AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Problem Solving: What is it and Why is it Important?

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Problem Solving

What, exactly, is Problem Solving?

Take a few moments to write down your current working definition of Problem Solving.

Problem Solving

Why is it important?

Now write down why it is important to include problem solving as a core element of the mathematics curriculum.

If you disagree with it being given such a central role, write down your reasons for this instead.

Problem Solving

When and How often

In your classroom currently:

When do you work on problems and problem solving?

How often do you spend at any one time?

Problem Solving

How good are you?

On a scale from 1 to 10, give yourself a score for how confident you feel in solving problems.

Problem Solving

The "Official" Definition

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

(ACARA, http://www.australiancurriculum.edu.au/mathematics/content-structure)

Problem Solving

Year Level Descriptions

In addition to this overall statement, ACARA also has descriptions for what Problem Solving looks like at each year level.

These are available in collated form both on Calculate and in the AMSI Teacher Journal. They are also reproduced in the following slides.

Problem Solving

Years F to 2

Year	Description	Keyword
F	Problem Solving includes using materials to model authentic problems, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer	Model, solve, discuss reasonableness
1	Problem Solving includes using materials to model authentic problems, giving and receiving directions to unfamiliar places, and using familiar counting sequences to solve unfamiliar problems and discussing the reasonableness of the answer	Model, communicate directions, solve, discuss reasonableness
2	Problem Solving includes formulating problems from authentic situations, making models and using number sentences that represent problem situations, and matching transformations with their original shape	Formulate, model, comparison matching

Problem Solving

Years 3-4

Year	Description	Keyword
3	Problem Solving includes formulating and modelling authentic situations involving planning methods of data collection and representation, making models of three-dimensional objects and using number properties to continue number patterns	Formulate, model,
4	Problem Solving includes formulating, modelling and recording authentic situations involving operations, comparing large numbers with each other, comparing time durations, and using properties of numbers to continue patterns	Formulate, model, record, compare

Problem Solving

Years 5-6

Year	Description	Keyword
5	Problem Solving includes formulating and solving authentic problems using whole numbers and measurements and creating financial plans	Formulate,
6	Problem Solving includes formulating and solving authentic problems using fractions, decimals, percentages and measurements, interpreting secondary data displays, and finding the size of unknown angles	Formulate, solve, interpret,

Problem Solving

Years 7-8

Year	Description	Keyword
7	Problem Solving includes formulating and solving authentic problems using numbers and measurements, working with transformations and identifying symmetry, calculating angles and interpreting sets of data collected through chance experiments	Formulate, solve, identifying “symmetries”, interpret
8	Problem Solving includes formulating, and modelling practical situations involving ratios, profit and loss, areas and perimeters of common shapes, and using two-way tables and Venn diagrams to calculate probabilities	Formulate, model, convert/translate information

Problem Solving

Years 9-10

Year	Description	Keyword
9	Problem Solving includes formulating, and modelling practical situations involving surface areas and volumes of right prisms, applying ratio and scale factors to similar figures, solving problems involving right-angle trigonometry, and collecting data from secondary sources to investigate an issue	Formulate, model, apply, solve, investigate,
10	Problem Solving includes calculating the surface area and volume of a diverse range of prisms to solve practical problems, finding unknown lengths and angles using applications of trigonometry, using algebraic and graphical techniques to find solutions to simultaneous equations and inequalities, and investigating independence of events	Apply, investigate,
10A	No additional comments	

Problem Solving

"*Problem solving* at its most general was defined as trying to achieve some outcome, when there was no known method (***for the individual trying to achieve that outcome***) to achieve it ...

complexity or ***difficulty alone did not make a task a problem***"

Schoenfeld, 2013

Problem Solving

Polya: How to Solve it.

First published in 1945, it says there are 4 stages in solving any problem

1) Understand the problem

2) Devise a plan

3) Carry out the plan

4) Look back

Problem Solving

A Satchel of Strategies

Or a Toolbox of Techniques.

These can be built up over the years.

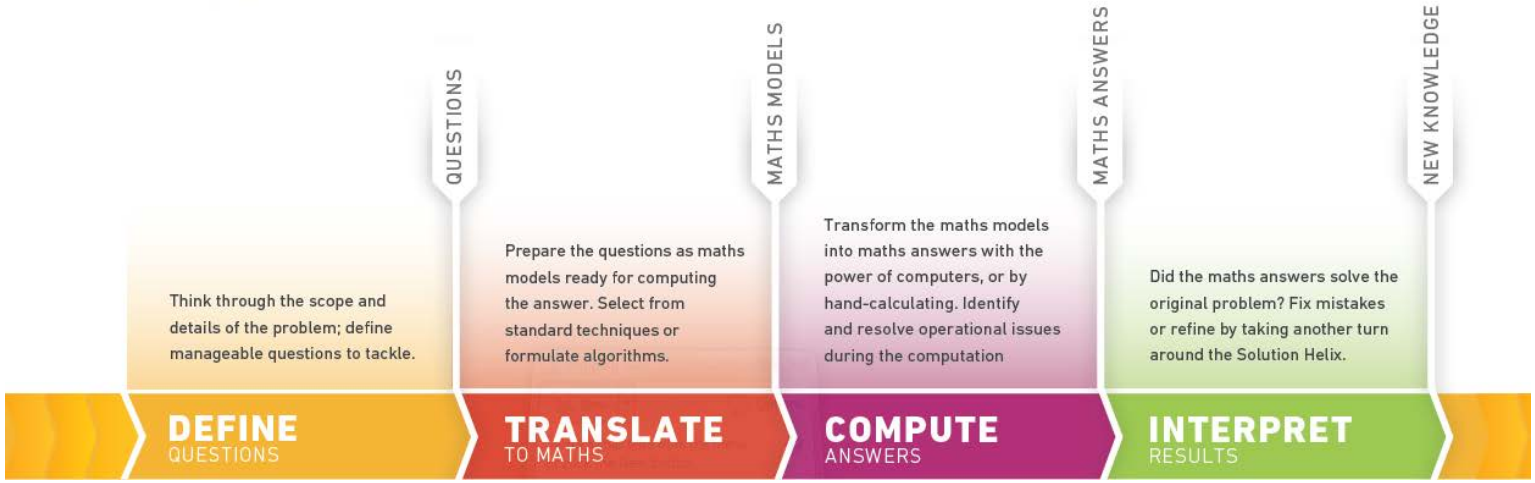
Often, a list of strategies, along with the 4 stages, are written on the classroom Maths Wall.

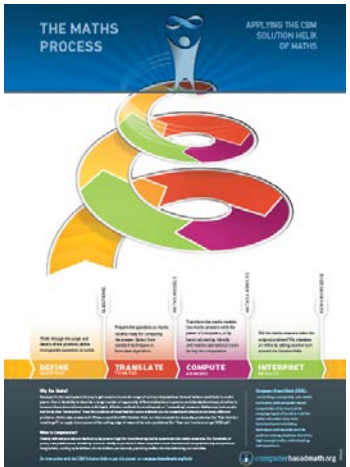
Some example strategies

- Guess and check
- Look for a pattern
- Draw a table
- Reduce to a simpler case
- Act it out
- Work backwards
- Draw a sketch
- Divide into subtasks
- Substitute simple values

Problem Solving

Wolfram's "Doing Maths"





Reproduced with permission. Copies of the poster are available at:
http://www.computerbasedmath.org/maths-process-poster/

Problem Solving

Polya again

	Connecting the other Proficiencies
1) Understand the problem	Understanding
2) Devise a plan	Reasoning
3) Carry out the plan	Reasoning & Fluency
4) Look back	Feedback, reflection and metacognition

Problem Solving

Why is it Important?

1) Mathematics is more than just calculation.

2) Real life, modern society and civilisation is full of problems large and small.

3) Promotes links between concepts and topics leading to greater understanding and new learning

Problem Solving

What makes a good Problem?

1. The problem has important, useful mathematics embedded in it.
2. The problem requires higher-level thinking and problem solving.
3. The problem contributes to the conceptual development of students.
4. The problem creates an opportunity for the teacher to assess what his or her students are learning and where they are experiencing difficulty.

These are regarded as the essential minimum for selection of all problems

Why is Problem Solving Important to Student Learning, NCTM Research Brief, April 2010

Problem Solving

What makes a good Problem?

5. The problem can be approached by students in multiple ways using different solution strategies.
6. The problem has various solutions or allows different decisions or positions to be taken and defended.
7. The problem encourages student engagement and discourse.
8. The problem connects to other important mathematical ideas.
9. The problem promotes the skillful use of mathematics.
10. The problem provides an opportunity to practice important skills.

These criteria add value at different times to the collection of problems being used.

Why is Problem Solving Important to Student Learning, NCTM Research Brief, April 2010

Problem Solving

Developing an Integrated Approach

Involves developing a mindset and a process to help students

a) build skill and capacity

b) encourage perseverance

c) reflect on thinking

d) recognise multiple solutions may be possible

Problem Solving

Developing an Integrated Approach

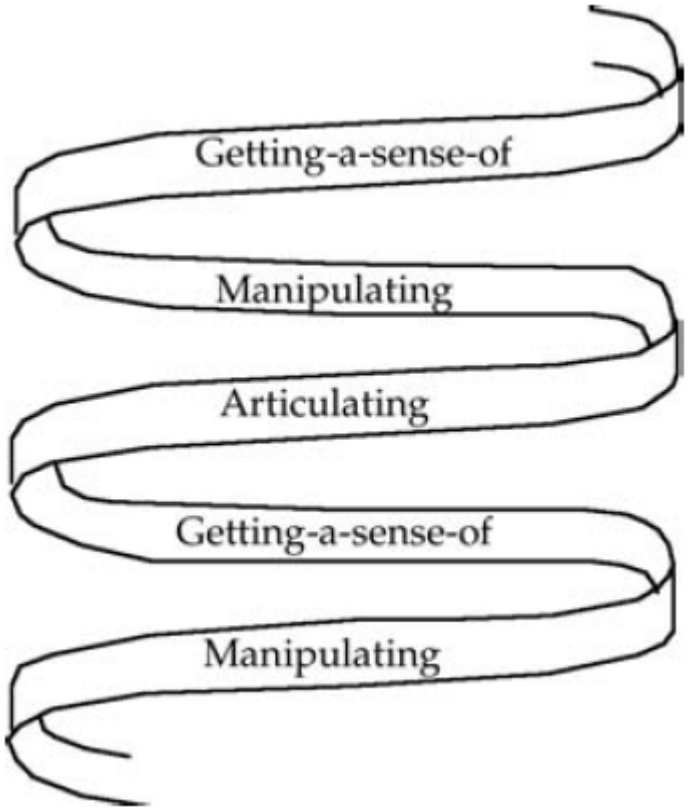
More than just a learning a strategy a week.

While having a set of strategies is important, so is choosing which strategy to use in a given situation and justifying this choice.

Problem Solving

Sense Making

MGA Spiral of developing understanding



Mason, J., et al, (2005), Developing Thinking in Algebra, SAGE Publications

Problem Solving

The Zone of Proximal Development (ZPD)

Developed by Vygotsky in the 1930's in Soviet Union.

Not until 1970's that ideas reached the West.

Used to assist students in the acquisition of knowledge and skills

Useful also for formalising problem solving processes

Problem Solving

The Zone of Proximal Development (ZPD)

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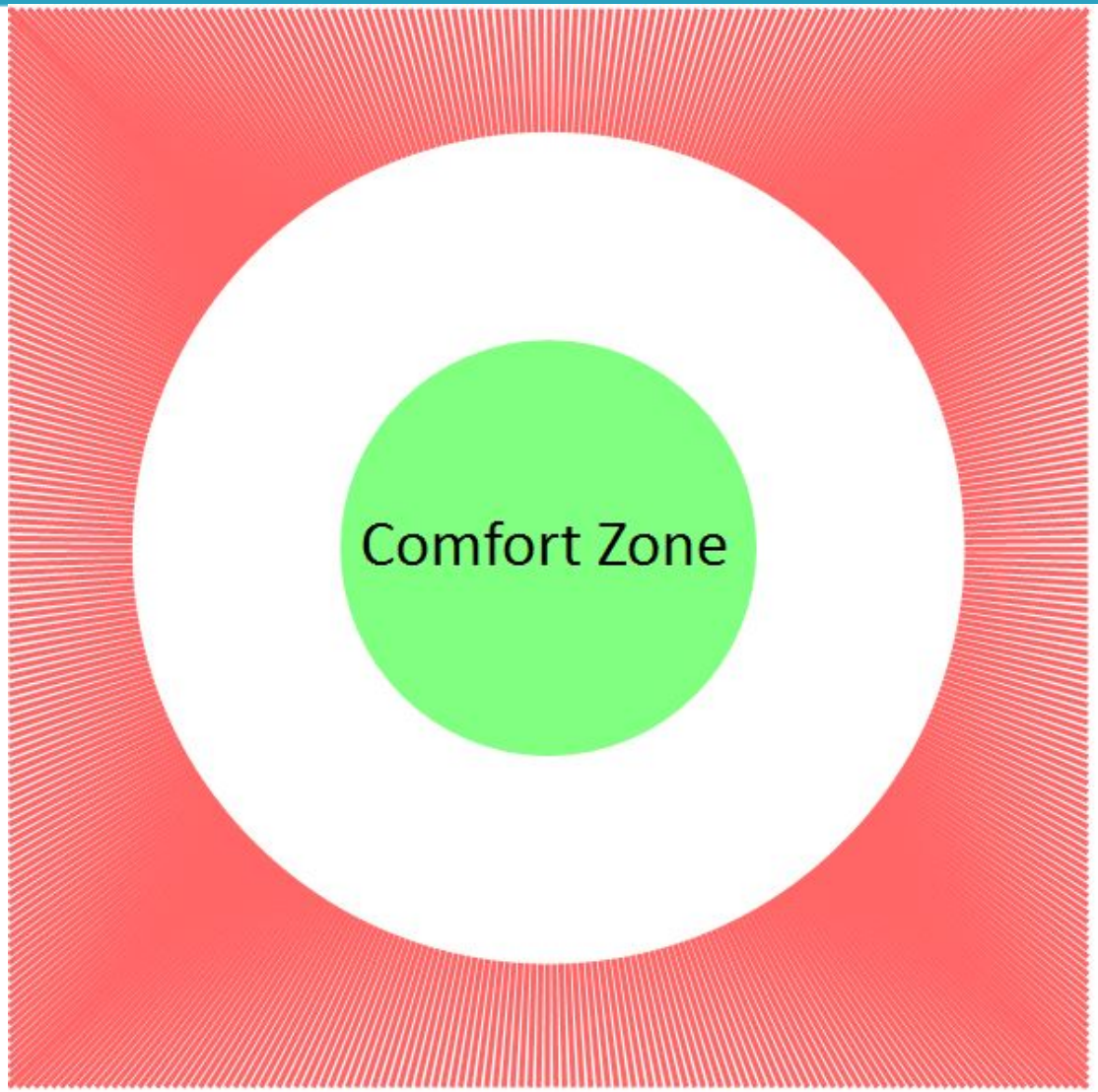

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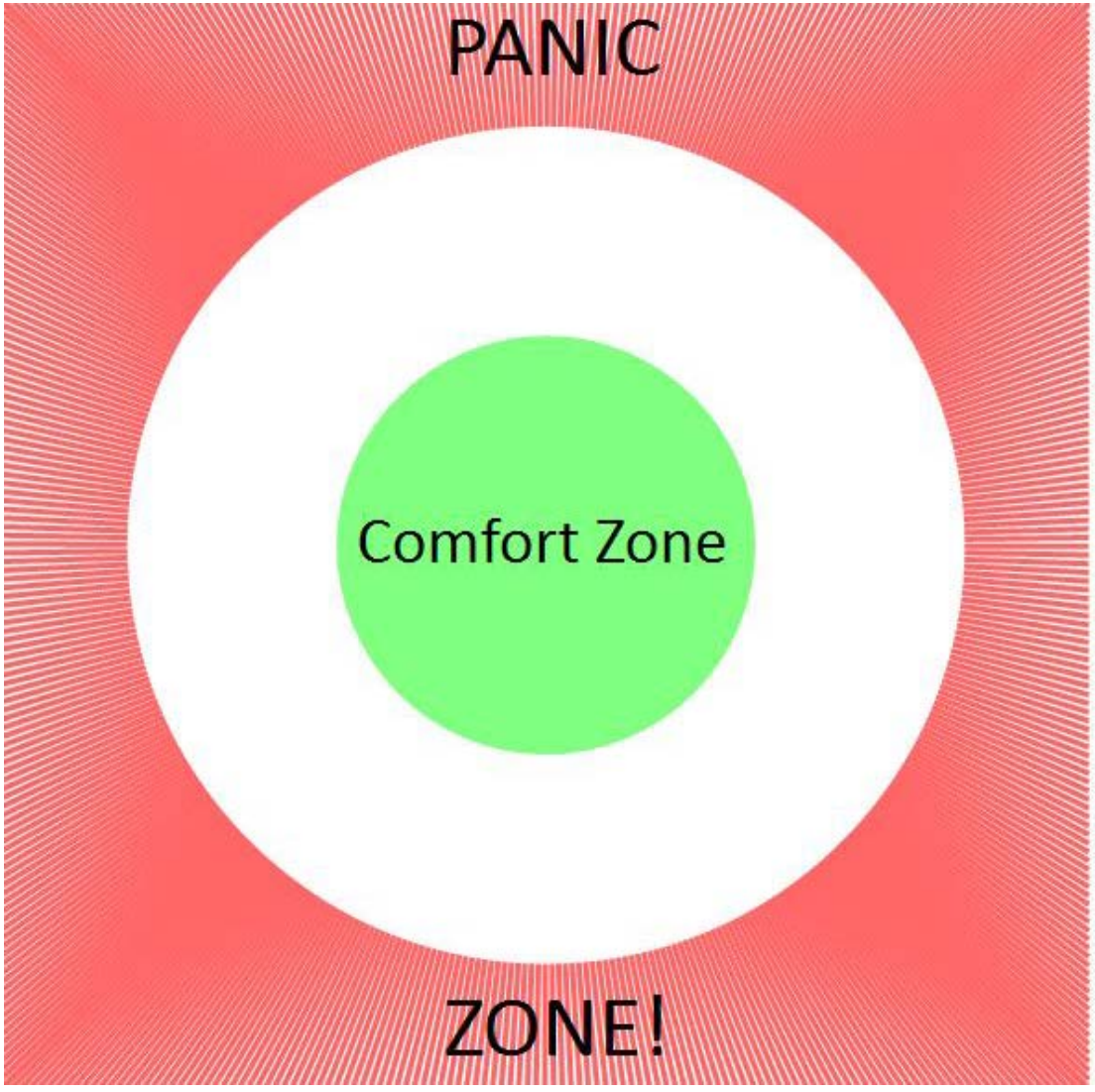

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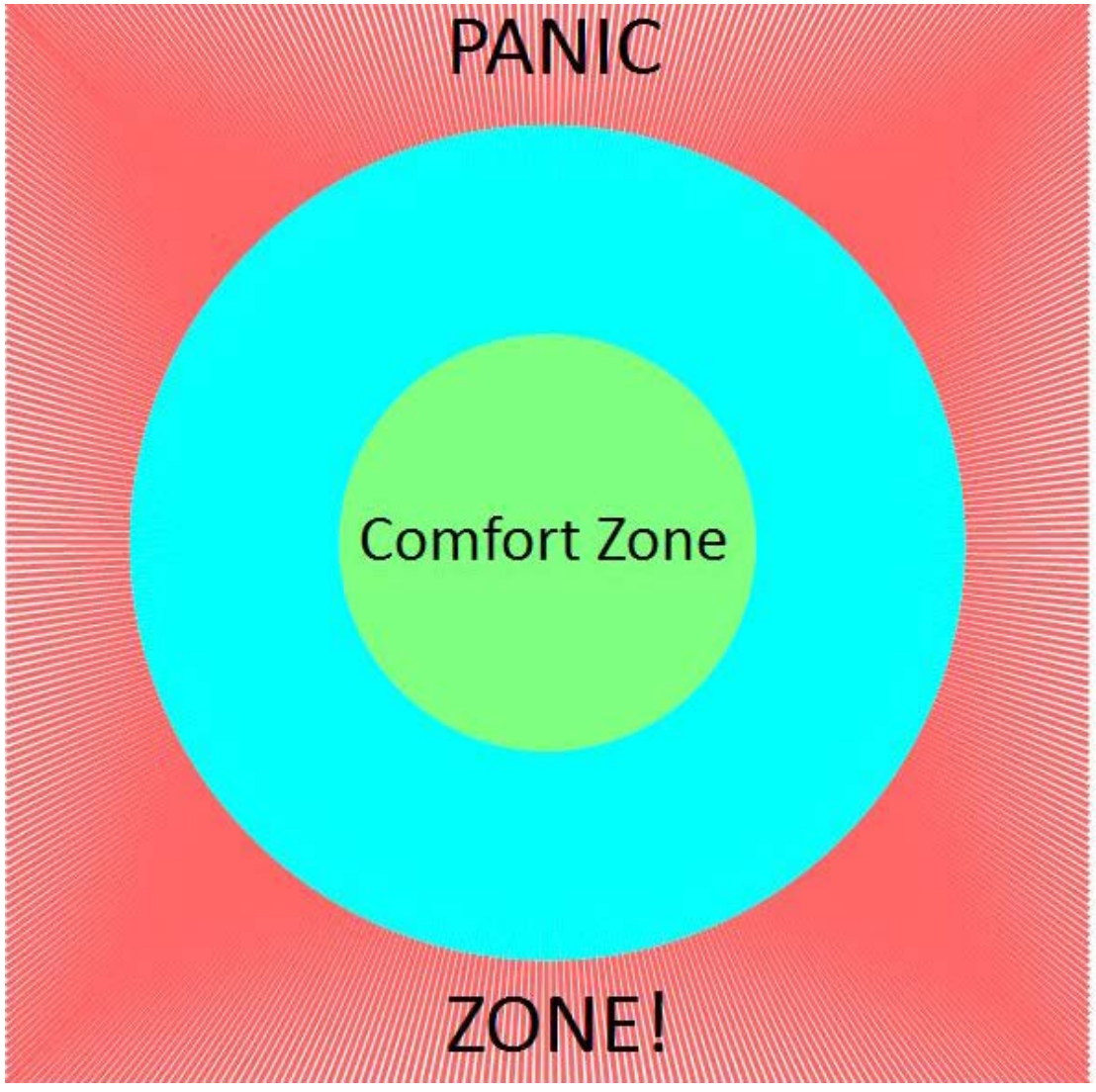

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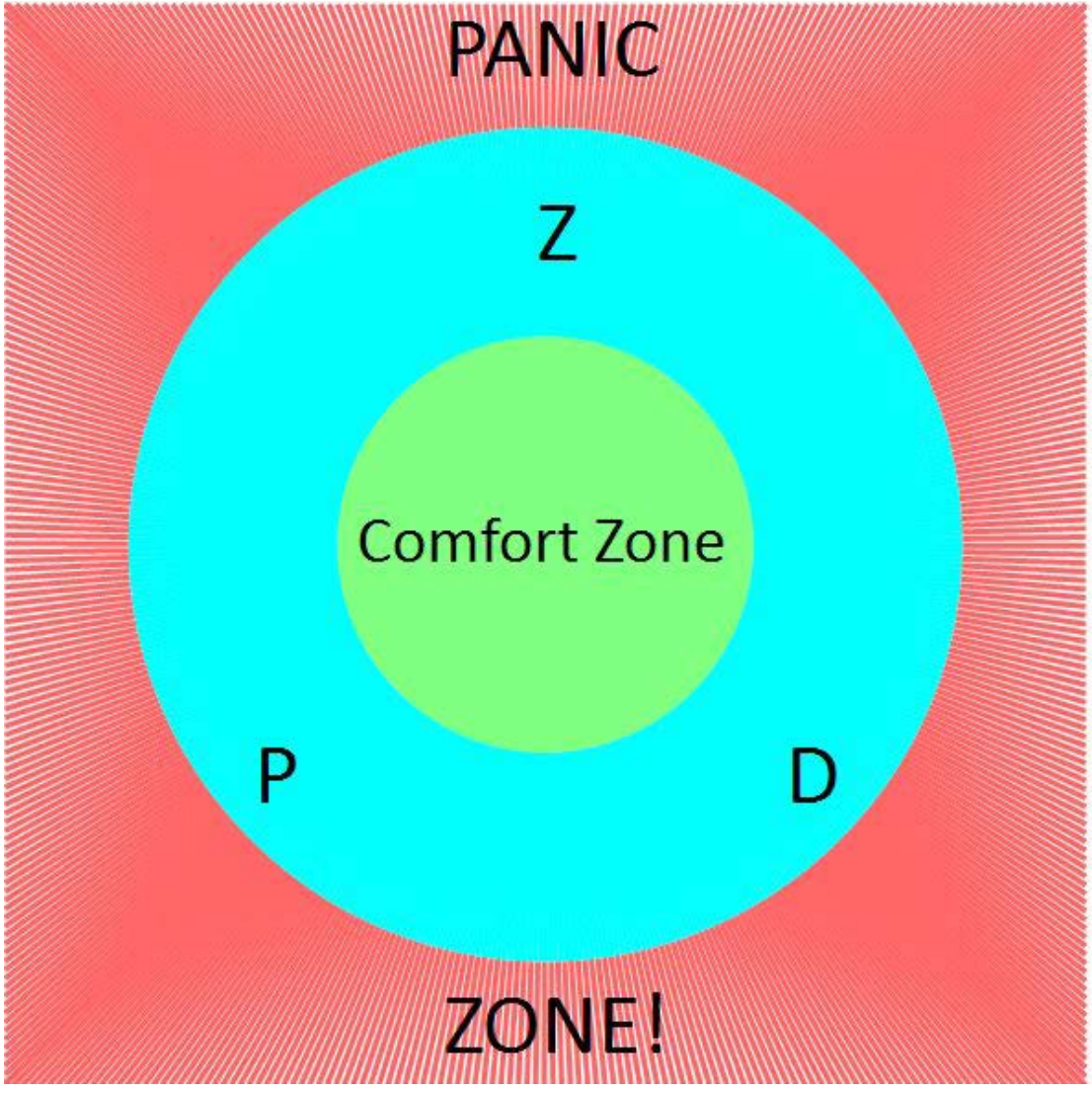

Problem Solving

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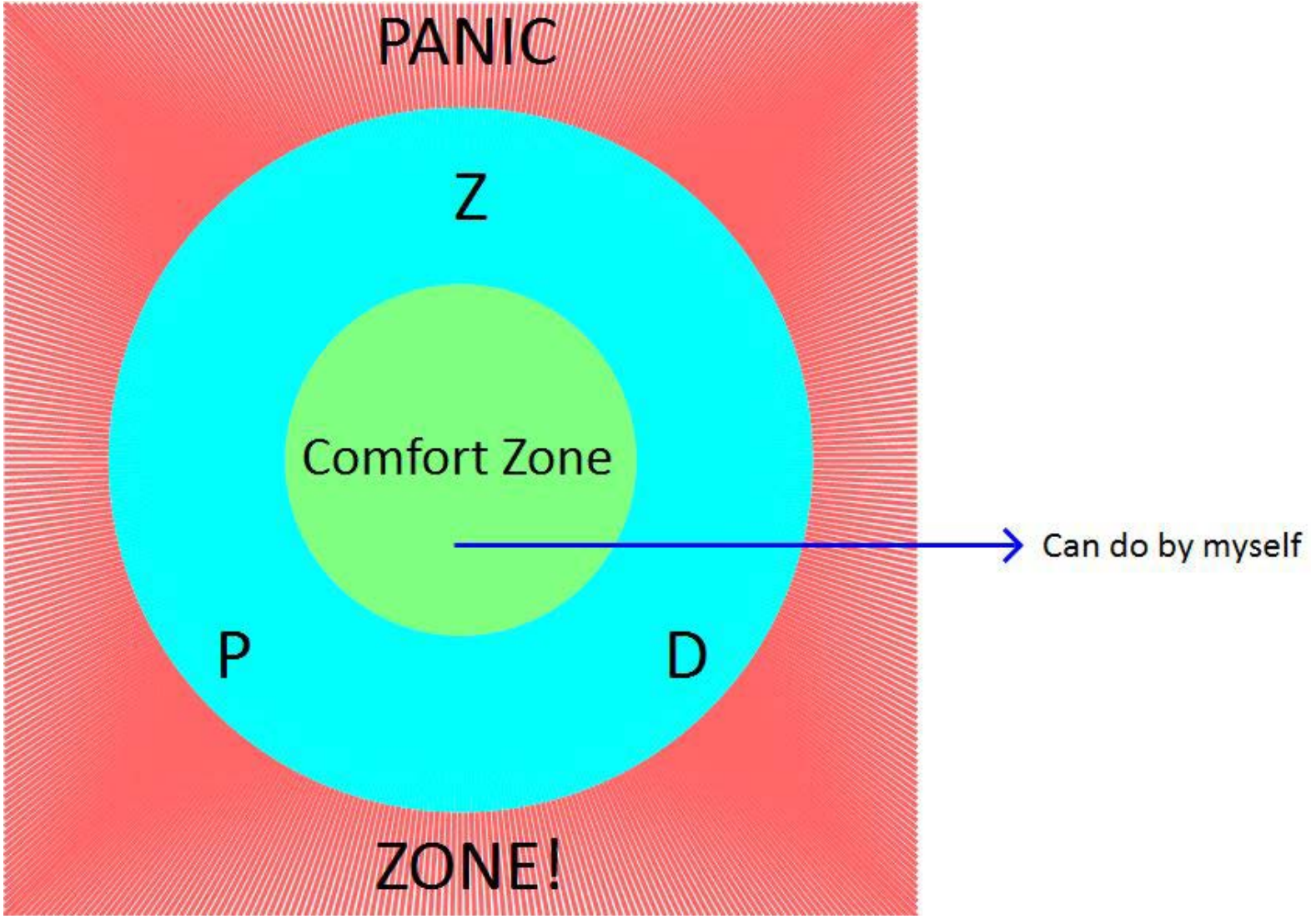

Problem Solving

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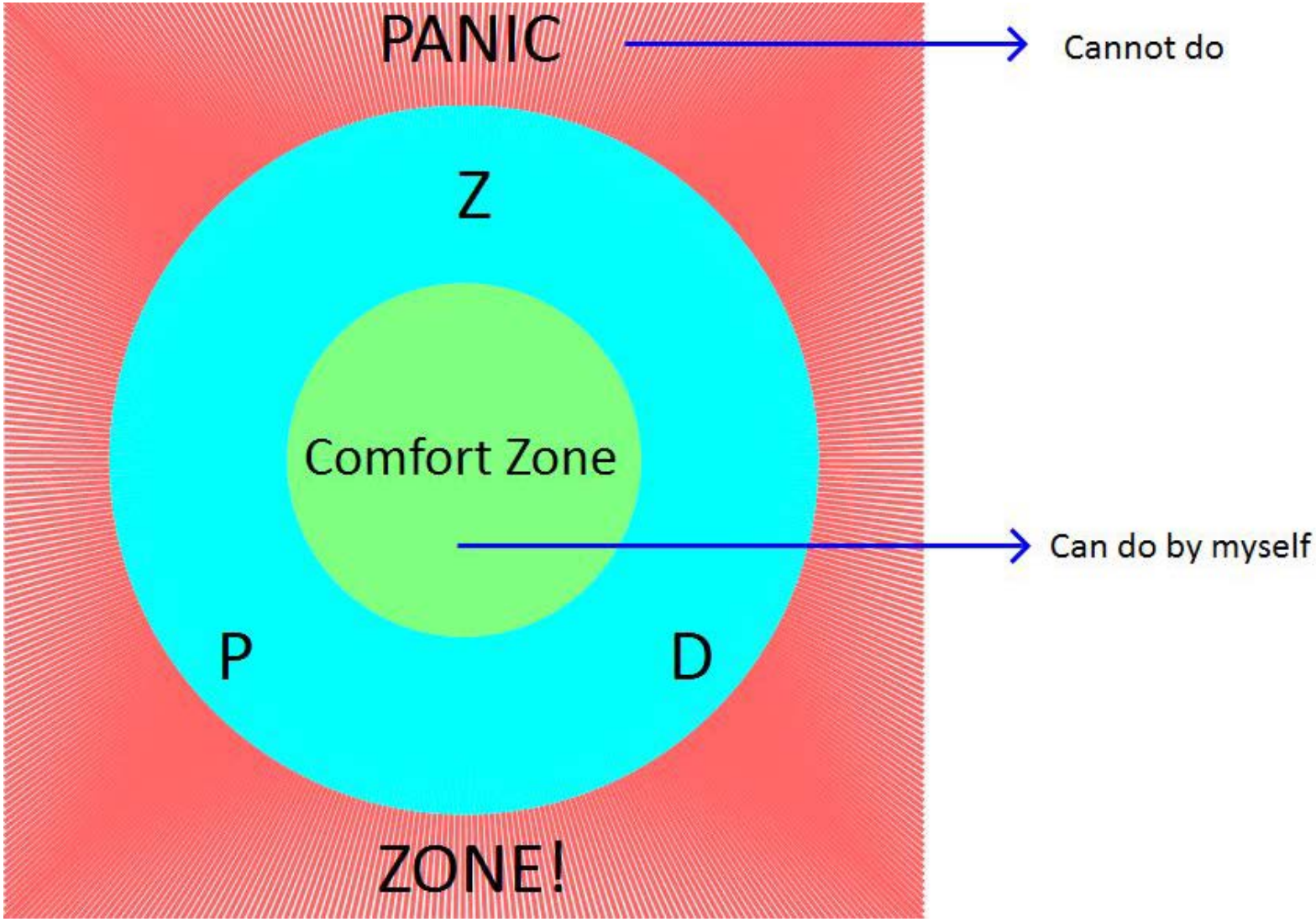

Problem Solving

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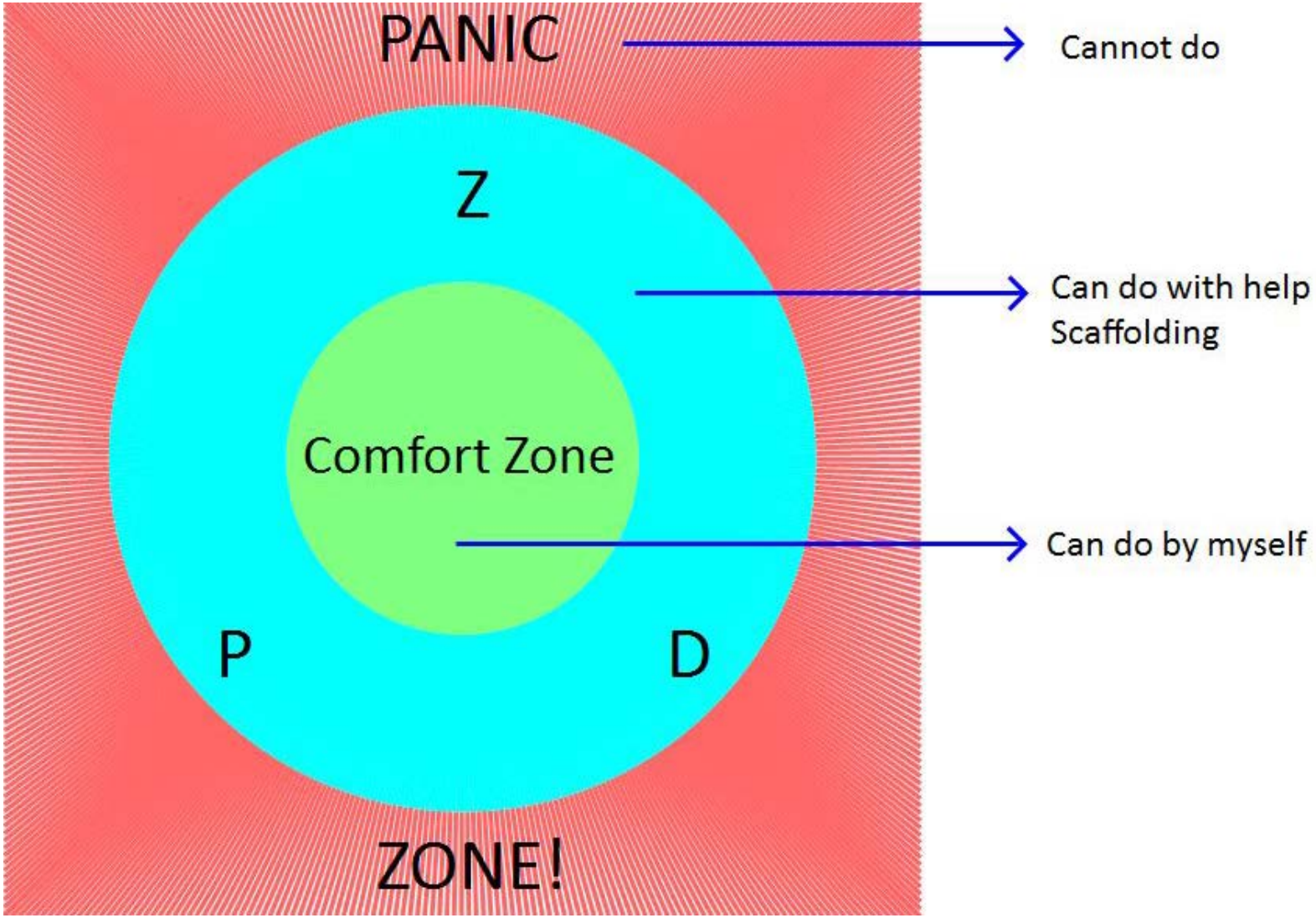
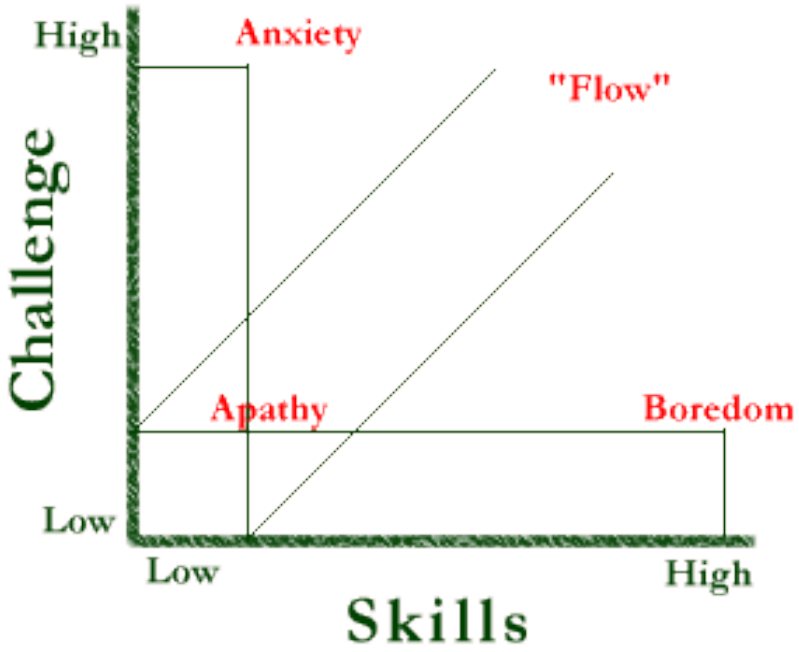


Problem Solving

“Flow”

How does it feel to be in "the flow"?
Completely involved, focused, concentrating - with this either due to innate curiosity or as the result of training
Sense of ecstasy - of being outside everyday reality
Great inner clarity - knowing what needs to be done and how well it is going
Knowing the activity is doable - that the skills are adequate, and neither anxious or bored
Sense of serenity - no worries about self, feeling of growing beyond the boundaries of ego - afterwards feeling of transcending ego in ways not thought possible
Timeliness - thoroughly focused on present, don't notice time passing
Intrinsic motivation - whatever produces "flow" becomes its own reward



http://austega.com/gifted/16-gifted/articles/24-flow-and-mihaly-csikszentmihalyi.html

Problem Solving

ZPD - Scaffolding

Breaking up tasks or problems into more manageable pieces.

Provision of hints and guides to assist students to continue progressing through the task.

Problem Solving

ZPD - Fading

The removal of scaffolding structures from questions and tasks.

Promoting student reliance on previously developed problem solving approaches.

The goal is for students to develop new approaches on their own.

Original discovery and Synthesis

Problem Solving

ZPD

ZPD is more than just the determination of effect size from pre and post testing.

It is an attempt to help students process how they are going with their problem solving attempts

Problem Solving

Being Stuck

What are the typical responses?

1) Give up completely
2) Put it away for a while
3) Keep working

Having rubrics or checklists can be useful

4) Collaboration

Thinking Mathematically, Mason, Burton & Stacey, 1985

Problem Solving

The Zone of Confusion

A term developed by Clarke et al

"as something which some teachers might find helpful in discussions with students about the different stages they might move through as they work on genuinely challenging tasks"

Clarke. D, et al, Australian Mathematics Teacher 70(1) 2014

Problem Solving

Analysing Success or Failure

Schoenfeld (1985) says that we need to know about the individual's:

1) knowledge
2) use of problem solving strategies
3) Monitoring and self-regulation (part of metacognition)
4) Belief systems (of self, of maths, of problem solving and the origins of these in prior mathematical experiences

Problem Solving

Metacognition

Two aspects:

1) Knowledge of cognition

   Strengths and weaknesses as a learner and problem solver

   Where, how and why to apply different strategies

2) Monitoring and Regulation of cognition

   Where am I going?

   How am I going?

   Where to next?

Problem Solving

Metacognition

Heuristic skills

“search strategies” for deciding which tools to apply to the problem.

This links directly to Polya’s list of strategies and the “toolkit” approach

Problem Solving

Multiple Approaches

Concrete

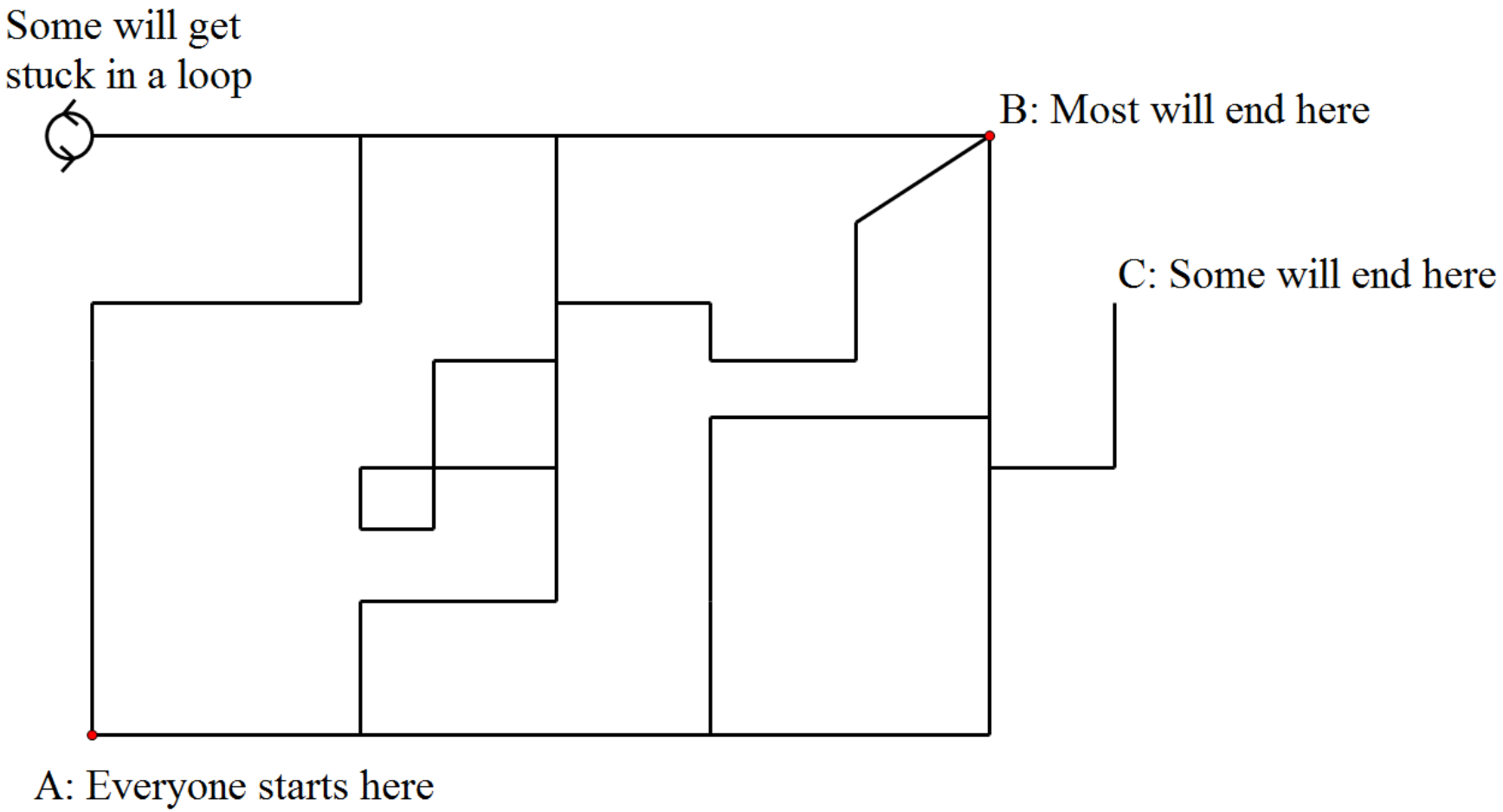
Representational

Abstract

Problem Solving

Multiple Approaches

Built in Differentiation:



Problem Solving

Playing with Problems

The rest of this session is devoted to exploring a selection of problems and critiquing them against the criteria outlined above.

HOMEWORK

This is the first of a 2 session presentation on Problem Solving. Over the next few weeks, you are encouraged to find or develop your own problem solving tasks to share with the group in session 2.