Problem Solving: In the Classroom

- Michael O'Connor: moconnor@amsi.org.au

Problem Solving

When to do it

Problem Solving, like the other three proficiencies, has been

> *"incorporated into the content descriptions … to ensure that the skills develop throughout the curriculum and become increasingly sophisticated over the years of schooling."*

http://www.australiancurriculum.edu.au/mathematics/content-structure

Problem Solving

Activity 1: What is my pattern?

Example 1

3

7

11

15

Rule: Add 4 to the previous number

Problem Solving

Activity 1: What is my pattern?

Example 2

1

3

7

15

Rule: 2 times the previous value plus 1

Problem Solving

Activity 1: What is my pattern?

Example 3

3

8

15

24

Rule: $x^2 + 2x$, where x starts at 1

Problem Solving

When to do it

Explicit references to the proficiencies are indicated by the verbs used in content descriptors. The AMSI Vertical View spreadsheet provides a guide for teachers to use.

	A	B	C	U	V
1	Ye	Strand & SubStran	Year Level	Primary Proficiency(ies)	Secondary Proficiency
2	F	**NUMBER AND ALGEBRA**	**Foundation**		
3	F	Number and Place Value	Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point(ACMNA001)	Understanding	
4	F	Number and Place Value	Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond (ACMNA002)	Understanding	
5	F	Number and Place Value	Subitise small collections of objects (ACMNA003)	Fluency	
6	F	Number and Place Value	Compare, order and make correspondences between collections, initially to 20, and explain reasoning (ACMNA289)	Fluency	
7	F	Number and Place Value	Represent practical situations to model addition and sharing (ACMNA004)	Problem Solving	
8	F	Patterns and Algebra	Sort and classify familiar objects and explain the basis for these classifications. Copy, continue and create patterns with objects and drawings (ACMNA005)	Reasoning	

Problem Solving

When to do it

Internationally there is considerable evidence to support the approach of teaching mathematics through problem solving rather than teaching problem solving as a set of skills on their own.

Problem Solving

When to do it

There are two approaches, that can be used

- Integrating into given topic areas
- As stand alone lessons.

These are complementary rather than mutually exclusive.

Problem Solving

When to do it

Individual teachers, departments and schools need to determine how they will incorporate problem solving into the day to day structure of lessons and topics.

What follows are some examples that other teachers have used to assist with the conversation.

Problem Solving

When to do it

As part of a daily set of practice questions:

Eg: Question 20 of Elementary Maths Mastery Lessons

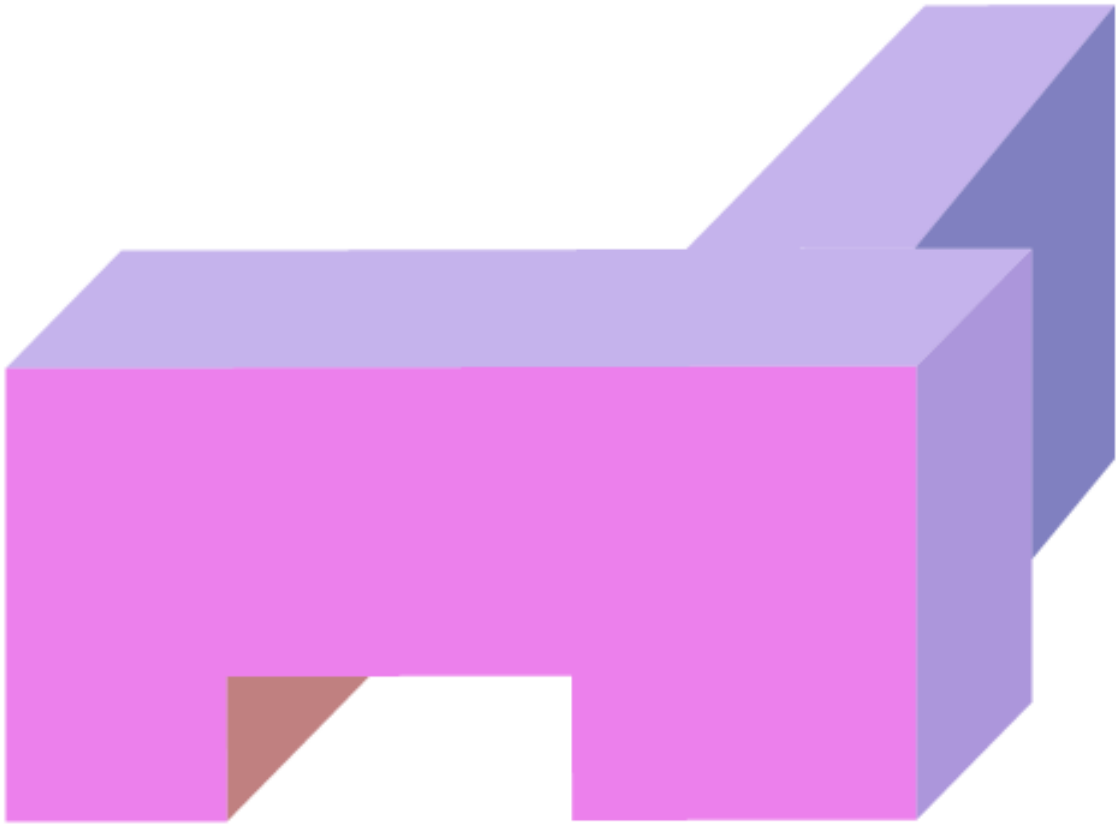
1	Find the least number of cuts needed to cut a log into 5 equal pieces.
2	Find the least number of cuts needed to cut a log into 3 equal pieces.
3	Find the least number of cuts needed to cut a log into 7 equal pieces.
4	Find the least number of cuts needed to cut a log into 11 equal pieces.
5	Can a log already cut into 2 pieces be cut into 3 equal pieces?

Designed to be used with students at years 5, 6 and 7

Problem Solving

Activity 2: Draw the view

Looking from the opposite direction



Problem Solving

When to do it

As an introduction to a topic, by exploration of new ideas.

For example:

Problem Solving

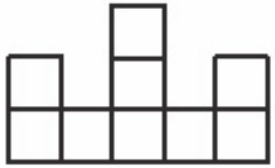
Activity 3: Which one is the same as?

This object was made using identical cubes.

Right side

Front

This is a drawing of the view from the front.



Front view

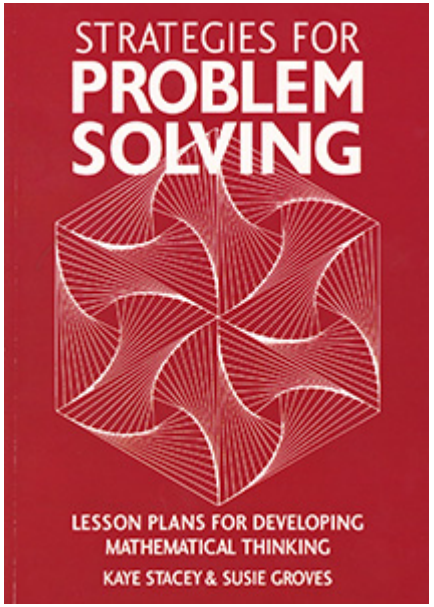
Which drawing shows the view from the right side?

Yr 5
NAPLAN

Problem Solving

When to do it

By using a formal set of lessons to develop skills spaced out across one or more year levels.



Designed to be used with students at years 7and 8

Problem Solving

When to do it

As a project or assignment at the end of a topic.

In this age of “Google it” make sure that students cannot simply bypass the thinking process by finding the answer somewhere in cyber-space.

Problem Solving

When to do it

For example, “The Gaoler Problem”:

In a gaol with an infinite number of cells, the gaoler:

1) Locks all the cells
2) Goes to every second cell and reverses it.
3) Goes to every 3rd cell and reverses them.
4) Goes to ever y 4th cell and reverses them.
5) And so on.

In the end, which cells are unlocked?

Problem Solving

When to do it

As an extension program for gifted and talented students.

Australian Mathematics Trust: Challenge & Extension Series

Plus many more

ENRICHMENT STUDENT NOTES

The Student Notes are supplied to students enrolled in the program along with other materials provided to their teacher. The six stages offer extension material for students from year 5 to year 10, in that order.

We are making these Notes available as a text book to interested parties for whom the program is not available. The notes refer to a problems booklet, which contains assessment problems for each year. If available, we will include a complimentary copy of a previous problems booklet but solutions to these are not available.

NEWTON
Price: AU$40.00

Recommended for students of about Year 5 and 6, topics include polyominoes, arithmetricks, polyhedra, patterns and divisibility.

DIRICHLET
Price: AU$40.00

Recommended for students of Year 6 or 7, topics include problem-solving techniques, tessellations, base five arithmetic, pattern seeking, rates and number theory.

EULER
Price: AU$40.00

Recommended for students of about Year 7, topics include elementary number theory and geometry, counting and pigeonhole principle.

GAUSS
Price: AU$40.00

Recommended for students of about Year 8, topics include Pythagoras' Theorem, Diophantine equations, counting techniques and congruences.

NOETHER
Price: AU$40.00

Recommended for students of about Year 9, topics include number theory, sequences and series, inequalities and circle geometry.

PÓLYA
Price: AU$40.00

Recommended for students of about Year 10, topics include polynomials, algebra, inequalities and Euclidean geometry.

Problem Solving

How to do it

Ask questions

AND

Wait for answers!

POSE

PAUSE

POUNCE

BOUNCE

Problem Solving

How to do it: ZPD

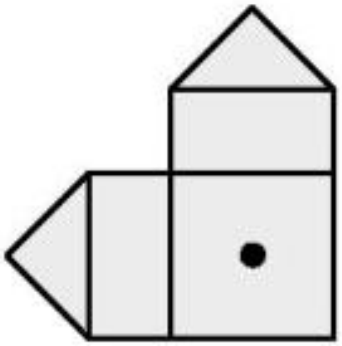
Asking questions and analysing the feedback allows the teacher identify a student's Zone of Proximal Development in real time.

Once the point is reached where the student needs a helping hand the teacher builds a scaffold to help them past the barrier.

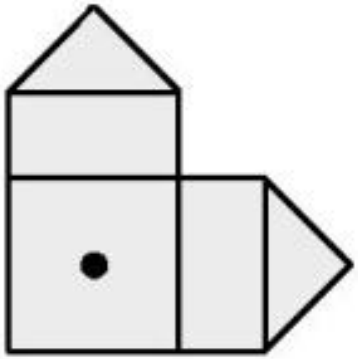
Problem Solving

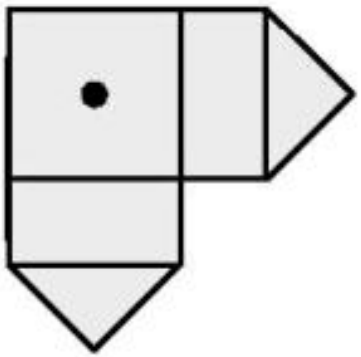
Activity 4:

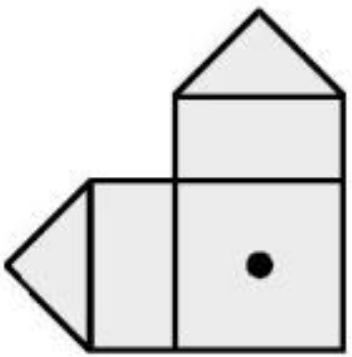
Look at this shape.

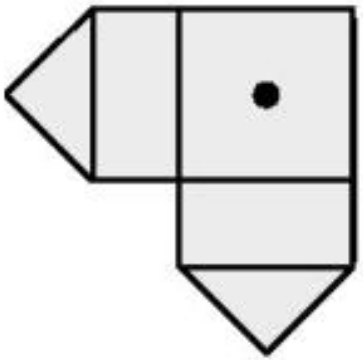


Which picture shows the shape rotated 90° to the right about the dot?









Problem Solving

Polya's Satchel of Strategies

Or a Toolbox of Techniques.

These can be built up over the years.

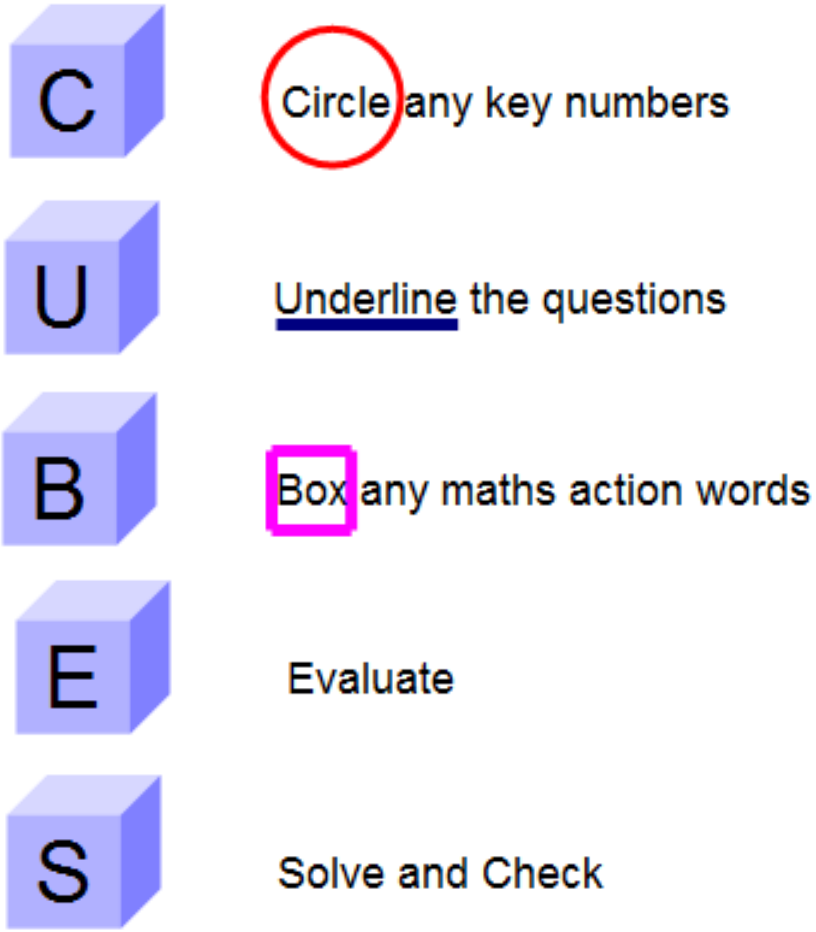
Often, a list of strategies, along with the 4 stages, are written on the classroom Maths Wall.

Some example strategies

- Guess and check
- Look for a pattern
- Draw a table
- Reduce to a simpler case
- Act it out
- Work backwards
- Draw a sketch
- Divide into subtasks
- Substitute simple values

Problem Solving

C.U.B.E.S.



Problem Solving

The G.I.F.T of L.I.F.E

There are two questions to ask when solving all mathematical problems

The first question is: **What do I know?**

The answer is **G.I.F.T.**

- **G**iven: in the question
- **I**mplied: by the context of the question, the meaning of the words or the diagrams
- **F**ound: in an earlier part of the question
- **T**heory: formulas, rules and the like

Problem Solving

The G.I.F.T of L.I.F.E

The second question is like it: **What can I do?**

The answer is: **L.I.F.E.**

- **L**ook, List, Label your information
- **I**dentify
- **F**ormulate
- **E**valuate

Problem Solving

When to do it

Every time a student becomes stuck on a question and needs help.

If students cannot answer something then it is a problem to be solved, not just a question to be answered.

So, almost all textbook questions can be approached in a “Problem Solving” manner.

Problem Solving

Introducing Concepts

Traditional “Textbook” Approach

1) Theory
2) Worked Examples
3) Exercise with
    - Fluency (Drill) Questions
    - Simple Worded Questions
    - Multi-part Worded Questions
4) Extended/Open Ended Challenge (maybe)

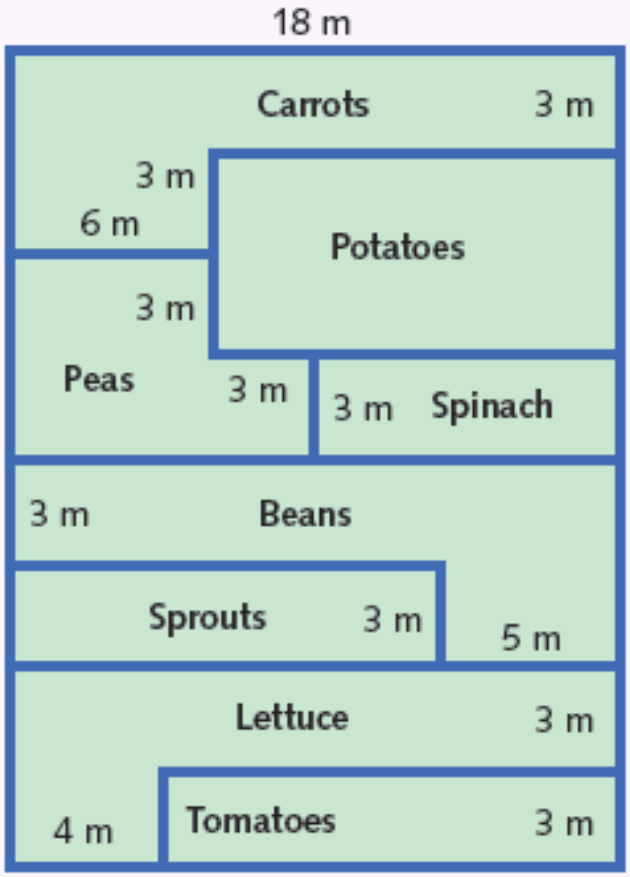
Problem Solving

Problems with Text Books

7 This is a plan of the Healey family's vegetable garden.

Use the measurements to calculate the perimeter of each garden bed.

Then calculate the length of fence required to enclose the whole garden.



The Healey family are planning their vegetable garden for next year. They want to know how much of their garden area they used for each vegetable. First, calculate the area used for each vegetable. Then add the areas together to see if they match the total area of the whole vegetable garden.

Problem Solving

Problems with Text Books

16 Sam throws a stone down to the ground from the top of a cliff s metres high, with an initial speed of u m/s. It accelerates at a m/s^2. The stone hits the ground with a speed of v m/s given by the formula $v^2 = u^2 + 2as$. Find the speed at which the stone hits the ground, correct to two decimal places, if:

a $u = 0$, $a = 9.8$ and $s = 50$

b $u = 2$, $a = 10$ and $s = 40$

c $u = 5$, $a = 9.8$ and $s = 35$

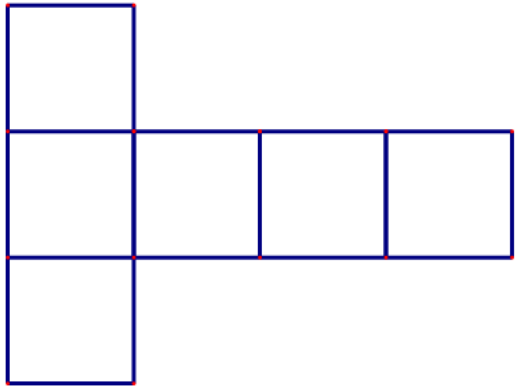
d $u = 10$, $a = 9.8$ and $s = 50$

Problem Solving

Activity 5: Which one is the same as?

Which of these nets make the same solid?

A



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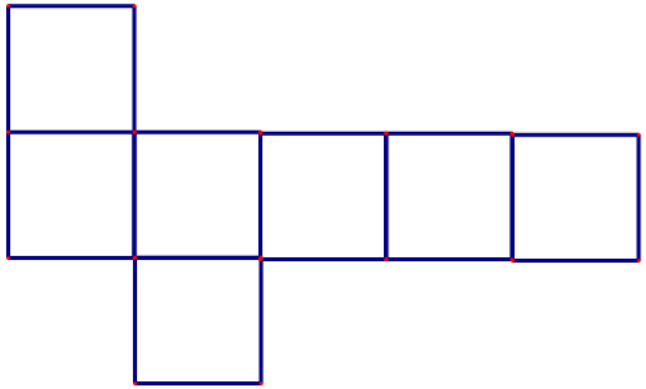

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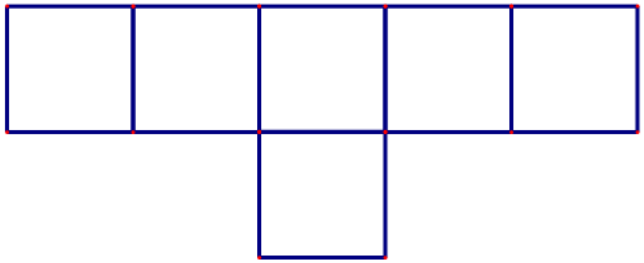
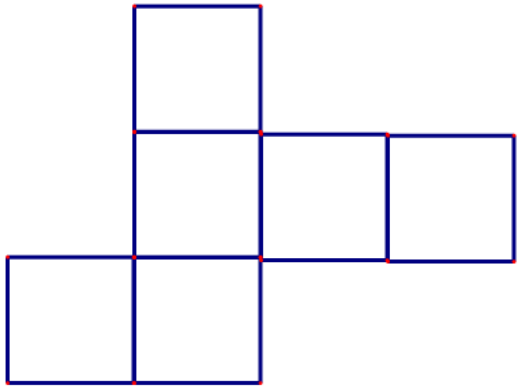


D



Problem Solving

Introducing Concepts Again

What are the alternatives to the “traditional” textbook approach?

Start with a problem and explore. The problem drives the learning of new knowledge and skills.

What other alternatives do you know of, or have you used?

Problem Solving

Finding Problems

Dedicated books on problem solving

(some examples given at the end of this presentation)

Puzzle books

FERMI questions on the web

Newspaper stories

Trivia questions/books

Observation

Problem Solving

Cooks vs Chefs

Cooks	**Chefs**
Follow recipes	Use recipes
In order	As guides
Without variation	Then experiment
End up with	End up with
The same result	Some failures
Everytime	But also some wonderful successes

Problem Solving

Dreyfus' Model of Skill Acquisition

This is an alternative to the Bloom and SOLO Taxonomies.
The example below is taken from
"Assessment for Teaching" by Griffin et al. p 41

Level	Behaviours
Novice	Follows a recipe
Competent	Attempts changes to recipe (e.g. substitutes and ingredient, adjusts amounts/times) when necessary
Proficient	Adapts recipe to suit situation
Expert	Invents own recipes

Problem Solving

Nutshell Statements: Griffin et al

Appendix 2.1: Progression of problem-solving development – nutshell statements	
Level I	Develop multiple strategies connecting abstract representations of objects and situations.
Level H	Generalise rules and/ or re-evaluate possible strategies in light of information provided.
Level G	Draw on variety of skills to develop strategies required to solve complex problems.
Level F	Identify complex logical relationships in a problem. Develop non-routine problem-solving strategies.
Level E	Select appropriate multi-step strategy to solve problems. Translate problem into a rule.
Level D	Apply a range of up to two-step strategies to solve problems.
Level C	Apply one-step strategies to solve problems in less familiar contexts.
Level B	Use one-step strategies to solve problems.
Level A	Follow instructions to solve problems in familiar contexts.

Problem Solving

Making Sense

Communication is paramount

Read aloud.

Question

Discuss

Argue

Explain

Write

Problem Solving

Always, Sometimes, Never

This is an approach to exploring properties of objects or ideas.

- Pick 2 even numbers. Add them together. What do you get?
- Now pick 2 odd numbers and do the same.
- Finally pick 1 even and 1 odd number and repeat.

Do you always get these results?

Try the same for multiplication.

Problem Solving

Revisiting and Going Deeper

From session 1 the Tic Tac Toe (or Leap Frog) and Triangular numbers problems can be given at several different year levels.

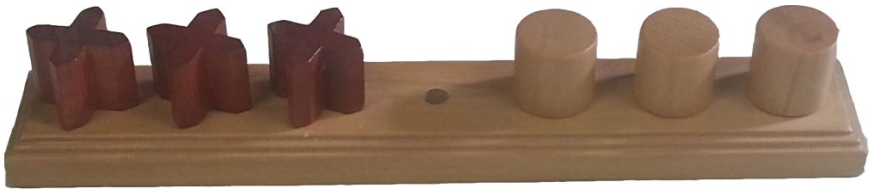
This emphasises:

1) Students can be introduced to the concepts at one level
2) There is plenty of depth to revisit at later times
3) Chances to reinforce/revise earlier learning

Problem Solving

When to do it

Puzzles: eg: Leap Frog - TicTacToe



What age level?

Where's the maths?

How deep can you go?

Problem Solving

When to do it

Stephen and Sophie both work as shelf stackers for their local Safeday Supermarket. Their manager wants them to build a display for one of the aisles using soup cans. He tells them they have to stack the cans to form a triangle.

They have 300 cans to use.

How many rows of cans will there be in the display and how many cans will be in the bottom row?

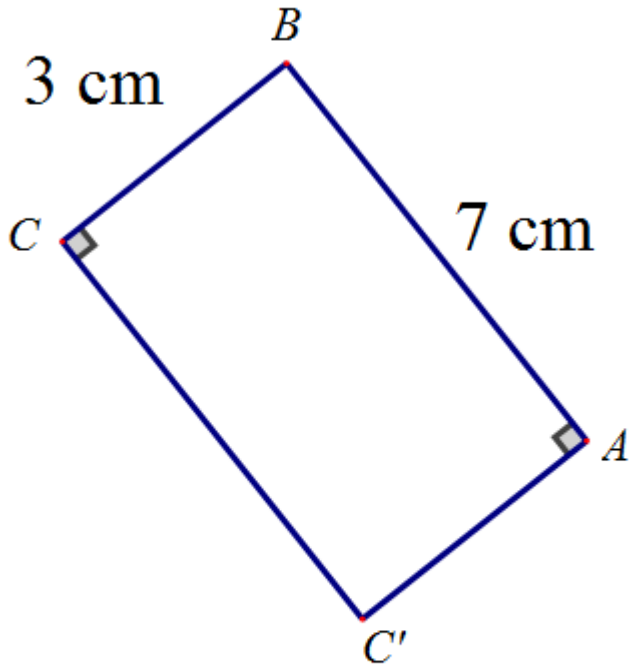
Problem Solving

Activity 6: What's my mistake?

$24 + 39 = 53$

$24 + 39 = 54$

$2^2 + 3^2 = 10$



The perimeter of this rectangle is 21 cm.

Problem Solving

Developing a Classroom Culture

1) Who does most of the talking in the lesson?
2) What questions do I (the teacher) ask?
3) Who answers the questions?
4) How well do I listen to student answers?
5) What do I do with the students' answers?
6) How do I facilitate the learning?
7) How confident are the students to take risks and make mistakes?
8) What does my body language communicate?

Adapted from NRICH: http://nrich.maths.org/10341/index?nomenu=1

Problem Solving

How long does a problem take?

Five Minutes? – Can it be Googled?

A lesson?

A week?

A semester?

A lifetime?

Fermat's Last Theorem

Problem Solving

How long should a solution be?

Two lines?

Five lines?

Half a page?

Several pages?

What are the elements of a good solution?

Problem Solving

Perseverance

"… students whose entire mathematical experience consisted of working exercises that could be solved in just a few minutes came to believe that "all problems can be solved in five minutes of less", and ceased working on problems that they might have been able to solve had they persevered."

Schoenfeld, 2013

Problem Solving

Powerful Learning Environments

Include:

Problematizing – students participate in the act of framing meaningful questions, which the class explores

Agency and authority – students are empowered to seek information, distil it, craft arguments, and explain them

Engle in Schoenfeld 2013

Problem Solving

Powerful Learning Environments

And:

Disciplinary accountability – students learn what it is to make claims and arguments that are consistent with disciplinary norms

Resources – when tools or information is needed, students have access to them

Engle in Schoenfeld 2013

Problem Solving

Collaboration

Solving problems is rarely an individual exercise, either in real life or in the mathematics classroom.

Provide opportunities for working alone, with small groups, and with large groups.

Terry Tao and the Polymath Project

Problem Solving

Blooms Taxonomy

Finding the "Level" of a Problem

1) Knowledge/memory	Fluency
2) Comprehension/Translation	Understanding
3) Interpretation	Understanding/Reasoning
4) Application	"Type 1" Problems
5) Analysis	"Type 2" Problems
6) Synthesis	"Type 3" Problems
7) Evaluation	Reflection/Metacognition

Adapted from NRICH

Problem Solving

Teacher Practice

To help our students become successful problem solvers, we need to practice and reflect on our own problem solving capacities.

Problem Solving

Activity 7: Extending Patterns

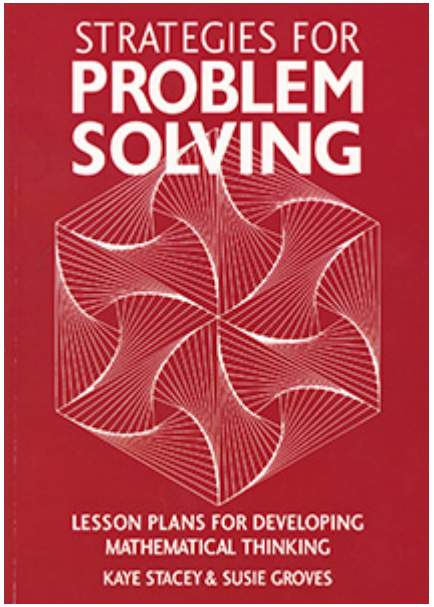
When bees build their homes, called hives, the make cells in the shape of hexagons.

- If they start with one cell in the middle of the hive, how many cells can they fit around the first one?
- How many more will they make in the next layer?
- How many in the layer after that?
- Can you see a pattern? Describe it.

Problem Solving

Resources

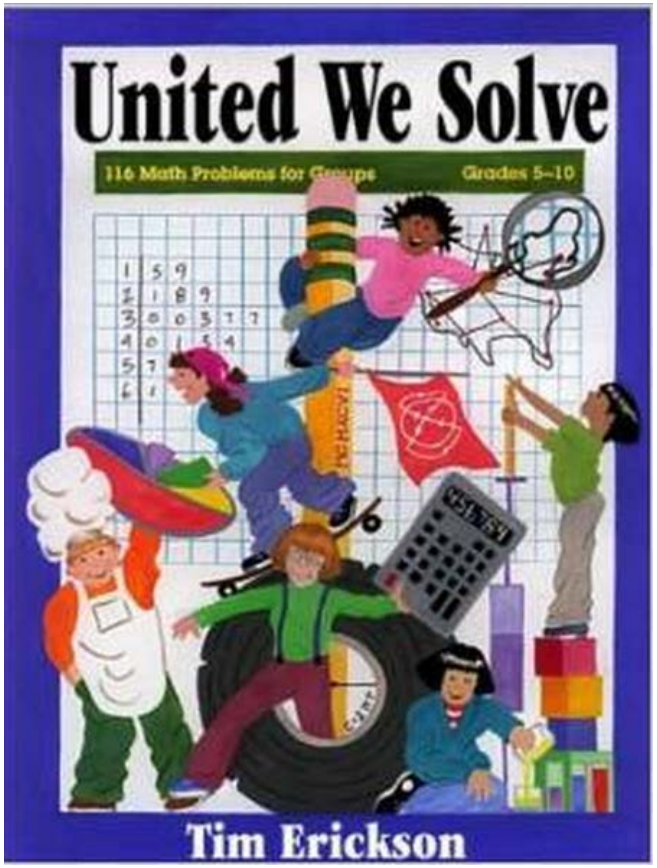
Objective Learning Materials.



Problem Solving

Resources

Objective Learning Materials

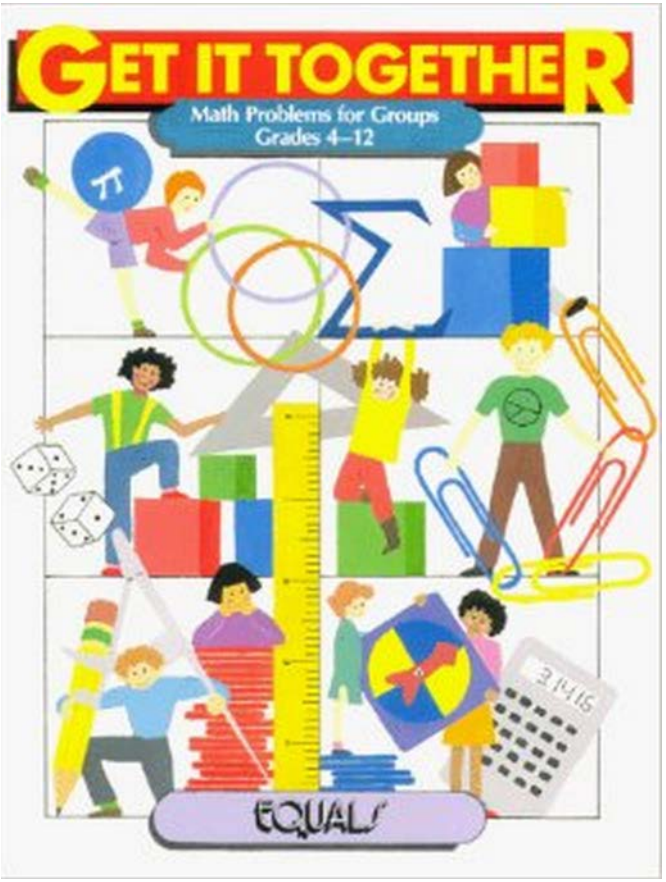


Problem Solving

Group Work

The “Get It Together” text has over 100 activities for groups. They focus on reasoning and problem solving skills where the necessary information is broken up into a series of clues that students use to piece the solution together.

Objective Learning Materials



Problem Solving

Resources

Maths 300 via SCOOTLE and Educational Services Australia (ESA)

Maths Task Centre from Black Douglas

Problem Solving

Resources

Literature and Links

Developing a Classroom Culture That Supports a Problem-solving Approach to Mathematics (NRICH, Cambridge, UK)

Victorian Dep't of Education and Training on ZPD and scaffolding

Problem Solving

More examples to play with

Now is a chance to have a look through the various reference books we brought along as well as for sharing your own examples.

If you did not submit any to us in advance please leave a copy for us to add to our database.

There are extra copies of the Problem Solving Evaluation Sheet from the first session a few weeks ago.

AMSI

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