rigour

If you teach at one level, those at lower levels can't work at it. For example, give Level 1 examples as well as Level 2 when working at Level 2.

GEOMETRY

Level 1: Visualisation (Recognition)

Figures are identified according to their overall appearance. That is, by their shape, not by their properties. Language is imprecise. Reasoning is dominated by perception. Visual prototypes used to identify figures

Parallel lines "like a door",

Cube: "like a box, dice"

Angle: "pointy triangle shape"

GEOMETRY

Level 2: Analysis

Figures identified in terms of their properties

- Figures still seen as wholes, but as a collection of properties rather than as typical shapes

- Properties are independent

Recognises that properties imply certain figures, but does not understand that one property may imply another

- a figure is not a rectangle because it is a square

Students' Level 2 Descriptions

"A square has four even sides" (lowest category)

"A square is a four-sided figure with all sides equal and all angles 90°"

"A rectangle is a four-sided figure with four right-angles and two pairs of parallel sides. The top and bottom are the same but different from the other two"

"Parallel lines are two lines the same distance apart that go on forever and never meet" (Pegg, 1995)

Level 3: Abstraction

Relationships between properties of a figure are established

Properties can be used to classify figures hierarchically

a square is a rhombus because it has the properties of a rhombus (plus some extra properties)

Understanding of class inclusion

GEOMETRY

Level 3: Abstraction

Student language is more sophisticated

E.g., “a square is a rectangle with all sides equal”, compared with:

a square is a sort of rectangle (level 2)

a square is like a rectangle but all sides are equal (level 2)

Students are able to discuss this rationally (i.e., with reasons)

GEOMETRY

Level 4: Deduction

The role of deduction is understood. Necessary and sufficient conditions are understood and ‘minimum property’ definitions can be given.

- a square is a rectangle with a pair of adjacent sides equal.

- A rectangle is a parallelogram with an angle of 90º.

The need for rote learning is minimised.

Proofs of theorems can be constructed.

GEOMETRY

Level 5: Rigour

Geometry becomes an abstract study based on systems of postulates/axioms

Students challenge the deductions of Euclidean geometry and look at alternate geometries

E.g., Projective geometry: parallel lines meet at infinity.

Can accept results contrary to everyday experience if the proof is mathematically valid

Not normally expected of school students

GEOMETRY

Attainment of Levels 1–3

Age parameters are not usually associated with van Hiele levels

Students can have attained Level 1 by age 5 or 6, but may still be at this level at age 14-15

Most children starting school will be at Level 1 and should be able to attain Level 2 by the end of primary school

Very few primary school children reach Level 3

GEOMETRY

Attainment of Levels 3 & 4

Level 3 should be attained by Year 10

indicates a reasonable grasp of geometry for everyday purposes

Level 4 is an appropriate goal for the end of secondary school (reasonable upper bound)

Only about 25% of 18 year-olds function at Level 4 (Pegg & Faithful, 1993)

GEOMETRY

Children's Misconceptions

Many misconceptions about space are 'learned misconceptions'

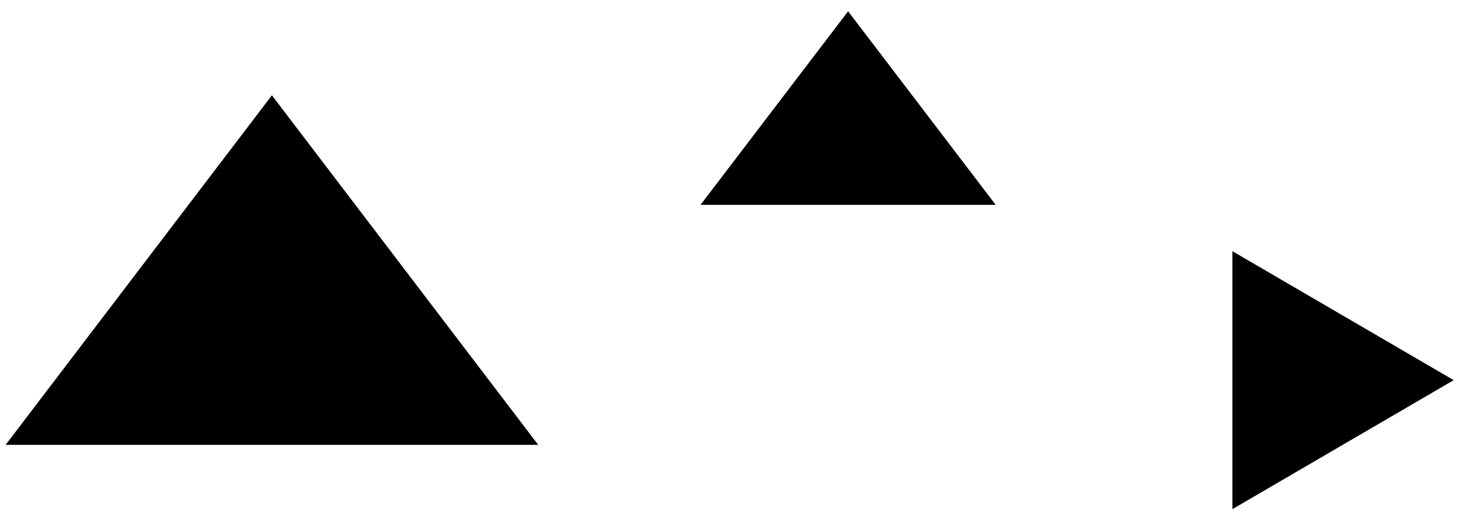
Children focus on the wrong characteristics and develop limited or false concepts

Geometrical figures are often presented in standard orientations making it difficult for children to generalise these concepts (i.e., it is a flaw in teaching).

GEOMETRY

Learned Misconceptions 1

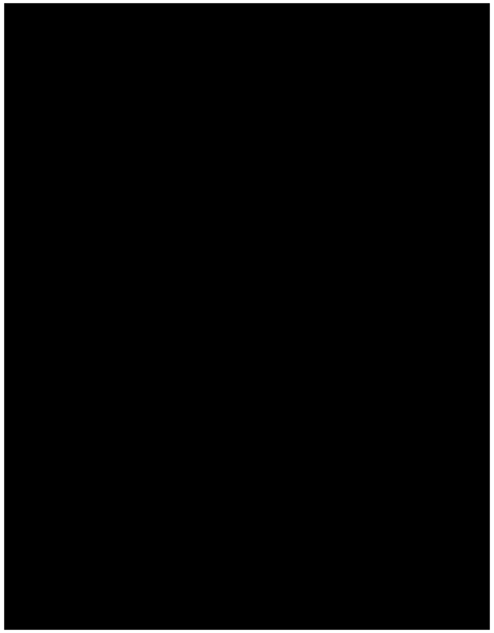
'It's not a triangle because it has fallen over'



GEOMETRY

Learned Misconceptions 2

‘Rectangles lie down’



GEOMETRY

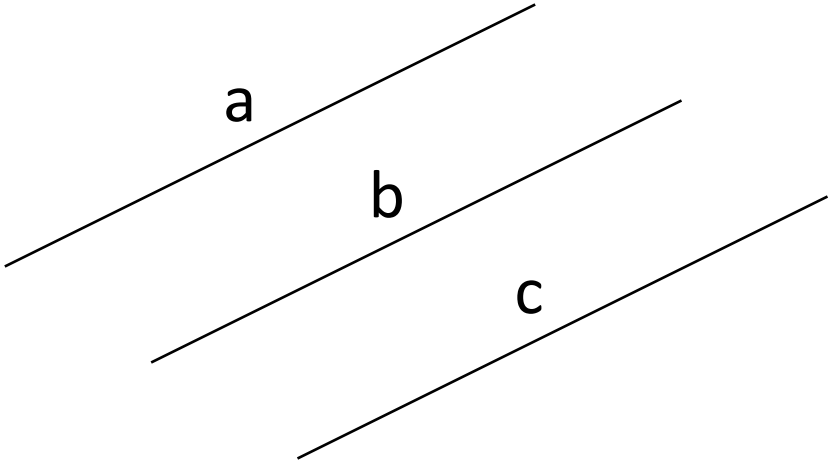
Learned Misconceptions 3

‘It’s too thin to be a rectangle. Rectangles are about twice the size of a square’

GEOMETRY

Learned Misconceptions 4

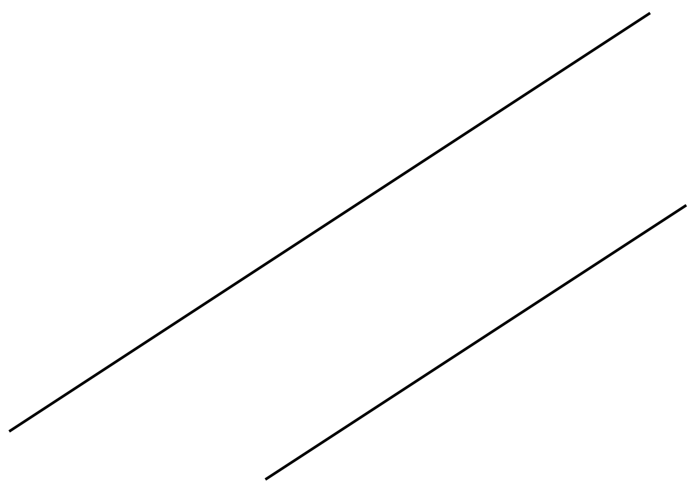
‘a is not parallel to c because b is in the way’



GEOMETRY

Learned Misconceptions 5

‘But parallel lines have the same length!’

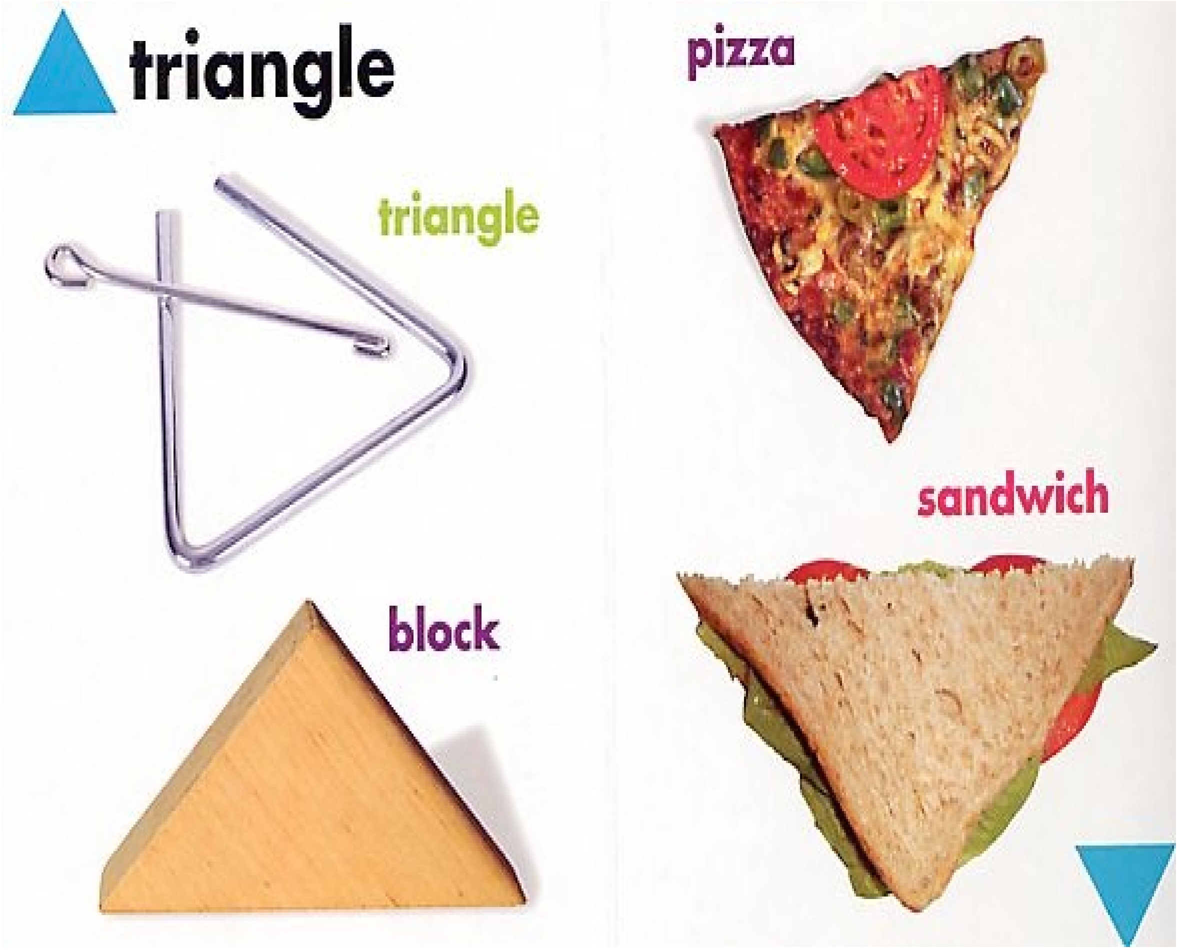


Avoiding Learned Misconceptions

A systematic attempt to avoid the misconceptions that a teacher knows may arise can be very effective

GEOMETRY

Things to watch for … (I)

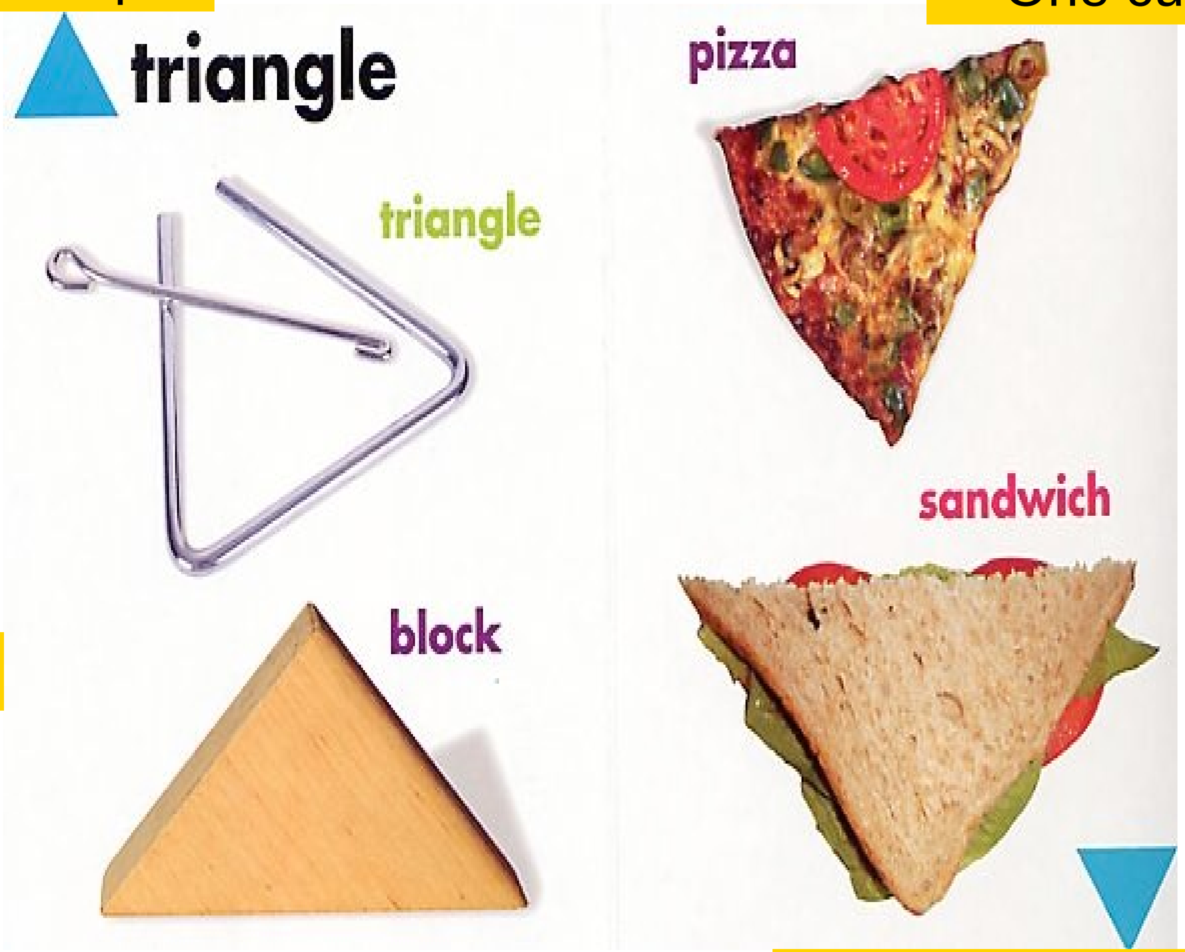


GEOMETRY

Things to watch for … (I)

Not “closed” shape

One curved side

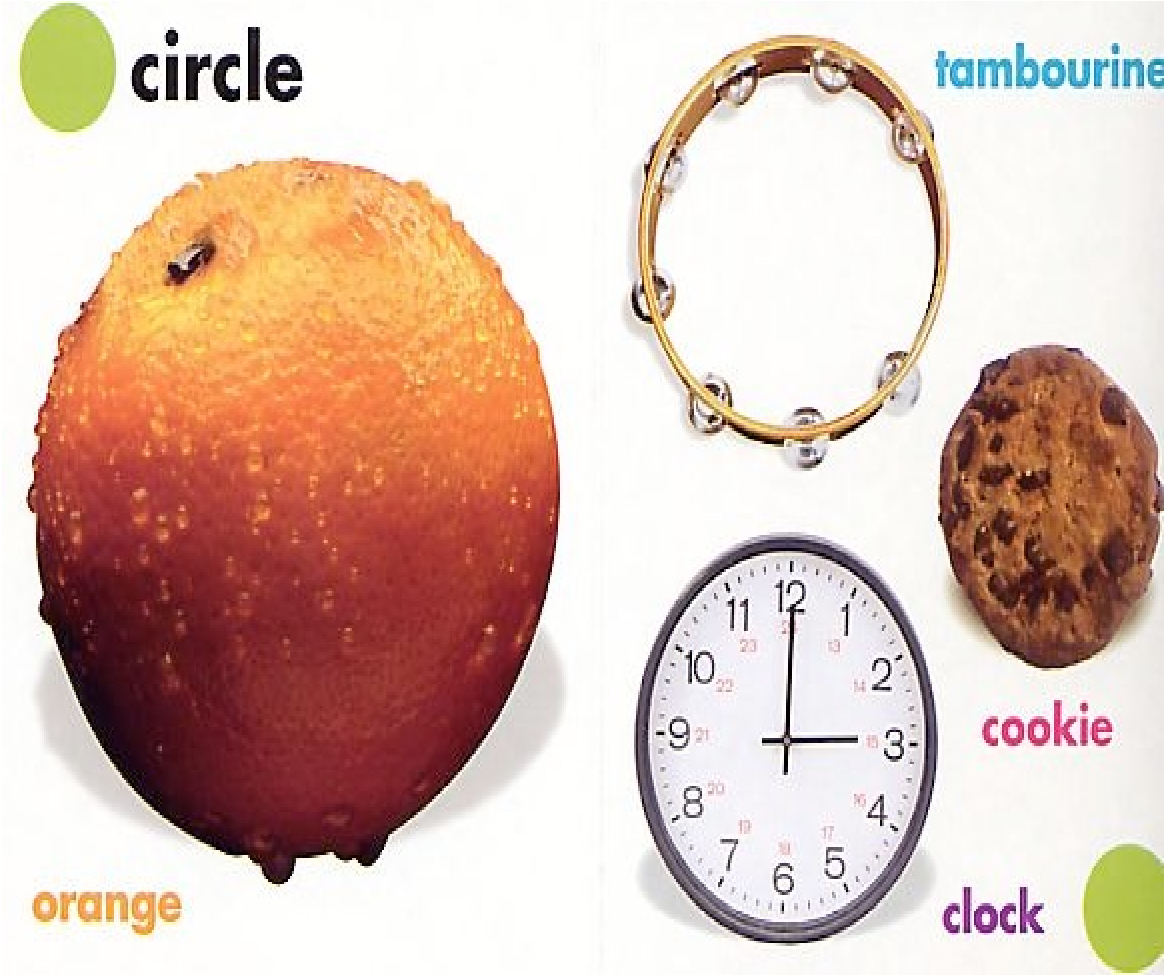


3D object

Sides not straight

GEOMETRY

Things to watch for … (II)



GEOMETRY

Things to watch for ... (II)

All 3D objects

Focus on angles here?

GEOMETRY

Making connections

We thought…
We said…
We wrote…
We saw…
We heard…
We know…
We drew…
We said…
We asked…
We felt…
We liked…
We learnt…

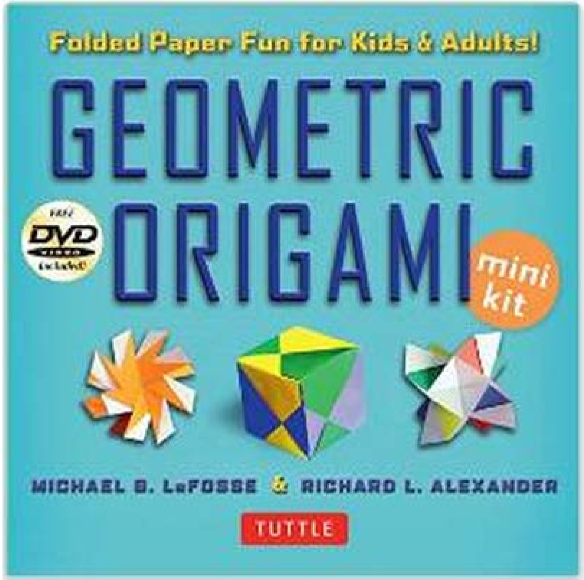
We didn't like…
We found out…
We already knew…
We remembered…
We used equipment…
We need to find out…
It was interesting when…
The tricky bit was…
We didn't know that…

It was cool when…
Congratulations to…
We discovered…
Our group worked well when…
A new word we learnt was…
The strategy we used was…
The important thing to remember is…

GEOMETRY

Origami

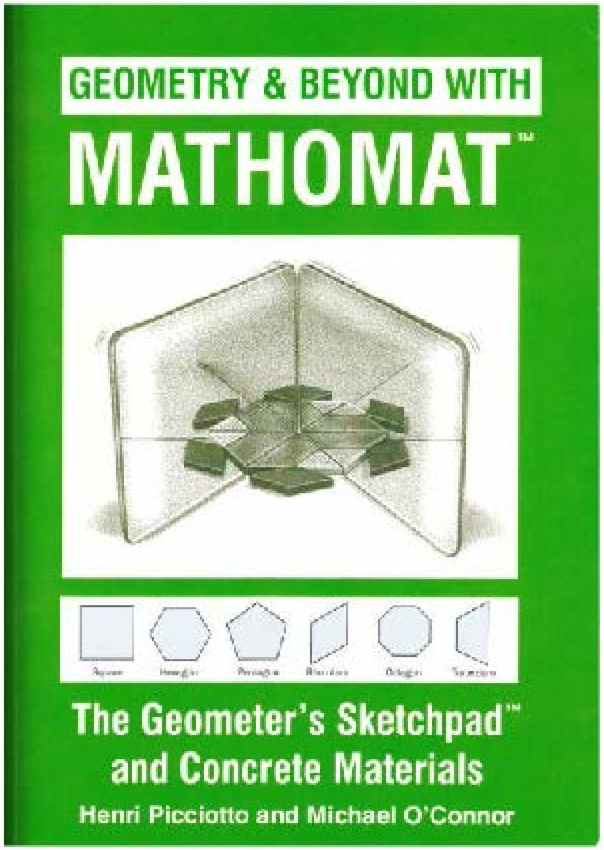
The instructions for the Harlequin Cube used in this session were taken from the Tuttle Geometric Origami mini-kit. There are many origami books and websites available to choose from. The focus should always be on the mathematics that can be demonstrated or extracted from the activity.



GEOMETRY

Some suggested authors

The Angle Measurement activity used in this presentation was taken from “Geometry and Beyond with Mathomat” published by OLM.

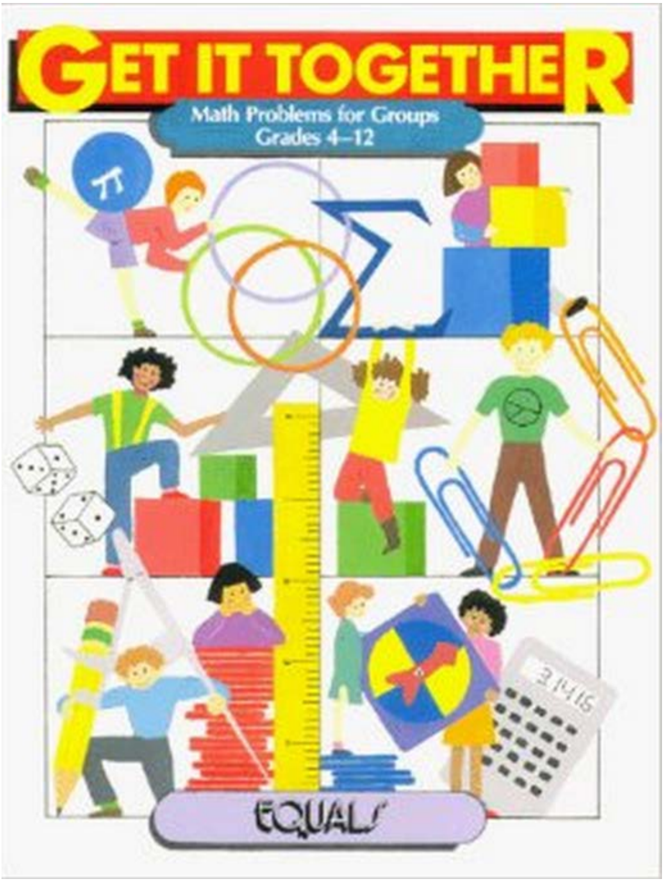


GEOMETRY

Reasoning

The “Get It Together” text has over 100 activities for groups. They focus on reasoning and problem solving skills where the necessary information is broken up into a series of clues that students use to piece the solution together.

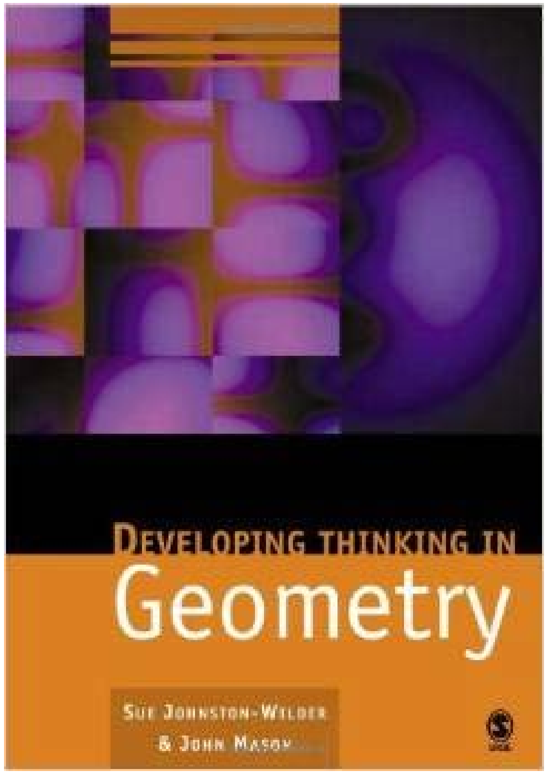
The “Length of AB” activity used in this session can be found on page 127.



GEOMETRY

Some suggested authors

The Developing Thinking series edited by Johnston-Wilder and Mason explores the development of mathematical understanding in three volumes each dedicated to one of the following areas of mathematics: Algebra, Geometry and Statistics



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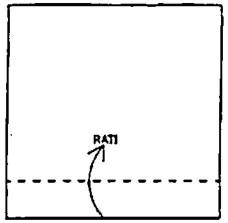
List of Activities and Sources

Activities used during this presentation were:

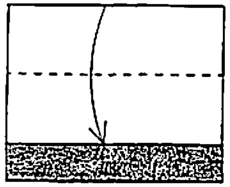
The Harlequin Cube: Geometric Origami Mini Kit, Tuttle Publishing
http://www.tuttlepublishing.com/origami-crafts/geometric-origami-mini-kit-book-and-kit-with-dvd

Angle Measurement: Geometry and Beyond with Mathomat, OLM
http://www.mathomat.com.au/teaching-maths-lesson-plans-geometry.php

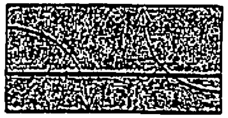
The Length of AB: Get it Together, Equals Publishing
http://store.lawrencehallofscience.org/Item/get-it-together



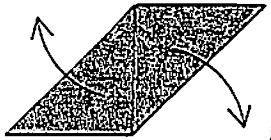
1. You will need six congruent squares to make one cube. It will be best to use two each of three different colors, for effect and ease of following these instructions. Valley-fold the bottom edge up, "RAT" (Right About There). Practically any level you choose will work!



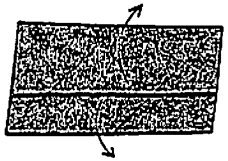
2. Valley-fold the opposite edge to meet the first.



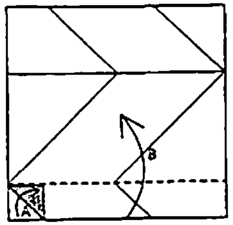
3. Valley-fold the right edge up to the top edge and the left edge down to the bottom edge.



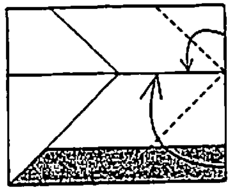
4. Your paper should look like this. Unfold the triangular flaps.



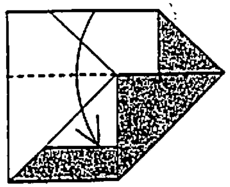
5. Unfold completely.



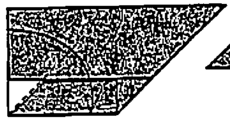
6. You will now use the crease pattern to sequentially weave the square closed. (A) Fold the bottom left corner over. (B) Fold up the bottom flap.



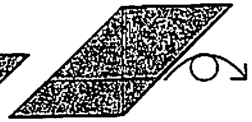
7. Fold up the bottom right corner. Fold down the top right corner.



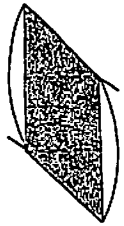
8. Fold down the top flap.



9. Fold the top left corner down and tuck it behind the bottom flap.

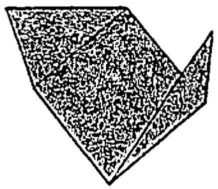


10. Turn over.

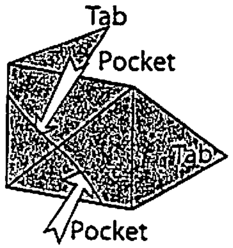


11. Valley-fold the acute corners to the obtuse corners to form a square tile.

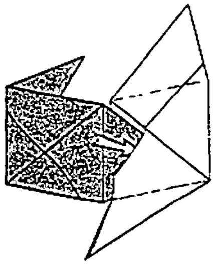
12. Move the triangular flaps away from the square center.



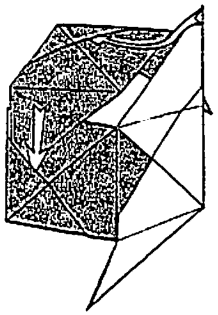
13. Set the triangle flaps at right angles to the square center. You will need six units.



14. Each unit has two tabs and four pockets. Only two of the pockets—indicated—are to be used during assembly.

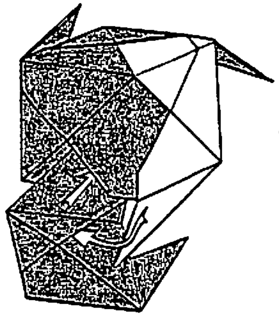


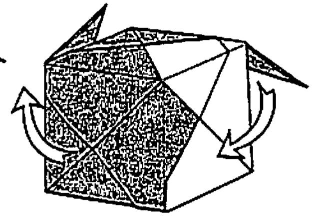
15. Two units are joined, perpendicular to each other, with the tab of the horizontal unit entering a pocket on the vertical unit.



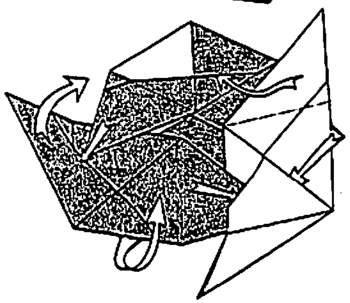
16. Complete a cube corner by adding a third unit to the top. Notice that the tab of the top unit enters a pocket on the horizontal unit and a tab from the vertical unit is tucked into a pocket on the top unit.

17. In this diagram, the purple is at the top, so a purple unit is added on the opposite side, at the bottom.

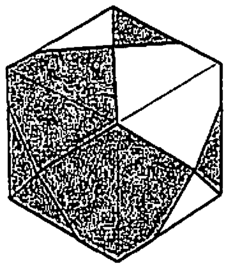




18. Rotate the cube to the open side.



19. Add the final pieces, arranging the colors opposite each other and tucking in all flaps.



20. The completed Harlequin Cube.

GEOMETRIC REASONING LESSON

2

Angle Measurement

A.C. Level:	5
A.C. Ref No's:	Estimates, measures and compares angles using degrees. Constructs angles using both a 180° and 360° protractor. ACMMG112
A.C. Substrands	Geometric Reasoning

Outcome

At the end of the activity students will know,

- That angles are a measure of turn
- That only some types of quadrilaterals will neatly fill around a point

Students will be able to

- Use protractors to measure angle sizes in degrees

Materials Required

Mathomat

1. What are the measures of the angles that share a vertex at the centre of

a. A Chrysler symbol?

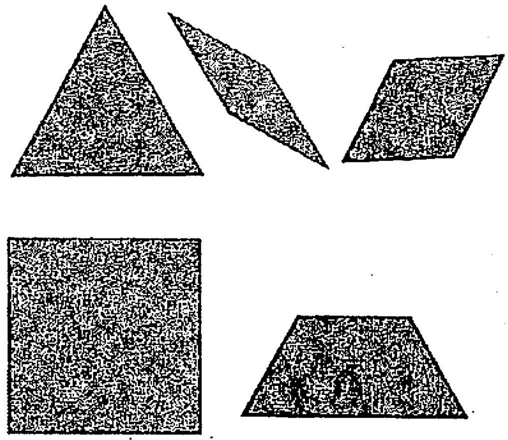
b. A Mercedes symbol?

c. A peace sign?

d. A clock, between consecutive hours?

e. A cross?

2. Find the measures of all the angles for each of the polygons shown below.
Write the angle measures in the shapes.



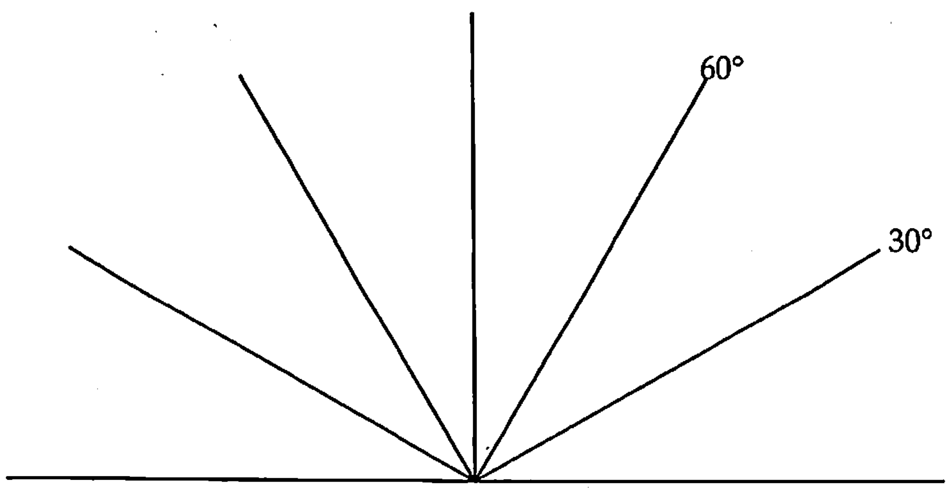
3. One way to measure angles is to place smaller angles inside larger ones. For example, six copies of the small angle on the thin rhombus fit inside the figure below.

This figure, called a protractor, can be used to measure all pattern block angles.

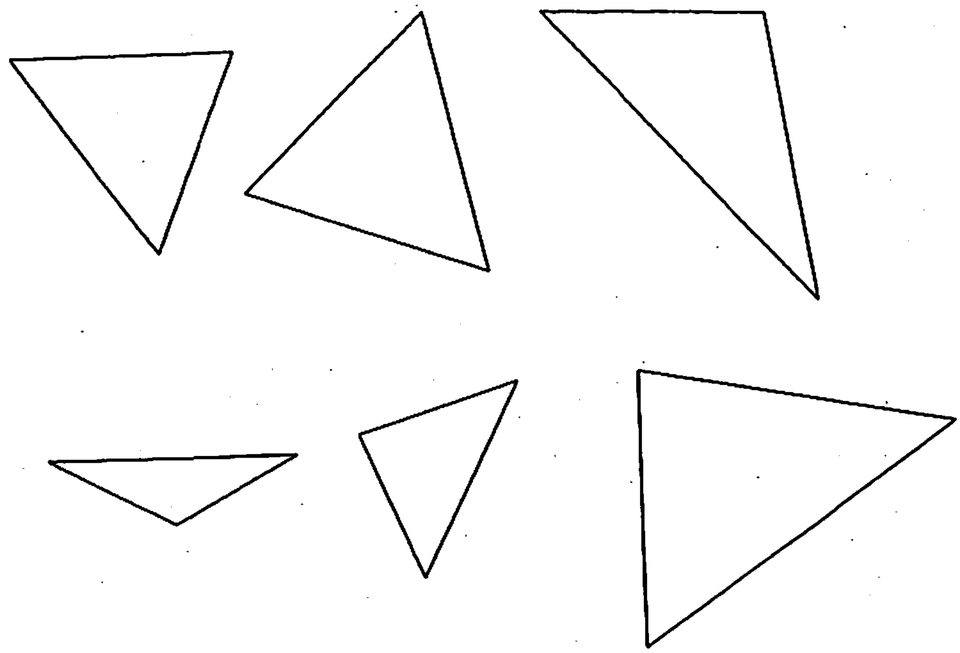
a. Mark the rest of the lines in the figure with numbers.

b. Use it to check the each of the angles on the given polygons.

c. Using the polygons, add the 15° lines between the 30° lines shown in the protractor.



4. Use the protractor on your Mathomat to measure the angles on the triangles below. For each one, add up the angles. Write the angle sum inside each triangle.



5. Use the protractor on your Mathomat to draw the following angles in the space below.

a. 20°

b. 50°

c. 100°

Discussion

Questions and key points

Which of the parts in question 1 did you find most difficult to answer?

What made the others easier?

Where to from here?

The world we live in is made up of all types of different shapes. In designing and building everything the objects we use everyday, artists, designers, builders and engineers have to measure and construct angles all the time.

Name, draw and measure the angles of other shapes you can see around you.

Making and Classifying 2 D Shapes

Using a sheet of A4 Paper:

Make	Describe (Understanding)	Relate to (Reasoning)
1) a square		
2) a Triangle		
3) a Trapezium		
4) a Parallelogram		
5) a Kite		
6) a Rhombus		

Constructions

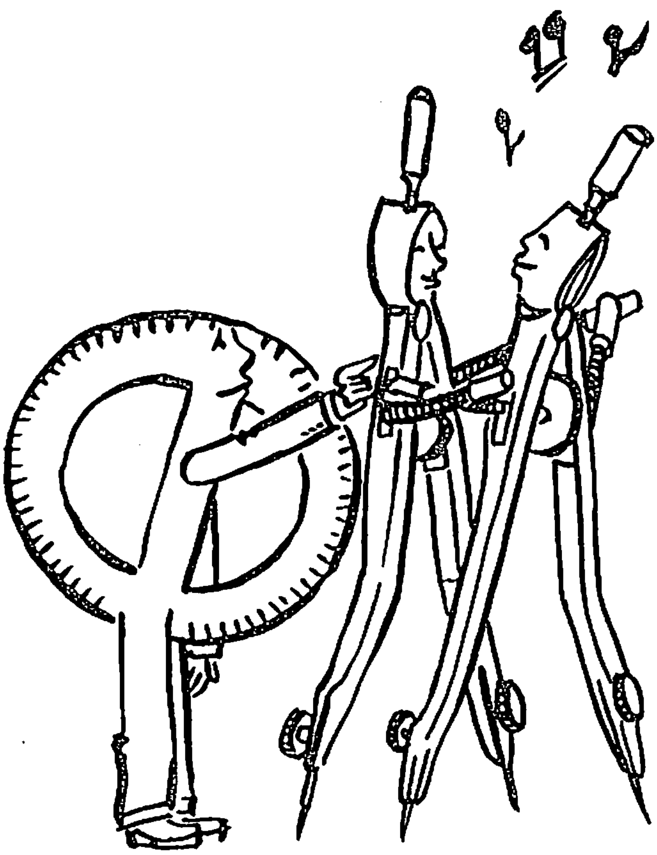
Concept Areas

Geometry. Formal constructions using the language of angles, congruence, similarity, and so forth. The Pythagorean theorem and special triangles.

For Each Group:

- Paper and pencil
- Compass
- Straightedge
- Protractors and calculators

The Geometric Supposer (computer software) is an intriguing alternative.



Description

These problems are most appropriate for high-school geometry, but they are accessible to junior high students with adequate preparation. Each problem asks the group to find some measurement in a geometrical construction. Each clue helps the group complete the drawing from which the answer must be taken. A typical clue might be, "line segment AB is tangent to circle C at its midpoint."

For easier geometry problems, see *Stick Figures* (page 50) or *Polygons* (page 62).

Geometry frightens many of us away from mathematics forever. Here, the group setting is the key. These problems give students a chance to use mathematical language with a purpose. If a student doesn't know what "tangent at the midpoint" means, the group will puzzle it out together. Students don't have to feel stupid, and because they are all working to solve the problem—and because they get to see it in a picture when they're done—it's more likely to stick.

Other Comments

Some students continue with their original drawing rather than redraw it, even when they decide on measurements for segments that make their construction terribly out of scale. For example, an isosceles triangle might turn out on the drawing to be radically scalene. In spite of that, students arrive at correct answers and don't seem to be bothered by a drawing that can't possibly represent what they say it does. So a good *individual* assignment at the end of one of these problems might be to make a good drawin[g] of the figure that led to the solution.

Possible Debriefing Questions

What does the figure look like, really? Have different groups draw it on the board. Compare and contrast.

Are there different diagrams that are both right?

Which clues were most useful in these problems

The Length of $\overline{AB}$

ΔABC is a right triangle (though $\angle ABC$ may not be a right angle).

Your job, as a group, is to figure out the length of segment $\overline{AB}$.

The Length of $\overline{AB}$

DCAB is a rectangle.

$\angle DCB$ is the same size as $\angle CBA$.

The Length of $\overline{AB}$

Line segment $\overline{BD}$ is four inches long.

$m\angle DCB + m\angle BCA = 90°$.

"$m\angle DCB$" means, "the measure of angle DCB," that is, its size in degrees.

The Length of $\overline{AB}$

The length of $\overline{BC}$ is five inches.

$\angle BAC$ is a right angle.

Draw a picture to help your group solve the problem.

The Length of $\overline{AB}$

The perimeter of ΔBDC is twelve inches.

Your job, as a group, is to figure out the length of segment $\overline{AB}$.

The Length of $\overline{AB}$

The area of ΔABC is six square inches.

$\overline{AB} \parallel \overline{CD}$.

GET IT TOGETHER

Math Problems for Groups
Grades 4–12