Number and Place Value

In the Australian Curriculum

•Janine McIntosh: janine@amsi.org.au

•Michael O’Connor: moconnor@amsi.org.au

Number and Place Value

What is Number?

How are whole numbers, fractions and decimals connected?

What do we understand by the term Place Value?

What order and sequence best facilitates understanding of place value?

Number and Place Value

In the Australian Curriculum

There is a Sub-Strand called Number and Place Value

There are also Number concepts in other sub-stands that will play important roles in understanding as students progress through the years.

Number and Place Value

F	Number and Place Value	Names numbers in sequences, initially to and from 20, moving from any starting point	ACMNA001
F	Number and Place Value	Connects numbers names, numerals, and quantities, including zero, initially up to 10 and then beyond	ACMNA002
F	Number and Place Value	Subitises recognises the number of objects in a collection without consciously counting) small collections of objects	ACMNA003
F	Number and Place Value	Uses words such as "more", "less", "same as" to compare and order items, to 20, and gives reasons	ACMNA289
F	Number and Place Value	Uses words such as "first" and "second" to indicate ordinal position	ACMNA289
F	Number and Place Value	Represents practical situations to model addition and sharing	ACMNA004

Why is subitising so important?

It is an indication, often the first indication, of internalisation of number generally and individual numbers specifically.

Number and Place Value

1	Number and Place Value	Develops confidence with number sequences to and from 100 by ones from any starting point. Skip counts by twos, fives and tens starting from zero	ACMNA012
1	Number and Place Value	Recognises, models, reads, write and order numbers to at least 100. Locates these numbers on a number line	ACMNA013
1	Number and Place Value	Counts collections to 100 by Partitioning numbers using place value. Understands the importance of grouping in tens	ACMNA014
1	Number and Place Value	Understands that 2 digit numbers are comprised of tens and ones	ACMNA014
1	Number and Place Value	Represents and solves simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts	ACMNA015
1	Number and Place Value	Develops a range of mental strategies for addition and subtraction problems	ACMNA015
1	Fractions and Decimals	Recognises and describes one-half as one of two equal parts of a whole	ACMNA016
1	Patterns and Algebra	Uses place-value patterns beyond the teens to generalise the number sequence and predicts the next number	ACNMA018
1	Patterns and Algebra	Investigates patterns in the number system, such as the occurrence of a particular digit in the numbers to 100	ACMNA018

Number and Place Value

2	Number and Place Value	Investigates number sequences, initially those increasing and decreasing by twos, threes, fives and ten from any starting point, then moving to other sequences	ACMNA026
2	Number and Place Value	Recognises, models, represents and orders numbers to at least 1000	ACMNA027
2	Number and Place Value	Groups, partitions and rearranges collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting	ACMNA028
2	Number and Place Value	Understands three-digit numbers are comprised of hundreds, tens and ones	ACMNA028
2	Number and Place Value	Partitions numbers to understand the connection between addition and subtraction	ACMNA029
2	Number and Place Value	Uses counting-on to identify the missing element in an addition problem	ACMNA029
2	Number and Place Value	Solves simple addition and subtraction problems using a range of efficient mental and written strategies such as commutativity for addition (order does not affect result – 5+3 is the same as 3+5), building to 10 (rainbow facts), doubles, 10s facts, and adding 10	ACMNA030
2	Number and Place Value	Models and represents simple addition problems using materials such as 10 frames, 20 frames and empty number lines	ACMNA030

2	Number and Place Value	Recognises and represents multiplication as repeated addition, groups and arrays	ACMNA031
2	Number and Place Value	Recognises and represents division as grouping into equal sets and solves simple problems using these representations	ACMNA032
2	Number and Place Value	Identifies the difference between dividing a set of objects into three equal groups and dividing the same set of objects into groups of three	ACMNA032
2	Fractions and Decimals	Recognises and interprets common uses of halves, quarters and eighths of shapes and collections	ACMNA033
2	Fractions and Decimals	Recognises that sets of objects can be portioned in different ways to demonstrate fractions	ACMNA033
2	Fractions and Decimals	Relates the number of parts to the size of a fraction	ACMNA033
2	Money and Financial Mathematics	Counts and orders small collections of Australian coins and notes according to their value	ACMNA034
2	Money and Financial Mathematics	Identifies equivalent values such as 2 x 5c coins has the same value as 1 x 10c coin	ACMNA034
2	Patterns and Algebra	Describes patterns by skip counting (in 2s, 5s, 10s) and identifies missing elements	ACMNA035

Number and Place Value

3	Number and Place Value	Investigates the conditions required for a number to be odd or even and identifies odd and even numbers	ACMNA051
3	Number and Place Value	Recognises, models, represents and orders numbers to at least 10 000	ACMNA052
3	Number and Place Value	Places four digit numbers on a number line using an appropriate scale	ACMNA052
3	Number and Place Value	Applies place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems. Recognises that 10 000 equals 10 thousands, 100 hundreds, 1000 tens and 10 000 ones	ACMNA053
3	Number and Place Value	Recognises and explains the connection between addition and subtraction by using portioning or by writing equivalent number sentences	ACMNA054
3	Number and Place Value	Recalls addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation	ACMNA055
3	Number and Place Value	Recognises that certain single-digit number combinations always result in the same answer for addition and subtraction and using this knowledge for addition and subtraction of larger numbers	ACMNA054

3	Number and Place Value	Recalls multiplication facts of two, three, five and ten and related division facts	ACMNA056
3	Number and Place Value	Represents and solves problems involving multiplication (for example, writing simple word problems in numerical form and vice versa) using efficient mental and written strategies and appropriate digital Technologies	ACMNA057
3	Fractions and Decimals	Partitions areas, lengths and collections to create halves, thirds, quarters and fifths, such as folding the same sized sheets of paper to illustrate different unit fractions and comparing the number of parts with their sizes	ACMNA058
3	Fractions and Decimals	Locates unit fractions on a number line	ACMNA058
3	Money and Financial Mathematics	Represents money values in multiple ways and counts the change required for simple transactions to the nearest five cents	ACMNA059
3	Money and Financial Mathematics	Recognises the relationship between dollars and cents and that not all countries use these denominations and divisions (for example Japanese Yen)	ACMNA059
3	Using Units of Measurement	Recognises and uses centimetres and meters, grams and kilograms and millilitres and litres.	ACMMG061

Number and Place Value

4	Number and Place Value	Investigates and uses the properties of odd and even numbers	ACMNA071
4	Number and Place Value	Reproduces five-digit numbers in words using their numerical representations, and vice versa	ACMNA072
4	Number and Place Value	Applies place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems	ACMNA073
4	Number and Place Value	Recognises and demonstrates that the place-value pattern is built on the operations of multiplication or division of tens	ACMNA073
4	Number and Place Value	Investigates number sequences involving multiples of 3, 4, 6, 7, 8, and 9	ACMNA074
4	Number and Place Value	Recalls multiplication facts up to 10 × 10 and related division facts	ACMNA075

4	Number and Place Value	Develops efficient mental and written strategies (for example, commutativity, doubling and halving) and uses appropriate digital technologies for multiplication and for division where there is no remainder	ACMNA076
4	Fractions and Decimals	Investigates equivalent fractions by exploring the relationship between families of fractions (halves, quarters and eights or thirds and sixths) by folding a series of paper strips to construct a fraction wall for example	ACMNA077
4	Fractions and Decimals	Counts by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line	ACMNA078
4	Fractions and Decimals	Coverts mixed numbers to improper fractions and vice versa	ACMNA078
4	Fractions and Decimals	Recognises that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation	ACMNA079

Number and Place Value

5	Number and Place Value	Identifies and describes factors and multiples of whole numbers and uses them to solve problems	ACMNA098
5	Number and Place Value	Uses estimation and rounding to check the reasonableness of answers to calculations	ACMNA099
5	Number and Place Value	Applies mental strategies to estimate the results of calculations such as estimating the cost of a supermarket trolley load	ACMNA099
5	Number and Place Value	Solves problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies for example, area model, the Italian lattice method or the partitioning of numbers	ACMNA100
5	Number and Place Value	Solves problems involving division by a one digit number, including those that result in a remainder	ACMNA101
5	Number and Place Value	Uses efficient mental and written strategies and applies appropriate digital technologies to solve problems	ACMAN291

5	Fractions and Decimals	Compares and orders common unit fractions and locates and represents them on a number line	ACMNA102
5	Fractions and Decimals	Investigates strategies to solve problems involving addition and subtraction of fractions with the same denominator for example using jumps on a number line or making diagrams of fractions as parts of shapes	ACMNA103
5	Fractions and Decimals	Recognises that the number system can be extended beyond hundredths (thousandths and beyond)	ACMNA104
5	Fractions and Decimals	Compares, order and represent decimals by locating them on a number line	ACMNA105
5	Fractions and Decimals	Recognises that the number of digits after the decimal place is not equivalent to the value of the fraction	ACMNA105
5	Patterns and Algebra	Describes, continues and creates patterns with fractions, decimals and whole numbers resulting from addition and subtraction	ACMNA107
5	Patterns and Algebra	Uses number lines or diagrams to creates patterns involving fractions or decimals	ACMNA107
5	Using Units of Measurement	Choose appropriate units of measurement for length, area, volume, capacity and mass, recognising that some units of measurement are better suited for some tasks than others, for example, km rather than m to measure the distance between two towns	ACMMG108

Number and Place Value

6	Number and Place Value	Identifies and describes properties of prime, composite, square and triangular numbers	ACMNA122
6	Number and Place Value	Represents composite numbers as a product of their prime factors and using this form to simplify calculations by cancelling common primes	ACMNA122
6	Number and Place Value	Selects and applies efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers	ACMNA123
6	Number and Place Value	Investigates everyday situations that use positive and negative whole numbers and zero. Locates and represents these numbers on a number line	ACMNA124
6	Number and Place Value	Solves everyday addition problems involving positive and negative integers without developing formal rules for the operations (for example, using a number line and counting to find the resulting outside temperature if it is 5° C at 7pm and drops by 8°C overnight)	ACMNA124
6	Fractions and Decimals	Compares fractions with related denominators and locates and represents them on a number line	ACMNA125
6	Fractions and Decimals	Solves problems involving addition and subtraction of fractions with the same or related denominators	ACMNA126
6	Fractions and Decimals	Models and solves additive problems involving fractions by using methods such as jumps on number line, or by making diagrams of fractions as parts of shapes (	ACMNA126
6	Fractions and Decimals	Finds a simple fraction of a quantity where the result is a whole number, with and without digital technologies	ACMNA127
6	Fractions and Decimals	Adds and subtracts decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers	ACMNA128

6	Fractions and Decimals	Multiplies decimals by whole numbers and perform divisions that result in terminating decimals. This includes interpreting and representing the remainder in division calculations appropriate to the context (eg: 6.5÷4 is sensibly expressed as 1.625km if dividing 6.5km into 4 equal legs or $1.63 if dividing $6.50 by 4)	ACMNA129
6	Fractions and Decimals	Use and explain the use of multiplication and division by powers of 10 to multiply decimal numbers mentally	ACMNA130
6	Fractions and Decimals	Connecting fractions, decimals and percentages as different representations of the same number moving flexibly between representations, and choosing the appropriate one for the problem being solved	ACMNA131
6	Money and Financial Mathematics	Investigate and calculate percentage discounts of 10%, 25% and 50% on sale items, with and without digital technologies	ACMNA132
6	Patterns and Algebra	Continue and create number patterns involving whole numbers, fractions and decimals.	ACMNA133

Number and Place Value

7	Number and Place Value	Investigate index notation and represent whole numbers as products of powers of prime numbers. Define and compare prime and composite numbers. Express whole numbers as products of powers of prime factors such as creating factor trees. Solve problems involving lowest common multiples and highest common factors	ACMNA149
7	Number and Place Value	Investigate and use square roots of perfect square numbers such as 25 and 36 and develop square root notation. Investigate between which two whole numbers a square root lies (eg: a √10 lies between 3 and 4)	ACMNA150
7	Number and Place Value	Apply the associative law: (2 + 3) + 5 = 2 = (3 + 5) and (2 x 3) x 5 = 2X (3 x 5), commutative law: 2 x 3 = 3 x 2 and 2 + 3 = 3 + 2 And distributive laws 18 x 17 = 8X (10 + 7) = 8 x 10 + 8 x 7 = 80 + 56 = 136	ACMNA151
7	Number and Place Value	Compare and order integers (positive and negative whole numbers), add and subtract integers with the aid of a number line. (eg: -2+-3=-5)	ACMNA280
7	Real Numbers	Compare fractions using equivalence, (for example, exploring equivalence among families of fractions by using a fraction wall or a number line 2/3 is the same as 4/6 and 6/9).	ACMNA152
7	Real	Locate and represent fractions and mixed numerals	ACMNA152

7	Real Numbers	Solve problems involving addition and subtraction of fractions, including those with unrelated denominators	ACMNA153
7	Real Numbers	Multiply and divide fractions and decimals using strategies including patterning and multiplication as repeated addition with both concrete materials and digital technologies, and identifying the processes for division as the inverse of multiplication.	ACMNA154
7	Real Numbers	Express one quantity as a fraction of another, with and without the use of digital technologies	ACMNA155
7	Real Numbers	Round decimals to a specified number of decimal places	ACMNA156
7	Real Numbers	Connect fractions, decimals and percentages and carry out simple conversions	ACMNA157
7	Real Numbers	Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies.	ACMNA158

Number and Place Value

8	Number and Place Value	Use index notation with numbers to establish the index laws with positive integral indices and the zero index	ACMNA182
8	Number and Place Value	Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies	ACMNA183
8	Real Numbers	Investigate terminating and recurring decimals	ACMNA184
8	Real Numbers	Investigate the concept of irrational numbers, including π	ACMNA186
8	Using Units of Measurement	Choose appropriate units of measurement for area and volume and convert from one unit to another.	ACMMG195

Number and Place Value

9	Real Numbers	Apply index laws to numerical expressions with integer indices. Use index laws to simplify algebraic expressions, using both positive and negative integral indices. Apply understanding of negative indices to calculations.	ACMNA209
9	Real Numbers	Express numbers in scientific notation. Represent extremely large and small numbers in Scientific notion and vice versa	ACMNA210
10	Patterns and Algebra	Apply the four operations to simple algebraic fractions with numerical denominators EG: Solve a wide range of linear equations, including those involving one or two simple algebraic fractions, and check solutions by substitution	ACMNA232
10A	Real Numbers	Define rational and irrational numbers and perform operations with surds and fractional indices. Understand that the real number system includes irrational numbers and that certain subsets of the real number system have particular properties. Apply the index laws to numeric and algebraic expressions and evaluating or simplifying them as required	10

Number and Place Value

Content

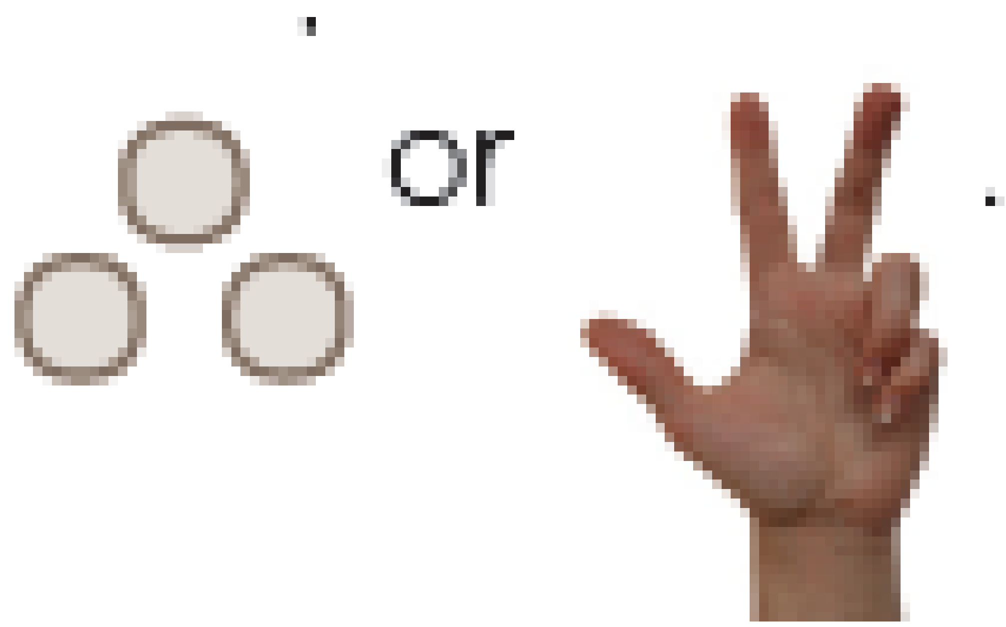
Number and Place Value

Prior knowledge

Children arriving for their first year of school may be able to:

- Recite the numbers up to 20 in order
- Write the numerals 0 to 9
- Grasp the connection between the

numeral ‘3’, the word ‘three’ and a picture such as



Number and Place Value

Hindu Arabic Numerals

Hindu-Arabic numerals exhibit some of the qualities that make mathematics so powerful, namely

- they can be used by understanding a small number of ideas, and
- they can be generalized beyond the original setting for which they were devised.

(The notation was developed to express whole numbers, but it extends to the representation of fractions and decimals.)

Number and Place Value

Hindu Arabic Numerals

Hindu-Arabic numerals are a decimal, or base-ten, place-value number system with the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 as fundamental building blocks.

Number and Place Value

Other Number Systems

What other number systems exist or have existed in the past?

What advantages does the Hindu-Arabic system have over the alternatives?

Number and Place Value

The Number System

Counting numbers or Natural numbers

{1, 2, 3, 4, 5, 6, 7, …}

Whenever you add or multiply two counting numbers, you get another counting number. This may not be true when you subtract or divide (e.g., 4-7 = -3, and –3 is not a counting number even though 4 and 7 are).

Number and Place Value

The Number System

Whole numbers

$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$

These are the counting numbers, together with 0.

Number and Place Value

The Number System

Integers

Positive and negative whole numbers

{… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, …}

Note that you can add, subtract and multiply two integers and you will always get an integer, but you may have trouble with division (e.g., $3 \div 6 = 1/2$).

Number and Place Value

The Number System

Rational numbers $\{\frac{m}{n}$, where m and n are integers$\}$

For example, $\frac{2}{3}$, $\frac{7}{11}$, $\frac{-34}{97}$, (all fractions)

$7 = \frac{7}{1}$, -1000, (all integers)

0,

$0.15 = \frac{15}{100}$, $0.555... = \frac{5}{9}$ (some decimals)

Number and Place Value

The Number System

Rational numbers are just fractions.

Integers can always be written as fractions

In decimal form, the rationals are those numbers which terminate or those which have a recurring block.

Division of a rational by a rational always gives a rational (except that you can't divide by 0).

Number and Place Value

The Number System

Irrational numbers

Cannot be written in the form $\frac{m}{n}$, i.e, not rational.

Examples include:

$\pi = 3.1415926535879...$,

$0.101001000100001000001...$,

$e = 2.71828...$,

$0.1234567891011121314151617...$

$\sqrt{2} = 1.4142...$

Number and Place Value

The Number System

Irrational numbers

The irrationals are those numbers that cannot be written as a fraction.

In decimal form, the irrational numbers are precisely the non-recurring non-terminating decimals.

There are actually more irrational numbers than there are rational numbers.

Number and Place Value

The Number System

Real numbers The rational numbers together with the irrational numbers make up the real numbers. These are the numbers we use in everyday life.

Complex numbers

[Learned in late high school or university, if at all.]

There are numbers which help us deal with things like $\sqrt{-5}$ (i.e. square roots of negative numbers). The real numbers can’t do this. These are the numbers that make digital devices possible.

Number and Place Value

Learning Numbers

Even though the Number System is built up logically in this way, it is not how we learn or develop an understanding of numbers.

Both individually and historically, number sense follows a different path.

Counting (including 0)

Fractions

Decimals (Rationals)

Negatives

Irrationals

Number and Place Value

Learning to Count - History

Early counting was done via one-to-one correspondence.

The word calculus means stone. It is the root of the word calculation because shepherds used pebbles to count their sheep.

Number and Place Value

Ten as the basis of our place value system

Hindu-Arabic notation is a place value system based on bundles of 10; so it is a decimal system.

The key to a place value system is the use of a place marker.

A place value system using 9 digits and a space or the word kha (for emptiness) as place marker was used in India the 6th century.

Number and Place Value

Ten as the basis of our place value system

By the 9th century the system had made its way to the Arab world (including Persia and Al-Andalus in what is now Spain).

The digit 0 evolved from “·” and was used in both Madhya Pradesh (Northern India) and the Arab world by the 10th century.

Leonardo Fibonacci learned to use the notation from merchants in Africa when he was a boy and wrote a book, Liber Abaci, in 1202 which popularized the system.

Number and Place Value

Learning to Count

Number is an abstract concept. Three is the concept of threeness.

Numerals

A numeral is a symbolic representation of a number.

So 7, 5 + 2, and VII are all numerals for the number we call seven;

10, 23 – 13 and X are all numerals for the number we call ten.

Number and Place Value

Learning to Count

Recitation of a list of numbers *is not necessarily counting*.

Many children can recite the number names when they arrive at school.

Some of them may be able to recite them in order up to 20 or more.

It is likely, however, that some of them are only just beginning to understand that each numeral and its corresponding number word represent a quantity that is fixed.

Number and Place Value

Learning to Count

For children to be considered as having the ability to count, there are certain behaviours that must be evident;

Each of these behaviours is essential to counting.

Number and Place Value

Learning to Count

Digits

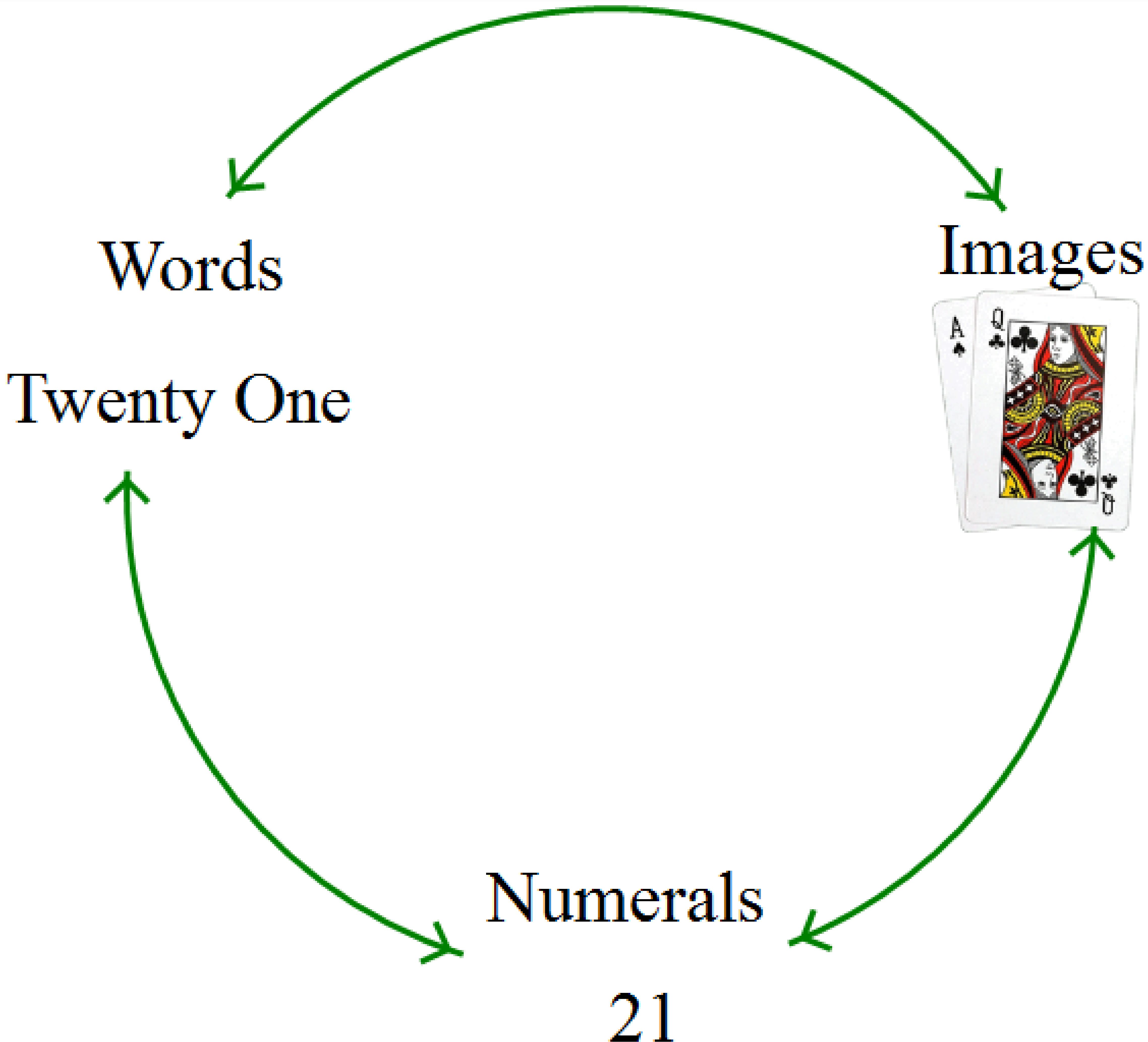
We call the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 digits. They are numerals consisting of a single symbol.

(It is not a coincidence that the same word means fingers.

Early counting was, and still is, done using fingers.)

Number and Place Value

Representation of a Number



Up until this slide we have been using words, numerals and symbols or images for numbers, interchangeably.

It is only with the numeric notation that place value has obvious meaning and relevance.

Number and Place Value

Representation of a Number

draw a picture

write the words

write a number

draw the number

Number and Place Value

Representation of a Number

The digits 0 to 9 are to numbers

what the letters of the alphabet are to words.

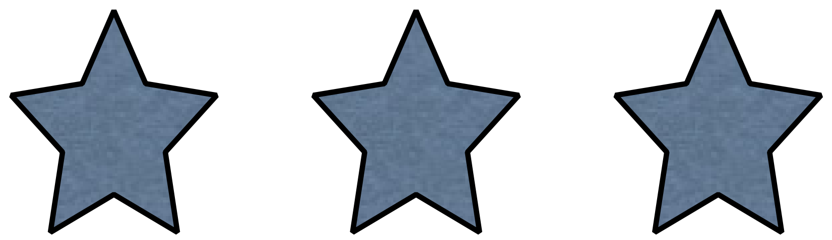
With just ten symbols we can write down any number imaginable

Number and Place Value

Learning to Count

One-to-one correspondence with number names and objects

What goes through your head when:



The understanding that we use different number names for each object included in a count is a major milestone.

A child who counts a set of objects by saying “four, two, one” has understood the need for one-to-one correspondence, as has the child who correctly counts “one, two, three”.

A child who counts “three, two, three” has not used a unique number name for each object and neither has the child who uses more number names than objects and counts “one, three, six, seven, two, four” when counting these stars

Number and Place Value

Learning to Count

Stable Order

To count accurately and reliably, it is important to say the number names in the right order and without skipping any.

The ability to assign the number names in order to objects being counted and without skipping any numbers, is known as the stable-order principle.

Number and Place Value

Learning to Count

Counting objects in any order or arrangement

The number of elements in a set does not depend on the way the objects are presented or the order in which they are counted.

Knowing that the order in which objects are counted has no relevance to the actual number of objects in the group of objects is known as the order-irrelevance principle.

Number and Place Value

Learning to Count

Cardinality

One of the deep observations about counting is that when you set up a 1-1 correspondence between the number names in their correct order and the set of objects you are trying to count, then the last number name you say is the cardinality (or size) of the set.

We can help children develop the understanding of cardinality by involving them in activities where they answer questions about ‘how many’.

Number and Place Value

Learning to Count

Ordering numbers

Ordering is the basis of our number system.

The ability to place quantities in order of increasing (or decreasing) size demonstrates a deep understanding of how the number system fits together.

Beginning with the idea of one more or one less, pre-school children become adept at moving around on a mental number line.

Classroom

Automaticity

Developing automaticity with number facts is the aim of many of the tasks we do with children.

The tasks themselves are not the aim.

To achieve this we need to help children pull apart numbers and put them back together again.

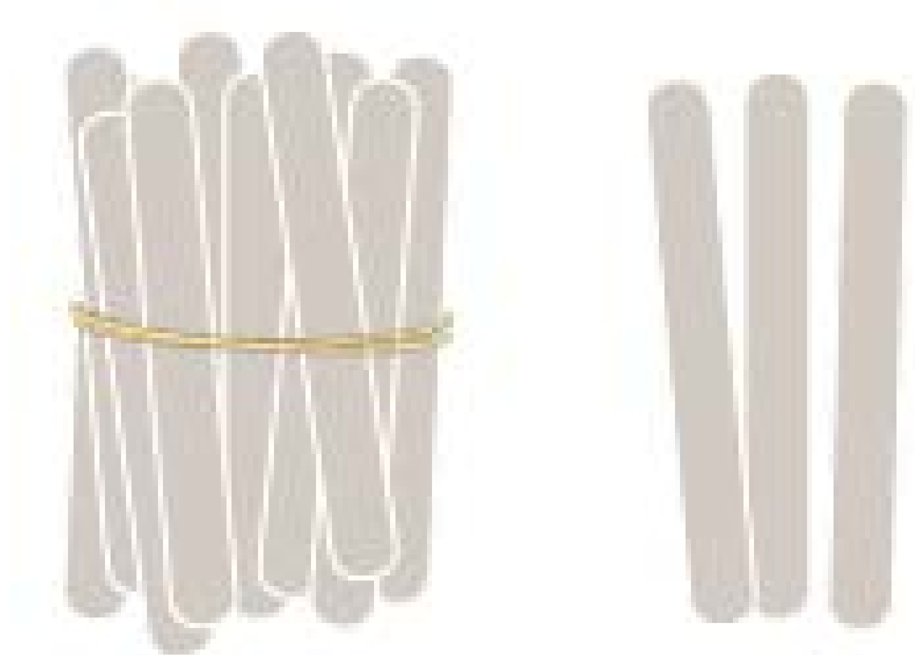
As children grow, tasks that require mental strategies need to become more cognitively demanding.

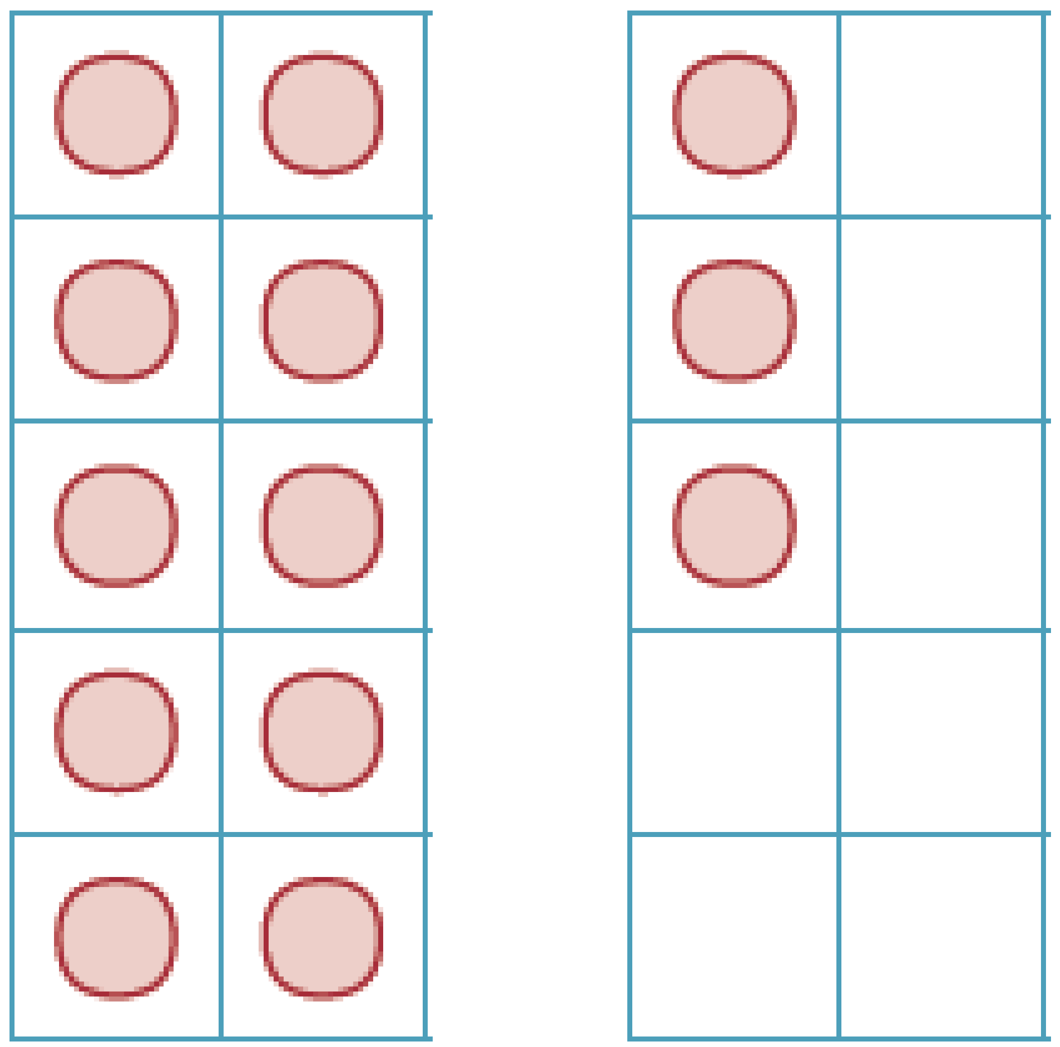
Children often develop these skills by themselves, other children need some gentle encouragement to do so.

Classroom

Ten as the basis of our place value system

Once the numbers below ten are established, the next goal is to look at the numbers from ten to twenty.





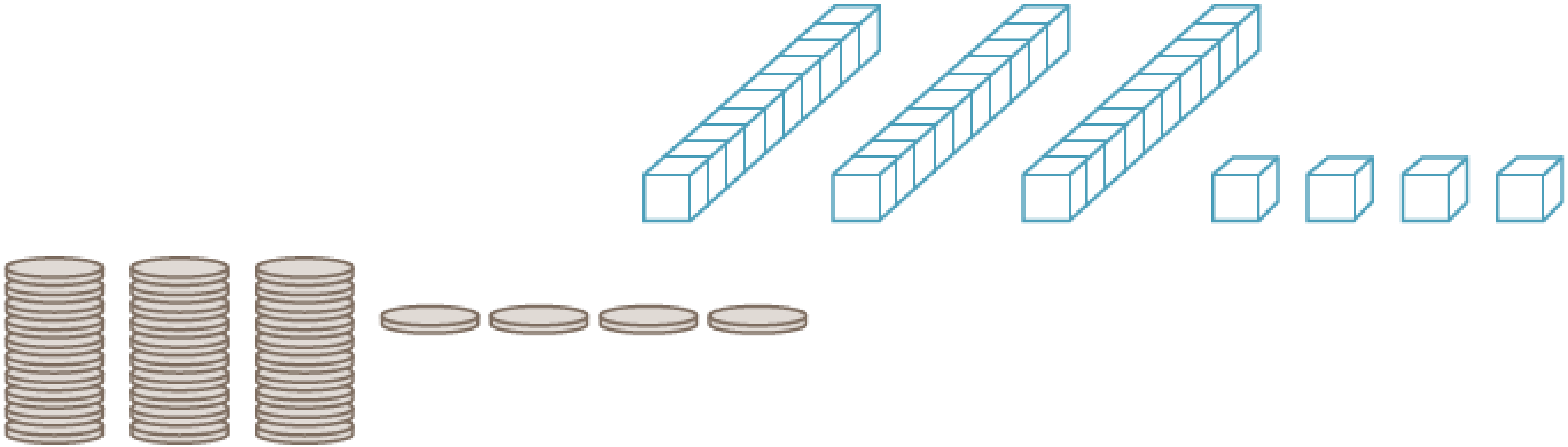
We want students to

- see the importance of ten
- start to use ten as a countable unit.

Classroom

Ten as the basis of our place value system

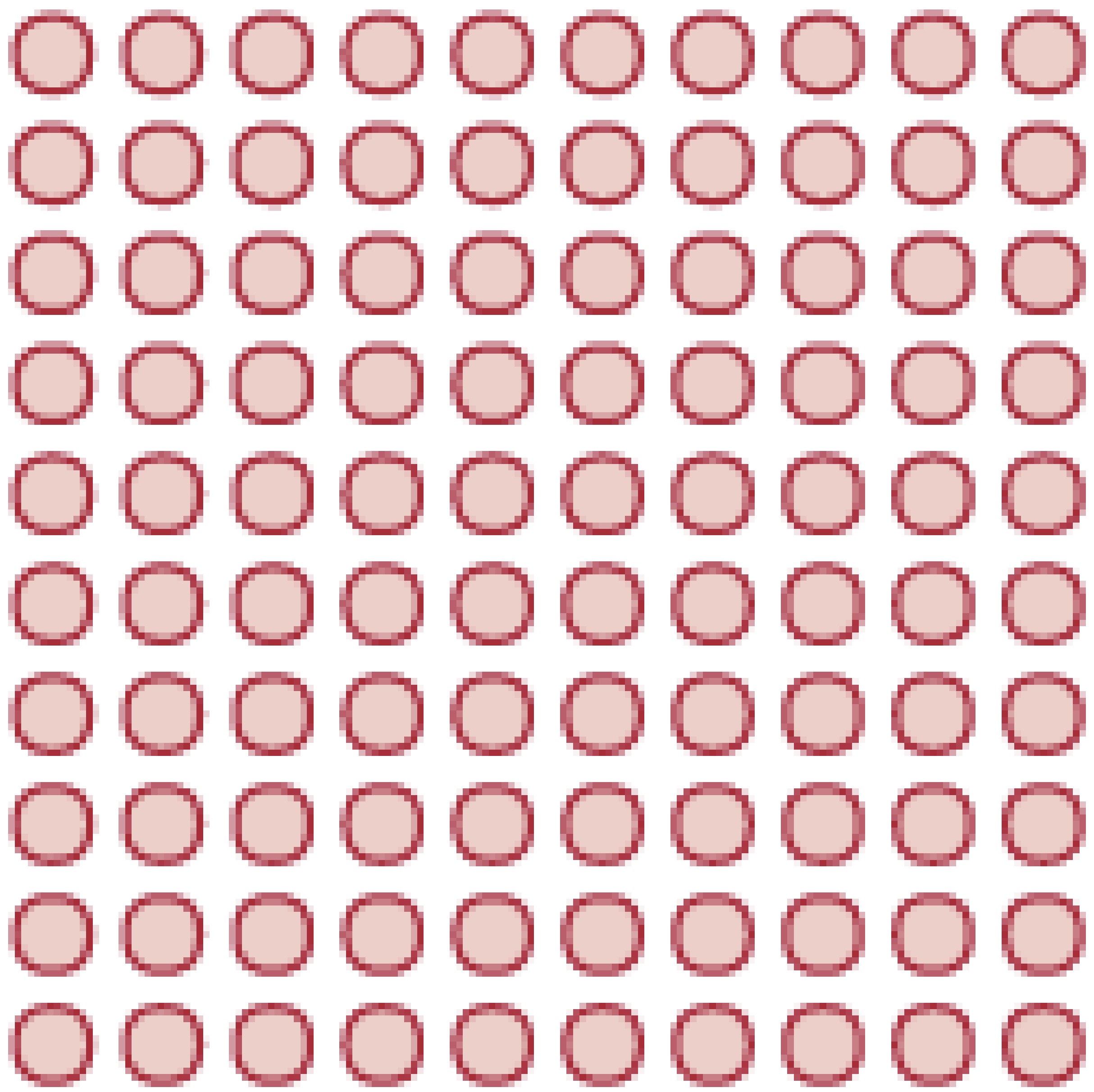
Use a variety of materials



Classroom

Ten as the basis of our place value system

- What does one hundred look like?



Classroom

Number Charts

100 chart

Make tables in word

Make puzzles out of them

Go over the hundred

Extend to thousands

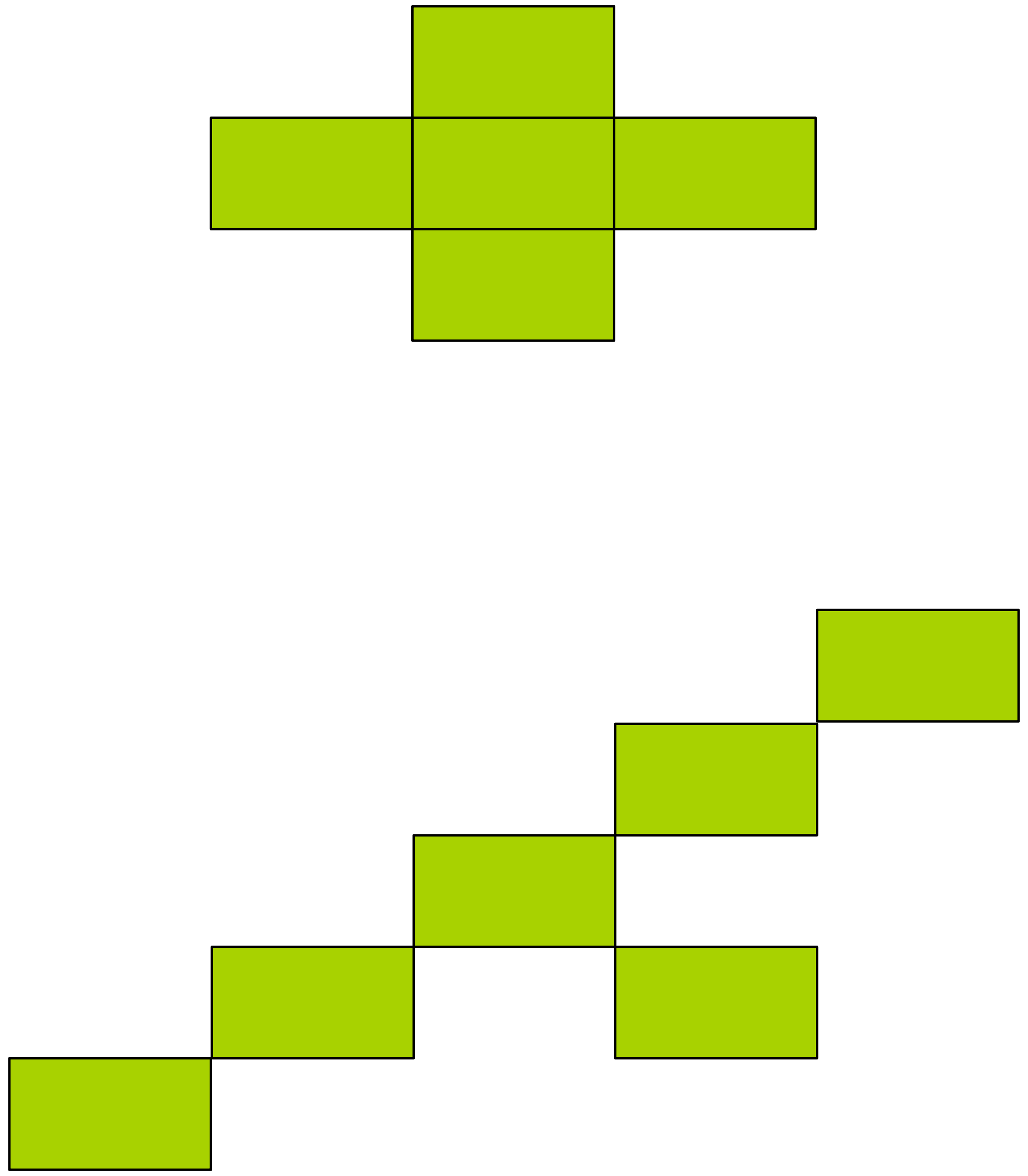
Number Ladder

Start at different numbers

Fill in the blanks

Classroom

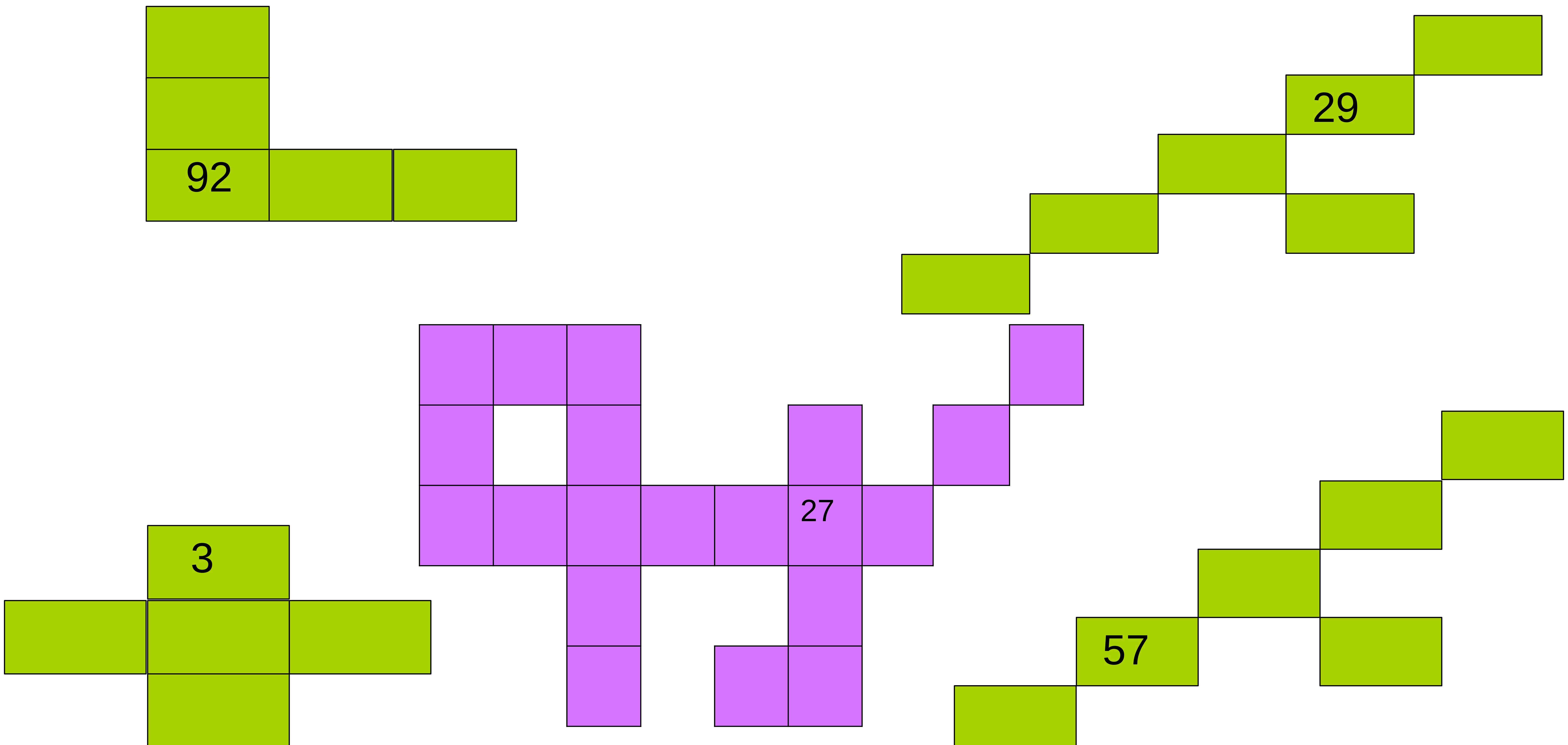
Number Charts



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Classroom

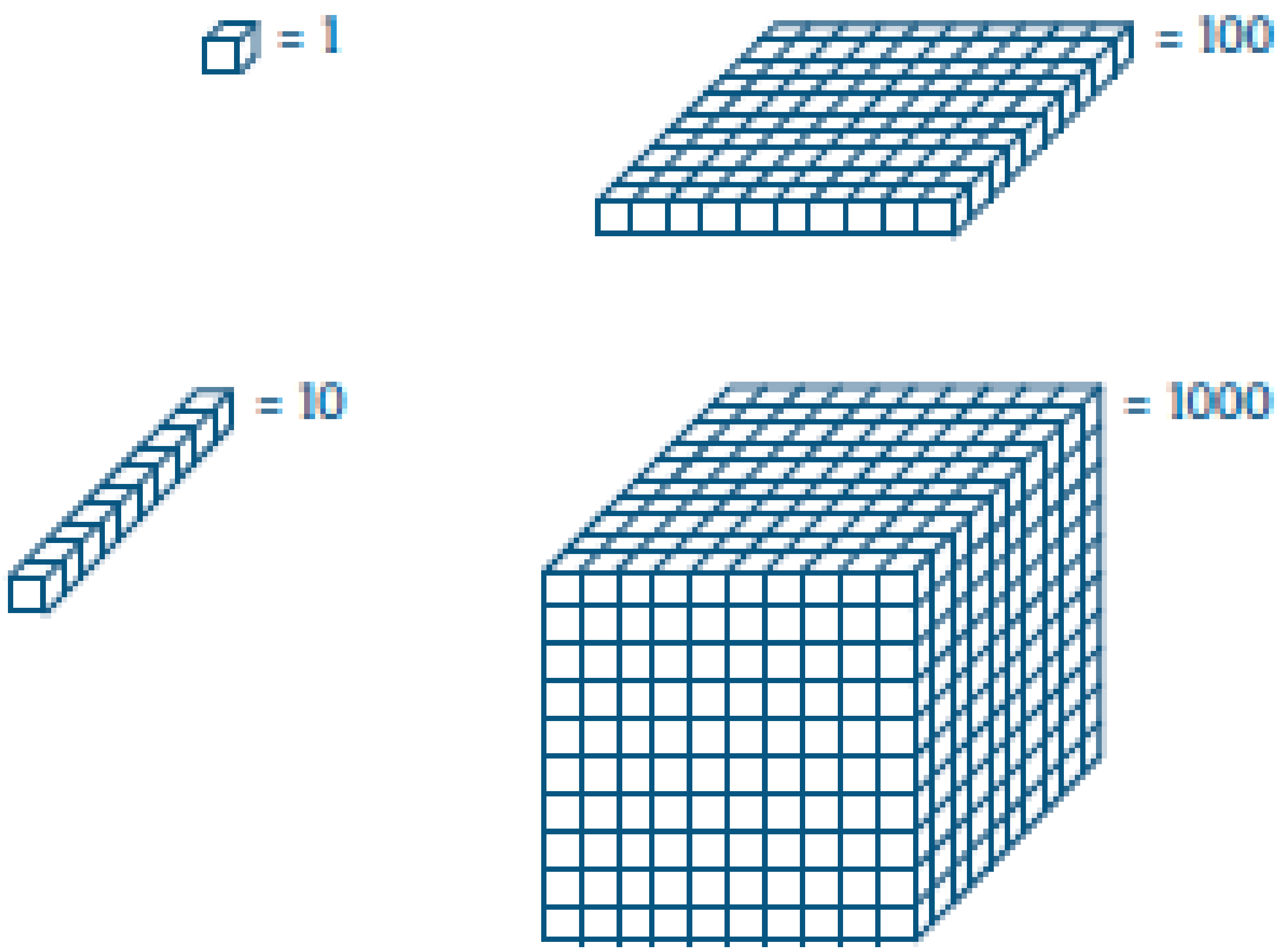
Number Charts



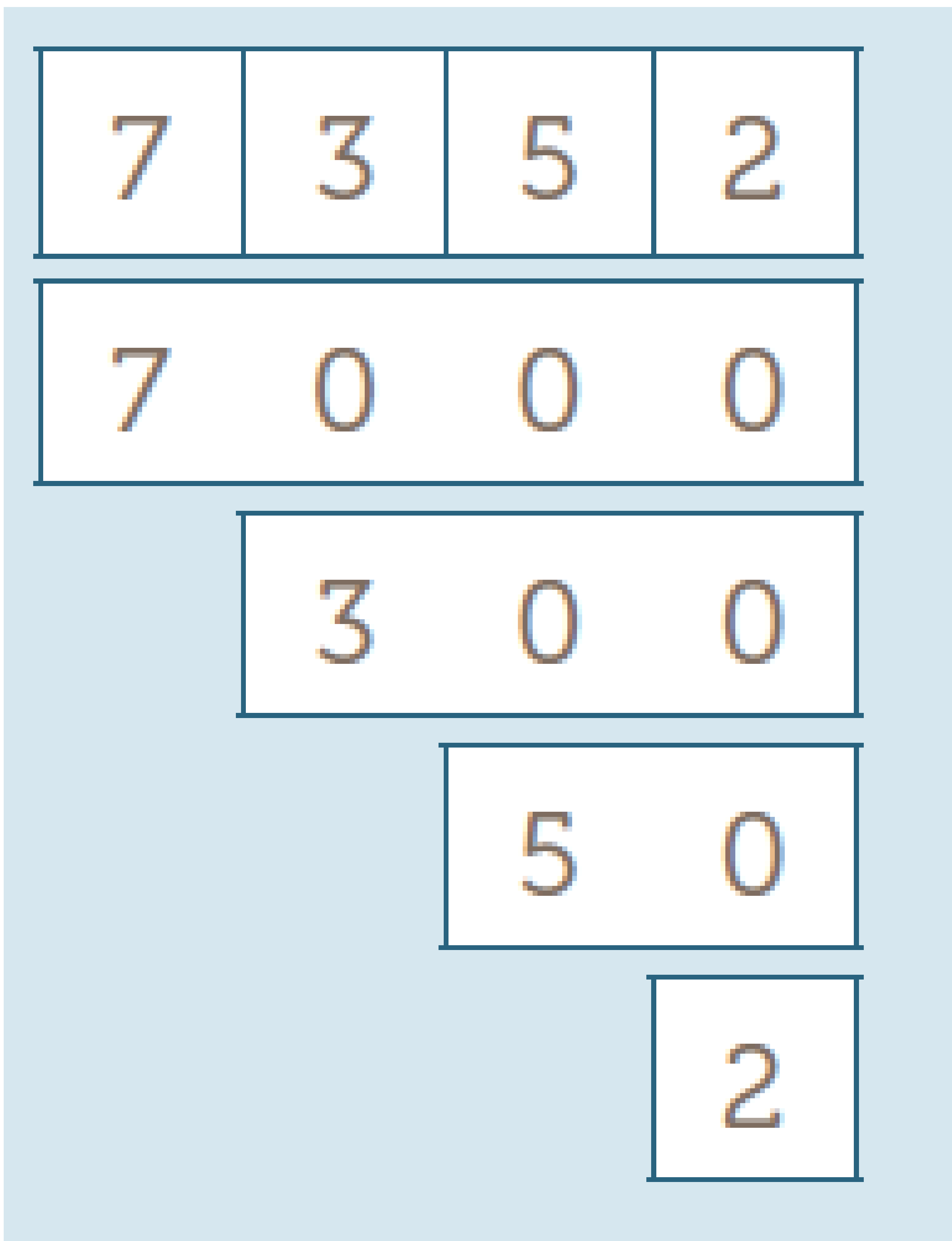
Try some for yourself

Classroom

Ten as the basis of our place value system



- What does one thousand look like?

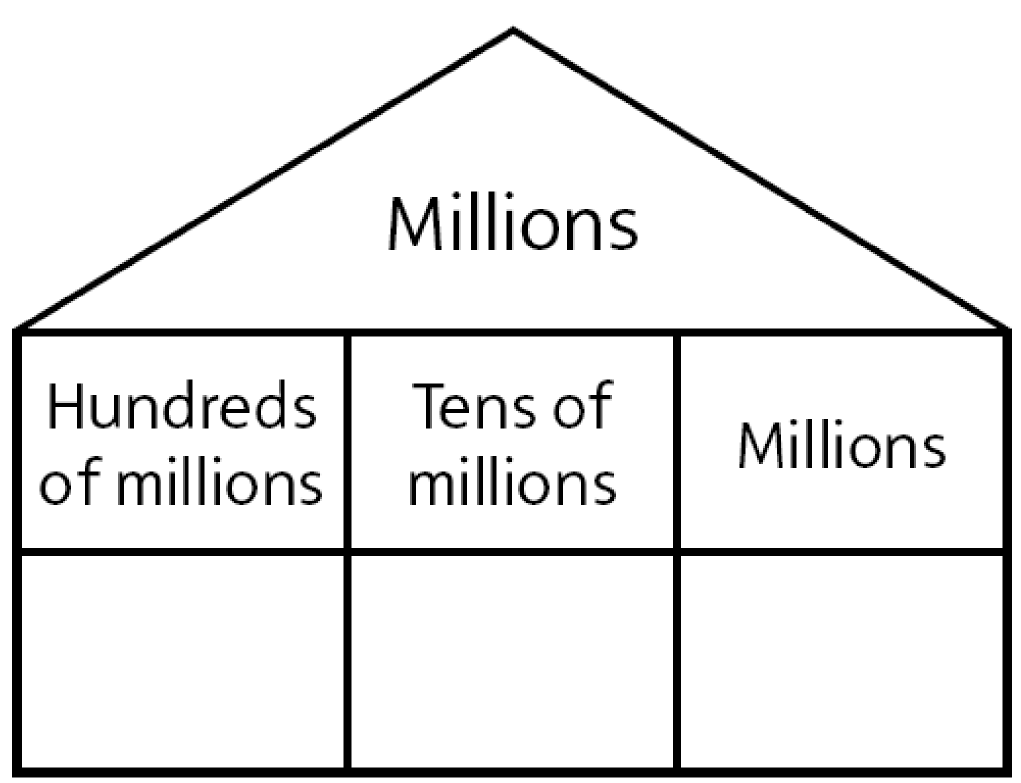


Classroom

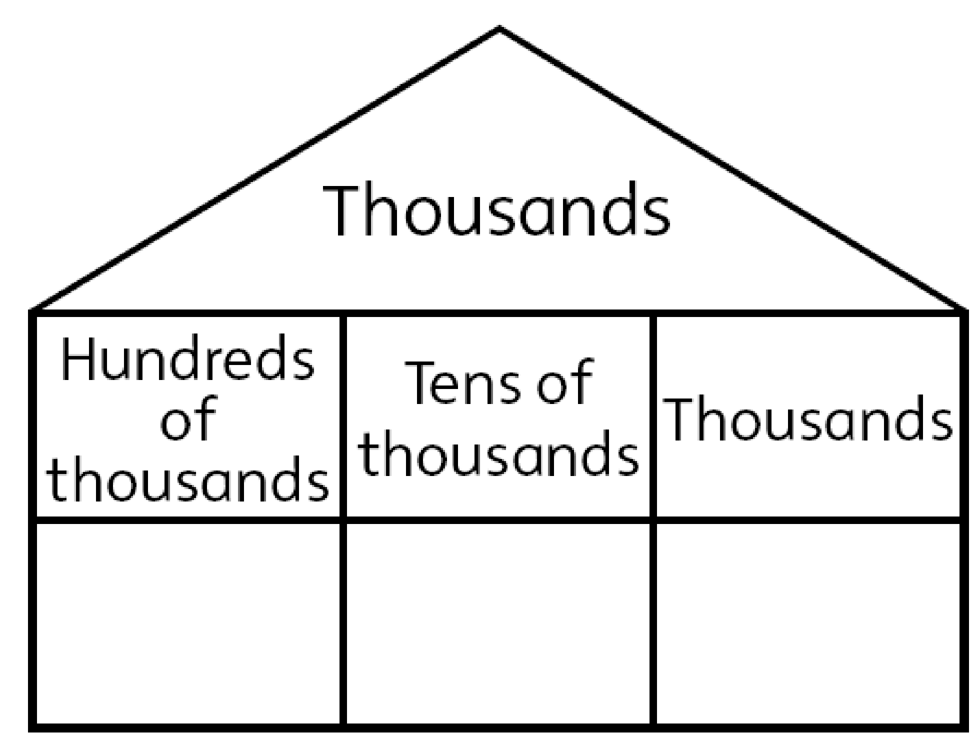
Ten as the basis of our place value system

Place Value houses

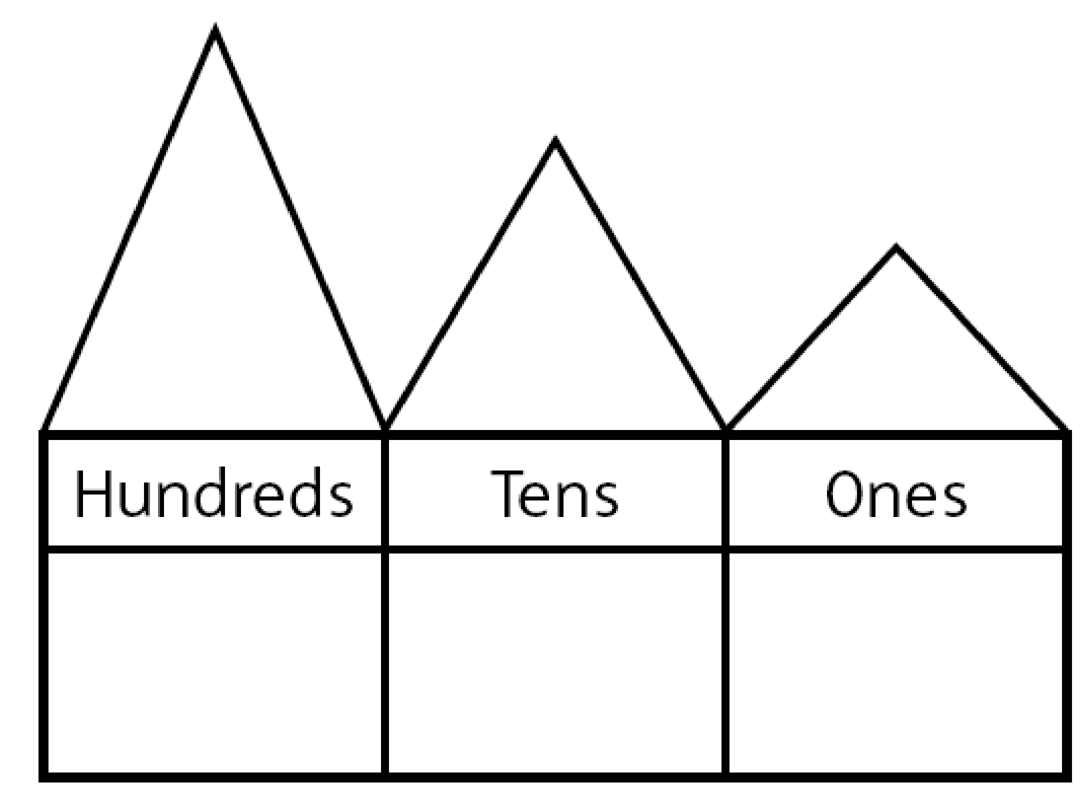
Mansions



Houses



Tents



Classroom

Ten as the basis of our place value system

Cushions?

Or the world's largest abacus?

Classroom

The Number Line

The number line can be used as a model for number from the first years of primary school through to the early years of secondary schooling. It is a model that can be consistently applied in most areas of number.

- Comparing
- Modelling
- Ordering
- Operations

Classroom

The Number Line

Like subitising and automatic recall of addition and multiplication facts, internalising the number line is a significant step toward understanding and being able to manipulate numbers.

Having an accurate internal number line gives confidence in the position and relationship between different numbers of all sorts.

Classroom

The Number Line

Introducing the number line

- Mark in zero and one other reference point
- Convention of negative numbers to the left, zero in the middle and positive numbers to the right
- Move towards children drawing their own

Step through the introduction of the number line very slowly

Do not assume this has been done before

Remind the children all the time, where is the one?

Classroom

The Number Line

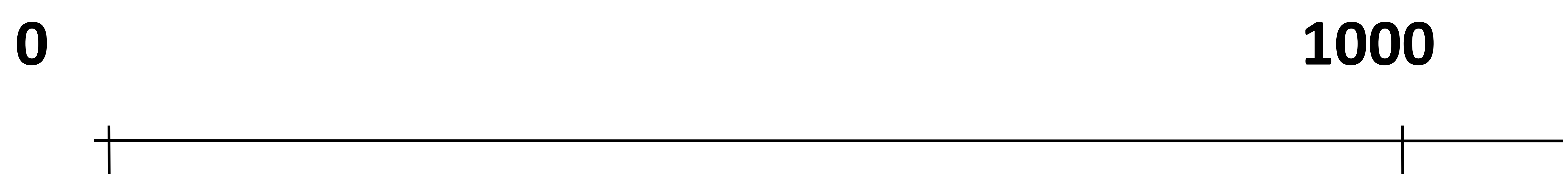
Use

- Masking tape on the floor
- String across the room
- Chalk in the playground
- Magnetized numbers on a blackboard or whiteboard
- Cash-register rolls
- Number ladder

Classroom

The Number Line

Put a cross where one hundred would be on this number line.



Classroom

Today’s Number

15

Tools session idea:

Today’s number is… from MCTP

Classroom

Today's Number

Use often to

- Reinforce number facts
- Engage students in thinking about strategies
- Allow students to pull apart and then reconstruct numbers

Use it to

- Settle and focus students
- Allow all students time to respond.
- Pre-test and post-test by counting number of facts in a minute with extra points for FAT facts.
- Insist on certain strategies. For example, students must include multiplying with fractions.

Classroom

Number Between

A good way to introduce the idea of the infinite nature of the place value system.

Give me a number between 0 and 1000.

Now give me a number between 0 and ________.

…..between 0 and ________.

Teaching Strategies

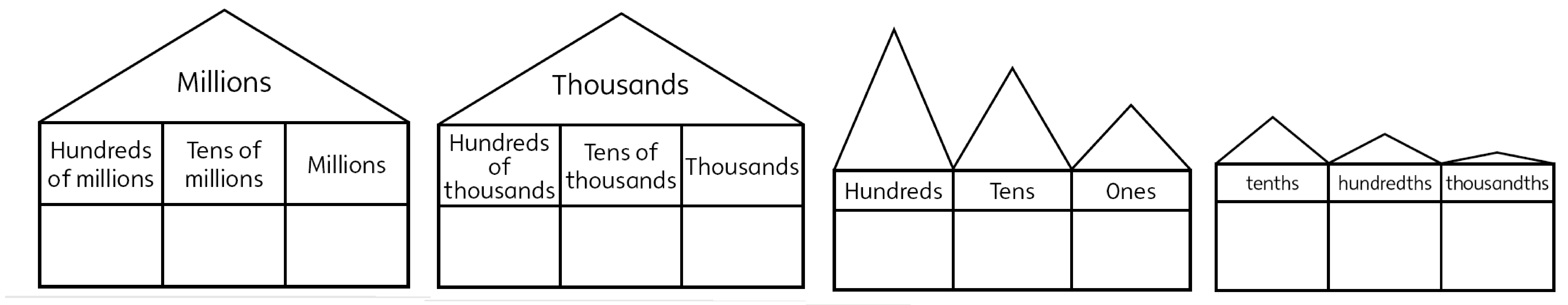
Number Between

- can be used for whole numbers, fractions and decimals
- shows students the density of decimals
- use a blank number line and choose two end points
- select two students and in turns, ask them to write a number in between the two end points
- rub off an end point once a new number has been written

Classroom

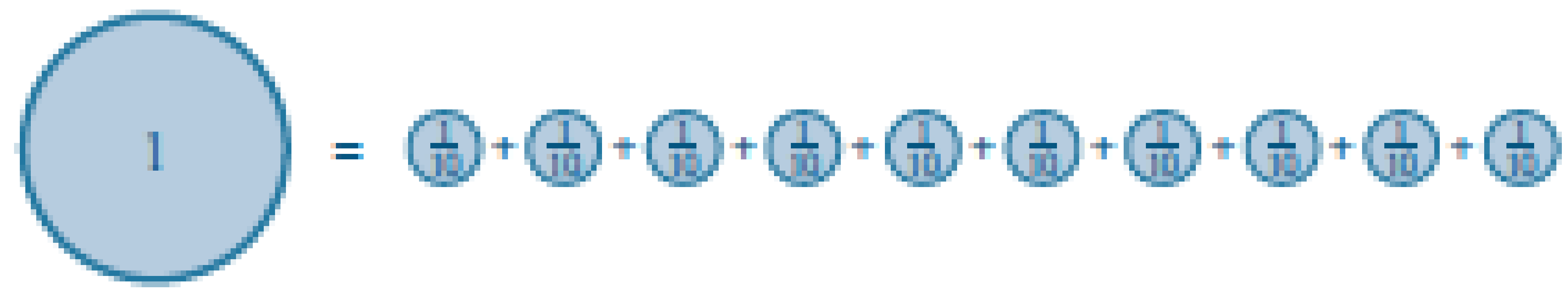
Ten as the basis of our place value system

Place Value houses



Classroom

Ten as the basis of our place value system

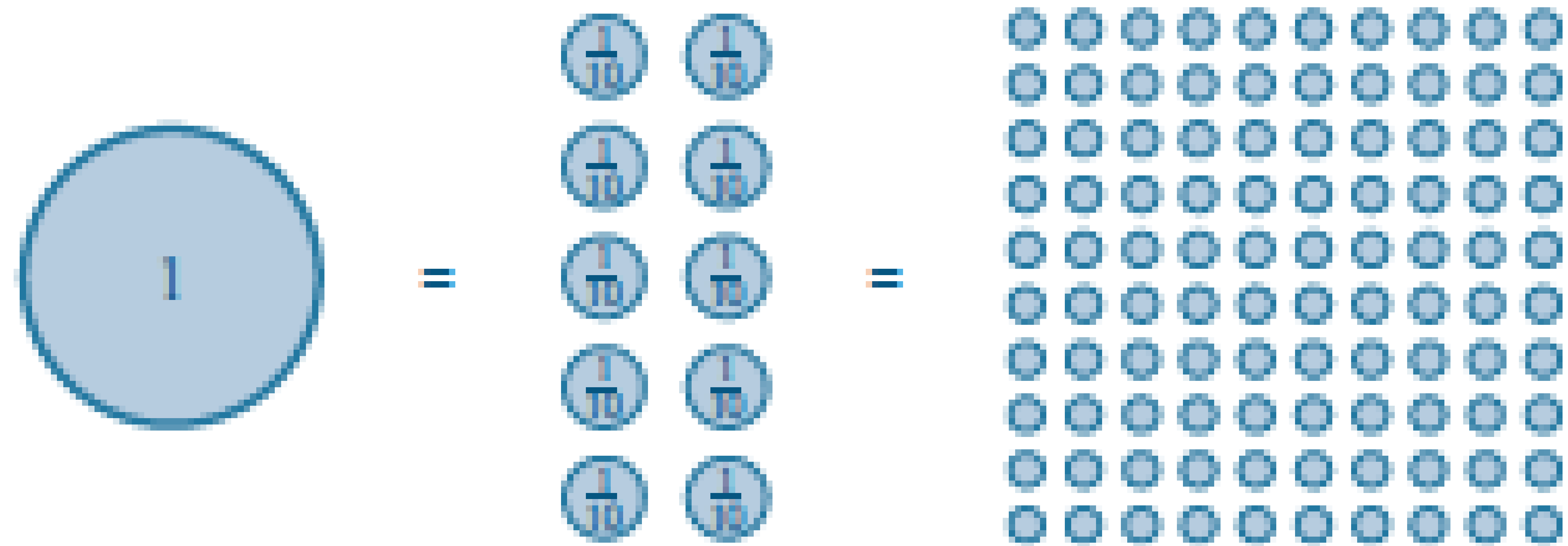


- What does one *tenth* look like?

Classroom

Ten as the basis of our place value system

- What does one *hundredth* look like?



Fractions

Fractions come before Decimals in the Curriculum

It is only at grade 4 that decimals are introduced and then it is after exploring the relationships between families of fractions.

Build up to tenths in fractions and only then bring in the concept of decimal notation.

It is also worth noting that it is also at grade 4 that the idea of place value is built on multiplication and division of ten (powers of ten)

Fractions

Fraction Families

Just like real families, fraction families are related to each other by their similarities.

The Halves Family (can be created just by paper folding)

<table>
<tr><td colspan="16">1</td></tr>
<tr><td colspan="8">$\frac{1}{2}$</td><td colspan="8">$\frac{2}{2}$</td></tr>
<tr><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{2}{4}$</td><td colspan="4">$\frac{3}{4}$</td><td colspan="4">$\frac{4}{4}$</td></tr>
<tr><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{2}{8}$</td><td colspan="2">$\frac{3}{8}$</td><td colspan="2">$\frac{4}{8}$</td><td colspan="2">$\frac{5}{8}$</td><td colspan="2">$\frac{6}{8}$</td><td colspan="2">$\frac{7}{8}$</td><td colspan="2">$\frac{8}{8}$</td></tr>
<tr><td>$\frac{1}{16}$</td><td>$\frac{2}{16}$</td><td>$\frac{3}{16}$</td><td>$\frac{4}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{6}{16}$</td><td>$\frac{7}{16}$</td><td>$\frac{8}{16}$</td><td>$\frac{9}{16}$</td><td>$\frac{10}{16}$</td><td>$\frac{11}{16}$</td><td>$\frac{12}{16}$</td><td>$\frac{13}{16}$</td><td>$\frac{14}{16}$</td><td>$\frac{15}{16}$</td><td>$\frac{16}{16}$</td></tr>
</table>

Fractions

Fraction Families

The Thirds Family (needs measurement and paper folding)

Start by using a strip of paper that is 90 cm long. This gives thirds that are thirty cm long, or the length of a standard ruler. An extension at a later time is to use 120 cm strips of paper, and thus thirds of 40 cm.

<table>
<tr><td colspan="18">1</td></tr>
<tr><td colspan="6">$\frac{1}{3}$</td><td colspan="6">$\frac{2}{3}$</td><td colspan="6">$\frac{3}{3}$</td></tr>
<tr><td colspan="3">$\frac{1}{6}$</td><td colspan="3">$\frac{2}{6}$</td><td colspan="3">$\frac{3}{6}$</td><td colspan="3">$\frac{4}{6}$</td><td colspan="3">$\frac{5}{6}$</td><td colspan="3">$\frac{6}{6}$</td></tr>
<tr><td colspan="2">$\frac{1}{9}$</td><td colspan="2">$\frac{2}{9}$</td><td colspan="2">$\frac{3}{9}$</td><td colspan="2">$\frac{4}{9}$</td><td colspan="2">$\frac{5}{9}$</td><td colspan="2">$\frac{6}{9}$</td><td colspan="2">$\frac{7}{9}$</td><td colspan="2">$\frac{8}{9}$</td><td colspan="2">$\frac{9}{9}$</td></tr>
</table>

Fractions

Fraction Families

The Fifths Family (needs measurement and paper folding)
Strips of length 120 cm

Once the tenths have been explored it is possible to discuss how they can be written as decimals.

1									
$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$	
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

Number and Place Value

Decimals

Decimals are fractions written in Place Value notation.

$\frac{1}{10}$ is 0.1

$\frac{1}{100}$ is 0.01

$\frac{1}{1000}$ is 0.001

When the denominator of a fraction is a power of 10, writing the decimal form is easy and straight forward.

When the denominator is not a power of 10, conversion becomes more complicated.

Number and Place Value

Converting between Fractions and Decimals

Convert these fractions to decimals

$\frac{1}{2} = 0.5$ $\frac{1}{25} = 0.04$ $\frac{1}{7} = 0.142857...$ $\frac{489}{1000} = 0.489$

$\frac{38}{40} = 0.95$ $\frac{47}{64} = 0.73437$ $\frac{3}{5} = 0.6$ $\frac{1}{99} = 0.010101...$

What do you notice?

Fractions with denominators that are multiples of 2s and 5s convert to terminating decimals.

Decimals

Decimal Misconceptions

Longer is larger

- whole number thinking
- column over flow thinkers take the name from the left most column so 0.35 is 35 tenths
- reverse thinkers believe the value of the column increases as we move to the right. 0.35 is larger than 0.41 because 53 is larger than 14.

Shorter is larger

- any number of tenths is more than any number of hundredths. So 0.4 is larger than 0.83
- reciprocal thinkers think that 0.3728 is like $^{1}/_{3728}$. They would choose 0.3 as larger than 0.4 because $^{1}/_{3}$ is larger than $^{1}/_{4}$

Money thinking

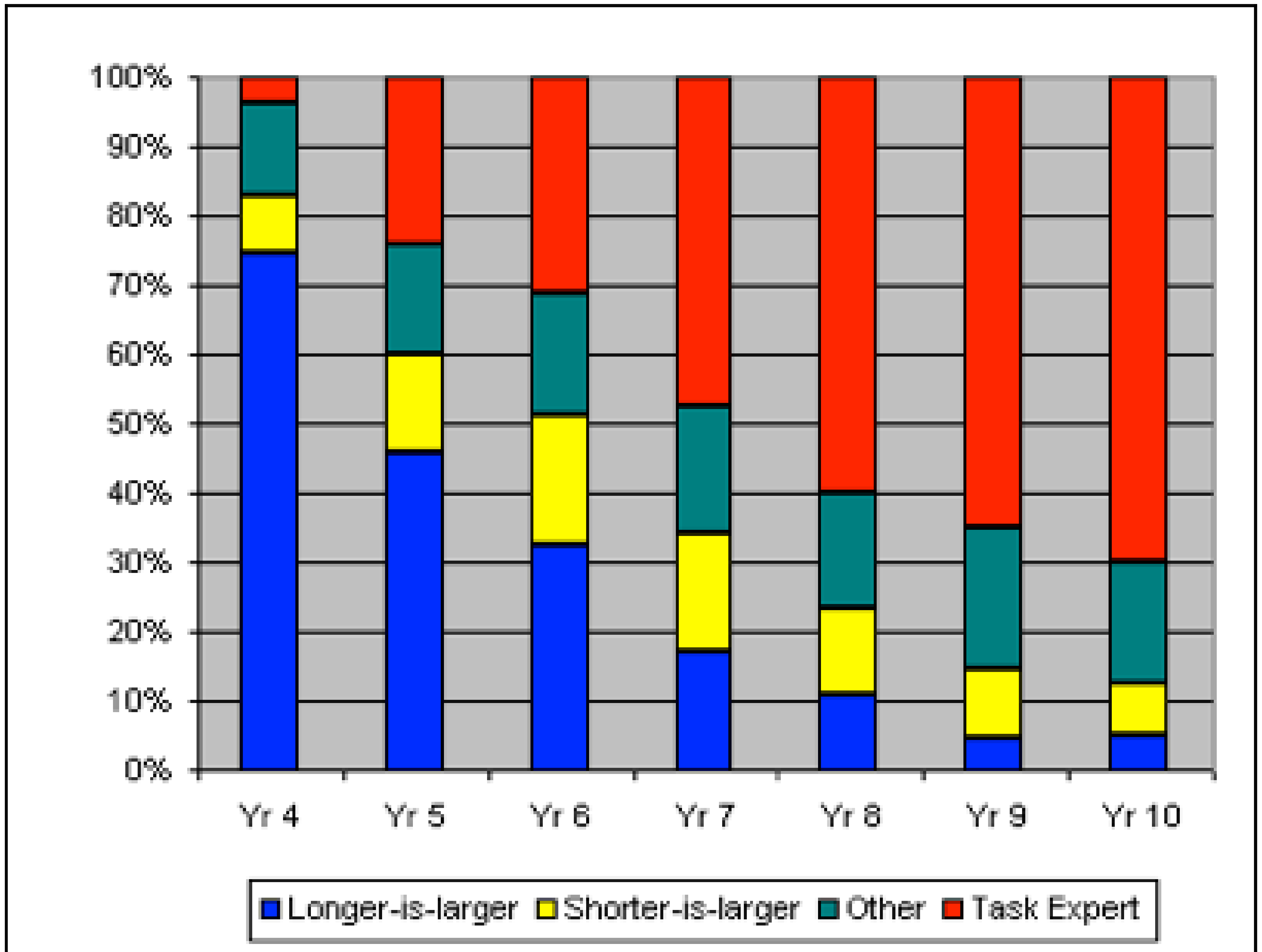
Zero does not matter

All decimals below zero (and past the negative numbers on the number line)

Decimals

Decimal Misconceptions

Our data is from over 3000 students aged from 10 to adult.



Acknowledgement: Dr Vicki Steinle,
'Teaching and Learning about Decimals'
The University of Melbourne.

Decimals

Prevalence of Misconceptions

Longer is larger: a longer decimal is larger than a shorter decimal

In Year 5 - 40% of students

By Year 10 - 5% of students

Misconceptions about Decimals

Longer is Larger

Whole number thinking

Students treat the portion of the number to the right of the decimal point as a whole number.

Therefore,

3.456 > 3.5 because 456 > 5

Misconceptions about Decimals

Longer is Larger

Column overflow thinking

Students interpret 0.12 as 12 tenths and 0.012 as 12 hundredths. They squeeze the number 12 into one column.

Therefore:

3.45 is three and forty-five tenths, and 3.5 is three and five tenths, so 3.45 > 3.5

Misconceptions about Decimals

Longer is Larger

Reverse thinking

Students assume that the column names on both sides of the decimal point are the same e.g.

..hundreds, tens, ones, . , tens, hundreds, thousands..

Therefore: 0.46 > 0.52 because 64 > 25

Decimals

Prevalence of Misconceptions

Shorter is larger: a shorter decimal is larger than a longer decimal

10% of all students from Year 5-Year 10

Misconceptions about Decimals

Shorter is Larger

Denominator focussed thinking

Students think that any number of tenths is greater than any number of hundredths, any number of hundredths is greater than any number of thousandths, and so on...

Therefore:

0.3 > 0.84 because tenths > hundredths

Misconceptions about Decimals

Shorter is Larger

Reciprocal thinking

Students view the decimal number to the right of the decimal point as something like the fraction formed by taking the reciprocal, e.g. 0.4 is viewed as 1/4.

Therefore:

0.12 > 0.3456 because 1/12 > 1/3456

Misconceptions about Decimals

Shorter is Larger

Negative thinking

Students display confusion between negatives and decimals, e.g. all decimals are below zero, or 0.0_ decimals are below zero.

Therefore:

If -12 > -18, then 0.12 > 0.18

Misconceptions about Decimals

Neither Longer or Shorter

Money thinking

Students relate all decimals to money e.g. 4.236 is shortened to 4.23 as in $4.23. Students appear to work well with decimals to hundredths but are not sure of the order of other numbers on the number line.

Therefore: 5.6786 = 5.67

Diagnosing Misconceptions

Quick Test

NAME:______________________

For each pair of decimal numbers, circle the one which is LARGER.

4.8	4.63
0.5	0.36
0.75	0.8
0.37	0.216
3.92	3.4813
5.62	5.736
0.6	0.85
0.426	0.3
2.516	2.8325
7.942	7.63
4.08	4.7
1.85	1.84
17.353	17.35

Diagnosing Misconceptions

Quick Test Analysis

Longer-is-larger misconceptions Typical pattern. The response to item 11 may indicate whether the student is using (X) Whole number thinking or (√) RH column overflow thinking				**Shorter-is-larger misconceptions** Typical pattern. The response to item 12 may indicate whether the student is using (X) Reciprocal thinking or (√) Denominator focussed thinking				**Apparent-expert behaviour** Typical pattern. The response to item 13 may indicate whether the student is using (X) Money thinking or (√) Taskexpert thinking Note that some money thinkers may guess and then look like a Taskexpert.			
1	4.8	4.63	X	1	4.8	4.63	√	1	4.8	4.63	√
2	0.5	0.36	X	2	0.5	0.36	√	2	0.5	0.36	√
3	0.75	0.8	X	3	0.75	0.8	√	3	0.75	0.8	√
4	0.37	0.216	X	4	0.37	0.216	√	4	0.37	0.216	√
5	3.92	3.4813	X	5	3.92	3.4813	√	5	3.92	3.4813	√
6	5.62	5.736	√	6	5.62	5.736	X	6	5.62	5.736	√
7	0.6	0.85	√	7	0.6	0.85	X	7	0.6	0.85	√
8	0.426	0.3	√	8	0.426	0.3	X	8	0.426	0.3	√
9	2.516	2.8325	√	9	2.516	2.8325	X	9	2.516	2.8325	√
10	7.942	7.63	√	10	7.942	7.63	X	10	7.942	7.63	√
11	4.08	4.7	?	11	4.08	4.7	√	11	4.08	4.7	√
12	1.85	1.84	√	12	1.85	1.84	?	12	1.85	1.84	√
13	17.353	17.35	√	13	17.353	17.35	X	13	17.353	17.35	?

Shorter is larger

Longer is larger

Need to discriminate

Diagnosing Misconceptions

Smart Tests

smartvic.com

Teaching Strategies

Left to Right Comparison

- Compare columns from left to right until one digit in a column is larger than any other in the same column e.g.

compare 45.789 with 45.77

Tens	Ones	Tenths	Hundredths	Thousandths
4	5	7	8	9
4	5	7	7	
same	same	same	top is larger - stop	

Therefore, 45.789 > 45.77

Teaching Strategies

Linear Arithmetic Blocks (LAB)

Use for representing decimals instead of MAB which children identify with whole number concepts

- uses length (not volume) to represent the size of a number

- relates to the number line

- shows equivalence

- shows density

- active engagement

Classroom

Hidden Number

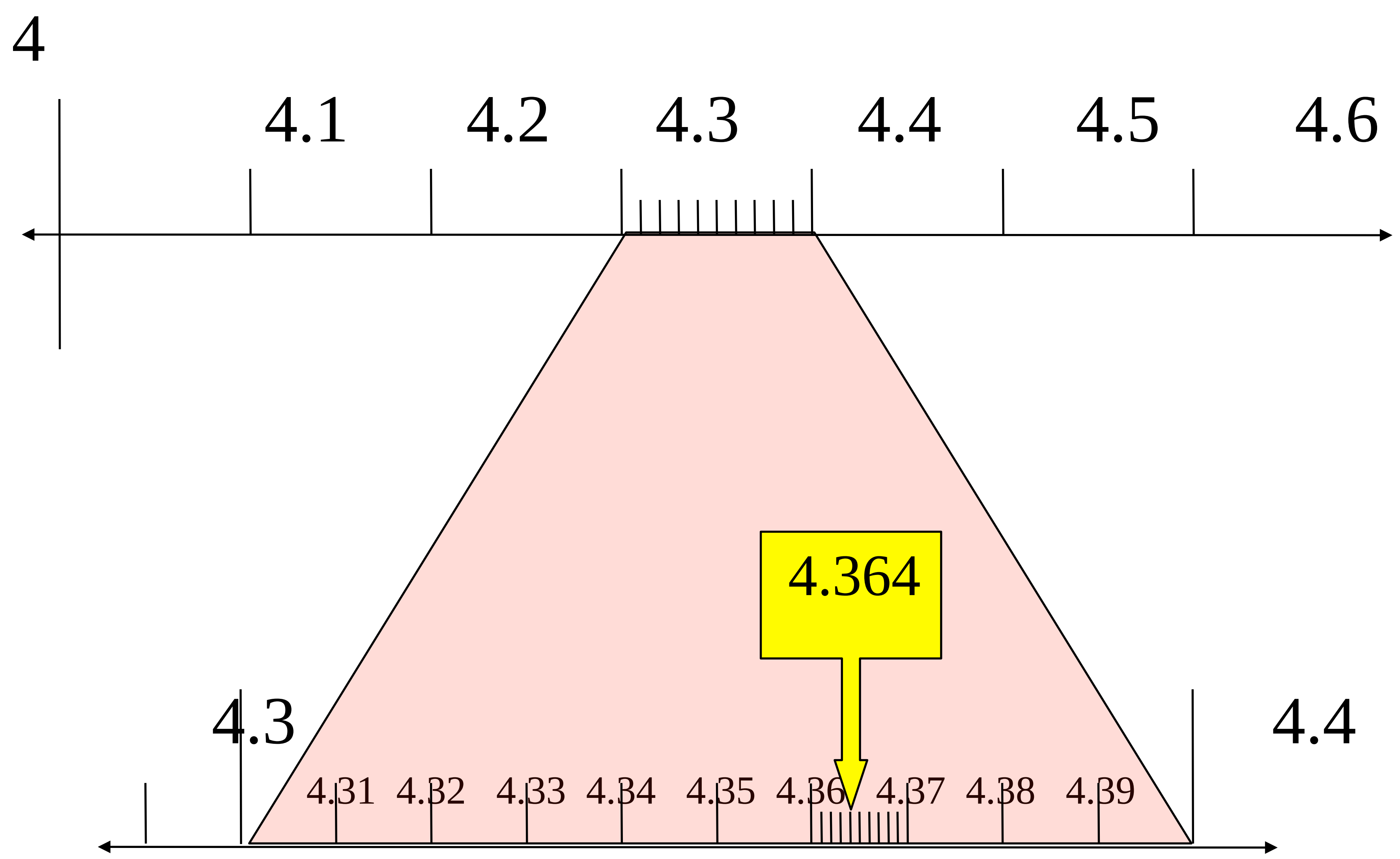
Used for building skills for comparing the relative size of decimal numbers.

Write two numbers ‘hidden’ behind doors.

Classroom

The Number Line

Decimals on the number line



We want students to get the sense of being able to zoom in or out of the number line.

Classroom

The Number Line

Watch for confusion...

- Defining decimals on number line

- A decimal number is both
  - A point **on** the number line

    AND
  - The distance from 0, a length

Classroom

The Number Line

Number Trails

A game for developing number patterns and sequences.

Suitable for all students.

Begin with whole numbers and addition or subtraction then introduce multiplication and division and decimals and fractions.

Teaching Strategies

Number Trails

Goal is to make the longest number trail you can

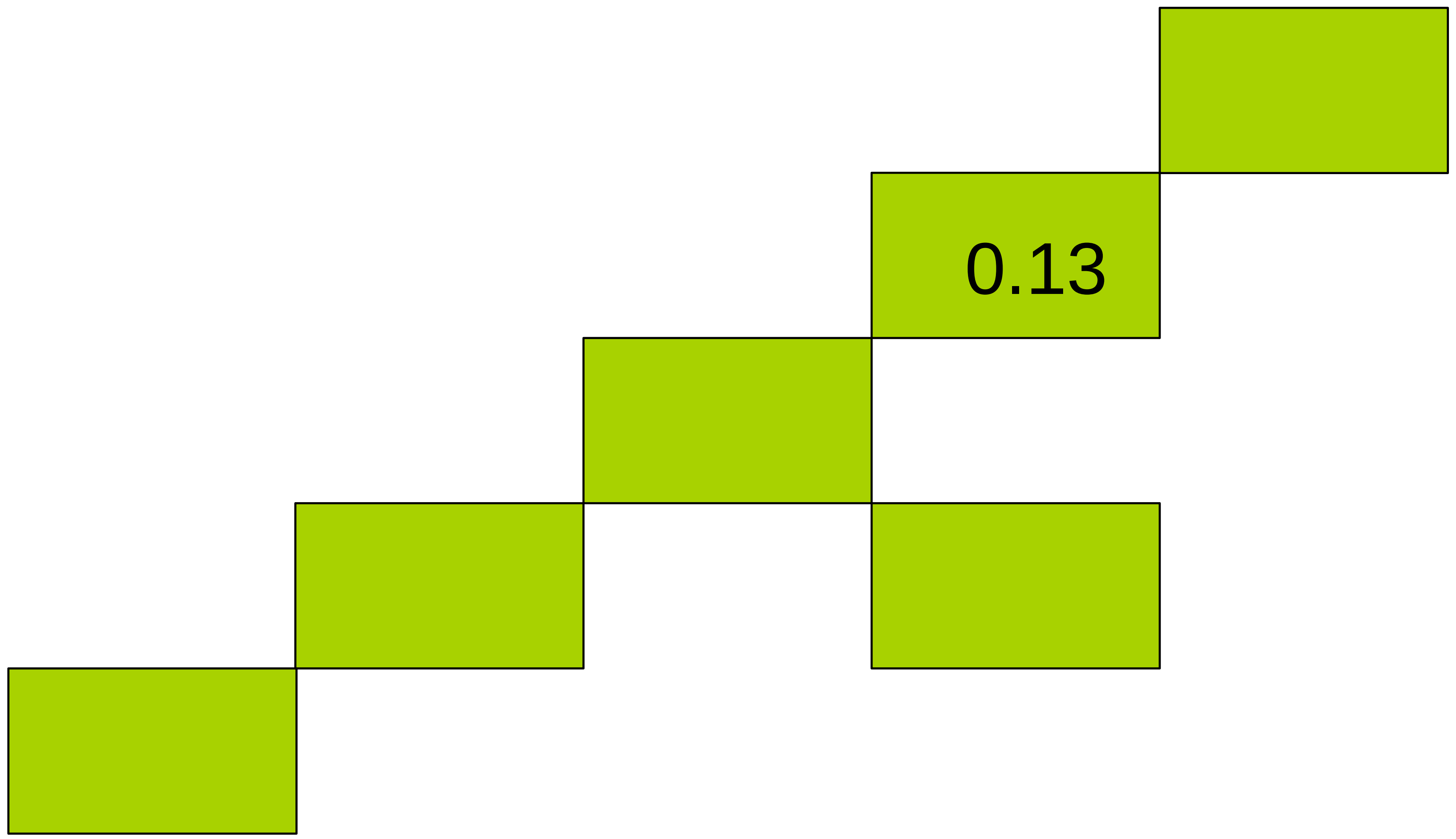
- On a board write: Start number:

  Doing number:

  Operation:
- Allow students two minutes to write as many numbers in the sequence as possible
- The student with the most responses reads their list
- Give this student a 30 second handicap in the next game

Teaching Strategies

Jigsaw Arrays



This is a section from a 10 x 10 grid.

Fill in the missing numbers.

Classroom

Today's Number

15.28

Tools session idea:
Today's number is… from MCTP

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

- An excellent activity to express and discuss understandings about a concept.

- Work in pairs, small groups or a whole group to sort cards into the following categories:
  - Always True
  - Sometimes True
  - Never True

- Some statements are designed to be quite easy while others are deliberately written to be provocative and to promote discussion.

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

Zero doesn't matter.

Discuss

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

There are a finite number of decimals.

Discuss

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

All decimals can be written as a fraction.

All fractions can be written as decimals.

Discuss

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

What is necessary for answering these types of questions?

What is sufficient, ie what is the minimum argument that justifies a point of view?

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

- The longer the decimal, the larger it is.
- The shorter the decimal, the larger it is.
- Zero doesn’t matter.
- To multiply a decimal by ten, you just add a zero.
- 0.9 is equal to 1
- There are a finite number of decimals.
- When you add two decimal numbers, you get another decimal number.
- We read the names of digits to the right of the decimal point individually.

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

- There is no decimal between 4.397 and 4.398
- All decimals can be written as a fraction.
- All fractions can be written as decimals.
- The decimal point separates the units column from the tenths column.
- The smallest possible decimal is 0.0001
- A number with 3 decimal places is larger than a number with 2 decimal places.
- Decimal numbers can be found between 0 and 1

Always, Sometimes, Never

https://www.ncetm.org.uk/resources/

Challenge

In groups of 4 or 5 come up with a new

Always, Sometimes, Never

statement to use in the classroom

Include the Year Level(s) you think it is appropriate for.

Teacher Understandings

Addition of Decimals

The algorithms for addition and subtraction of decimals are **similar to those for whole numbers.**

We can always relate the addition of decimal numbers to addition of fractions. However this is inefficient and the decimal notation gives us an easier way to calculate such sums.

The **decimal points in each number should be lined up** one under the other. The vertical addition algorithm is shorthand for adding hundreds to hundreds, tens to tens, ones to ones, tenths to tenths, and so on. It is important to line up the place value columns when lining up the decimal point.

Number and Place Value

Addition of Decimals

Example: 4.2 + 5.09

$$\begin{array}{r} 4.2 \quad + \\ \underline{5.09} \end{array}$$

Line up the decimal points vertically

$$\begin{array}{r} 4.2\textcolor{red}{0} \quad + \\ \underline{5.09} \end{array}$$

Make both decimals the same “length”

$$\begin{array}{r} 4.2\textcolor{red}{0} \quad + \\ \underline{5.09} \\ 9.29 \end{array}$$

Add columns starting from the right

Teacher Understandings

Subtraction of Decimals

When **subtracting** one decimal from another, write the numbers **one under the other** as with whole number subtraction, **making sure the decimal points are aligned.**

There are two standard subtraction algorithms. The following slides show how they work when subtracting 16.532 from 23.84

Teacher Understandings

Subtraction of Decimals

Method 1:

$$\begin{array}{l} 23.84 \quad - \\ \underline{16.532} \end{array}$$

Line up the decimal points vertically

$$\begin{array}{l} 23.84\textcolor{red}{0} \quad - \\ \underline{16.532} \end{array}$$

Make both decimals the same “length”

$$\begin{array}{l} 23.8\;4\;\;{}^{1}\textcolor{red}{0} \quad - \\ \underline{16.5\;3_{\textcolor{red}{1}}\;\;2} \end{array}$$

Starting from the right, borrow and payback “10s” between the top and bottom lines

$$\begin{array}{l} 2\;{}^{1}3.8\;4\;{}^{1}\textcolor{red}{0} \quad - \\ \underline{1_{\textcolor{red}{1}}\;6.5\;3_{\textcolor{red}{1}}\;2} \\ \quad 7.3\;0\;\;8 \end{array}$$

Subtract column by column, again starting on the right

Teacher Understandings

Subtraction of Decimals

Method 2:

$$\begin{array}{r} 23.84 \; - \\ \underline{16.532} \end{array}$$

Line up the decimal points vertically

$$\begin{array}{r} 23.84\color{red}{0} \; - \\ \underline{16.532} \end{array}$$

Make both decimals the same "length"

$$\begin{array}{r} {}^{1}\cancel{2}\,{}^{1}3.8\,{}^{3}\cancel{4}\,{}^{1}\color{red}{0} \; - \\ \underline{1\;6.5\;3\;2} \\ 7.3\;0\;8 \end{array}$$

Starting from the right, use trading on the top line

Subtract column by column, again starting on the right

Number and Place Value

Reflection: What have we learned today?

What is Number?

How are whole numbers, fractions and decimals connected?

What do we understand by the term Place Value?

What order and sequence best facilitates understanding of place value?

AMSI

The Team

Schools Manager	Janine McIntosh	janine@amsi.org.au
Outreach Manager	Michael O'Connor	moconnor@amsi.org.au

ACT
NSW
NT
QLD
SA
TAS
VIC
WA