LEVEL: Year 6	CONTENT: Problem Solving	FOCUS: Patterns
In the Classroom		
PURPOSE	• Identify the qualities of a good mathematician • Demonstrate the qualities of a good mathematician • Explain and record thinking using a systematic approach • Identity the different mathematical operations • Explain the order of operations • Use brackets to modify the order of operations • Explain the use of brackets • Record and solve equations that involve the use of brackets and different operations	
WARM UP	**Good Mathematician** Brief discussion about the qualities of good mathematicians - remind students that everyone is a good mathematician and encourage students to use the strategies that they have suggested **24 Game** Show students some examples of the 24 game (Open PowerPoint Four fours.ppt) Students need to use the four numbers to make a total of 24 **Challenge** – How would you accurately record the solution using symbols?	
INTRODUCTION	For this activity students are challenged to create equations using up to four fours and any mathematical operations to make the numbers from 0 to 100; telling the class that the previous class was only able to fin 25 correct solutions encourages them to perform better and to also work collaboratively as a whole class, building on one another's solutions to quickly increase the overall list.	
EXPLICIT TEACHING & LEARNING	**Four Fours** • Show students the Four Fours problem • Using up to four fours and any mathematical operation can you make the numbers from 0 to 100 • Introduce the order of operations concept, reminding students that multiplication or division are first before addition or subtraction (depending on the order) • Assist students to use the order of operations protocols to assist them to accurately • Create a shared document showing numbers where students have found solutions (or multiple solutions) **Challenge** – Are there any other operations we can use to help us complete this challenge?	
DISCUSSION/KEY QUESTIONS	• What do you know about this problem? • What is an equation? • What does the equal sign mean? • What are the different mathematical operators? • In what order do you solve operations? • What do brackets do in an equation? • How can you change the order of operations? • What happens if all the same operators are used in the equation? • What are other operations we can use? • Is there more than one way to find the same solution?	
DELIBERATIVE PRACTICE	The focus of this activity is for students to not just to find solutions for each of the numbers, but to accurately use their knowledge of the order of operations to accurately record each equation	
REFLECTION	Discussion with students about the ways they demonstrated that they were a Good Mathematician	
RESOURCES	**24 Game Cards** https://www.ru.ac.za/media/rhodesuniversity/content/sanc/documents/Maths%2024%20-%20cards.pdf **Four Fours Challenge** https://nzmaths.co.nz/resource/four-fours-challenge	

Curriculum Connections

CONTENT

AUSTRALIAN CURRICULUM
YEAR 6 – NUMBER & ALGEBRA
Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123)
Elaborations – applying strategies already developed for solving problems involving small numbers to those involving large numbers; applying a range of strategies to solve realistic problems and commenting on the efficiency of different strategies
Explore the use of brackets and order of operations to write number sentences (ACMNA134)
Elaborations – appreciate the need for rules to complete multiple operations within the same number sentence

NSW MATHEMATICS K-10 SYLLABUS: STAGE 3 – MULTIPLICATION & DIVISION 2
Outcomes

- describes and represents mathematical situations in a variety of ways using mathematical terminology and some conventions
- selects and applies appropriate problem-solving strategies, including the use of digital technologies, in undertaking investigations
- gives a valid reason for supporting one possible solution over another
- selects and applies appropriate strategies for multiplication and division, and applies the order of operations to calculations involving more than one operation

Content

- Select and apply efficient mental and written strategies, and appropriate digital technologies, to solve problems involving multiplication and division with whole numbers (ACMNA123)
    - select and use efficient mental and written strategies, and digital technologies, to multiply whole numbers of up to four digits by one- and two-digit numbers
    - select and use efficient mental and written strategies, and digital technologies, to divide whole numbers of up to four digits by a one-digit divisor, including where there is a remainder
        - estimate solutions to problems and check to justify solutions (Problem Solving, Reasoning)
    - use mental strategies to multiply and divide numbers by 10, 100, 1000 and their multiples
    - solve word problems involving multiplication and division, eg 'A recipe requires 3 cups of flour for 10 people. How many cups of flour are required for 40 people?'
        - use appropriate language to compare quantities, eg 'twice as much as', 'half as much as' (Communicating)
        - use a table or similar organiser to record methods used to solve problems (Communicating, Problem Solving)
    - recognise symbols used to record speed in kilometres per hour, eg 80 km/h
    - solve simple problems involving speed, eg 'How long would it take to travel 600 km if the average speed for the trip is 75 km/h?'
- Explore the use of brackets and the order of operations to write number sentences (ACMNA134)
    - use the term 'operations' to describe collectively the processes of addition, subtraction, multiplication and division
    - investigate and establish the order of operations using real-life contexts, eg 'I buy six goldfish costing $10 each and two water plants costing $4 each. What is the total cost?'; this can be represented by the number sentence $6 \times 10 + 2 \times 4$ but, to obtain the total cost, multiplication must be performed before addition
        - write number sentences to represent real-life situations (Communicating, Problem Solving)
    - recognise that the grouping symbols () and [] are used in number sentences to indicate operations that must be performed first
    - recognise that if more than one pair of grouping symbols are used, the operation within the innermost grouping symbols is performed first
    - perform calculations involving grouping symbols without the use of digital technologies, eg
      $5+(2\times3)=5+6=11$; $(2+3)\times(16-9)=5\times7=35$; $3+[20\div(9-5)]=3+[20\div4]=3+5=8$
    - apply the order of operations to perform calculations involving mixed operations and grouping symbols, without the use of digital technologies, eg
      $32+2-4=34-4=30$ addition and subtraction only, therefore work from left to right
      $32\div2\times4=16\times4=64$ multiplication and division only, therefore work from left to right
      $32\div(2\times4)=32\div8=4$ perform operation in grouping symbols first

	$(32+2)\times 4 = 34\times 4 = 136$ perform operation in grouping symbols first $32+2\times 4 = 32+8 = 40$ perform multiplication before addition ▪ investigate whether different digital technologies apply the order of operations (Reasoning) recognise when grouping symbols are not necessary, eg 32 + (2 × 4) has the same answer as 32 + 2 × 4
WHAT CAME BEFORE	Student require a knowledge of the four basic operations $(+, -, \times, \div)$ and how to apply these to solve simple problems involving one and two-digit numbers. An understanding of that the equals sign means "is the same as" will also be beneficial, along with knowledge of how to use symbols to record equations.
WHAT COMES NEXT	Some equations created by students may use brackets, where in fact brackets are NOT needed. Revisiting some of the solutions submitted by students and rewriting them in their most efficient form will help embed students understanding of the order of operations and use of brackets, for example: $(4 \times 4) + (4 \div 4) = 17$ could be correctly written as $4 \times 4 + 4 \div 4 = 17$ Providing students with just one equation and encouraging them to add brackets to find as many solutions as possible will also help embed students' knowledge, for example: $4 + 4 \times 4 - 4 = 16$ $(4 + 4) \times 4 - 4 = 28$ $4 + 4 \times (4 - 4) = 4$
VOCABULARY	order of operations, brackets, parentheses, addition, subtraction, multiplication, division, symbol, equation, expression, equals sign, operator, powers (orders) and roots
MISCONCEPTIONS	• Some students may think that all equations are solved left to right • Some students may believe that addition comes before subtraction or division comes before multiplication as that is often how the order of operations is sometimes remembered in the mnemonic BODMAS • Some students will record solutions as one continuous equation with multiple equals signs, for example: $4 + 4 = 8 + 4 = 12$; this equation is now incorrect as $4 + 4 \neq 12$ • Important that students understand that equations are like a balance; what is on one side must be equal to what is on the other side
WHAT PROFICIENCIES ARE TO BE UTILISED? **Understanding** **Fluency** **Problem Solving** **Reasoning** **Communicating (NSW)** **Justifying (NSW)**	**Year 6** **Fluency** includes representing integers on a number line, calculating simple percentages, using brackets appropriately, converting between fractions and decimals, using operations with fractions, decimals and percentages, measuring using metric units and interpreting timetables **Problem Solving** includes formulating and solving authentic problems using fractions, decimals, percentages and measurements, interpreting secondary data displays and finding the size of unknown angles **Reasoning** includes explaining mental strategies for performing calculations, describing results for continuing number sequences, explaining the transformation of one shape into another and explaining why the actual results of chance experiments may differ from expected results. **Communicating** - selects and applies appropriate strategies for multiplication and division, and applies the order of operations to calculations involving more than one operation
ASSESSMENT	**Exit Pass –** Show students a four by four equation with an incorrect solution and ask them to explain what the students has done

Teacher Knowledge	
STORIES AND ANECDOTES	The opportunity to run this lesson with multiple classes can encourage healthy competition. Sharing solutions across classes can also help students as a Year level to reach the ultimate goal of finding all the solutions from 0 to 100. Parents or students in the other year levels may also need to be involved in order to find some solutions to the more trickier numbers.
METHODS FOR ENGAGEMENT	Keeping a running list possible equations in a place where all students can see helps students to work collaboratively. Not only do students not spend time finding solutions to numbers that have already been solved, seeing other solutions can often prompt thinking for other numbers, for example: $4 \times 4 \times 4 - 4 = 60$ Using this equation as the base, I can quickly discover that: $4 \times 4 \times 4 - \sqrt{4} = 62$ $4 \times 4 \times 4 + \sqrt{4} = 66$ $4 \times 4 \times 4 + 4 = 68$
IMPROVISATION OF MATERIALS	Use the numbers 1, 2, 3 and 4 and any operation to make the numbers from 0 to 100
EASIER AND HARDER QUESTIONS	To make the problem easier allow students to use more than 4 fours. Alternatively, to make the problem more challenging, each equation must use exactly 4 fours, for example: $4 + 4 = 8\ would\ NOT\ be\ allowed$ $Instead\ (4 + 4) \times (4 \div 4) = 8\ would\ be\ accepted$
KNOWLEDGE OF MISCONCEPTIONS	• Some students may think that all equations are solved left to right • Some students may believe that addition comes before subtraction or division comes before multiplication as that is often how the order of operations is sometimes remembered in the mnemonic BODMAS • Some students will record solutions as one continuous equation with multiple equals signs, for example: $4 + 4 = 8 + 4 = 12$; this equation is now incorrect as $4 + 4 \neq 12$ • Important that students understand that equations are like a balance; what is on one side must be equal to what is on the other side
AN AWARENESS OF THE CURRICULUM FOR DETECTING GAPS IN STUDENT KNOWLEDGE	Important that when students are sharing possible equations that they avoid creating continuous equations with multiple equal signs. If a student submits a solution that has an error, explain to the student how this solution can be recorded accurately, for example: Student suggests that $4 + 4 = 8 \times 4 = 32$; Model that this solution could be written: $4 + 4 = 8$ $8 \times 4 = 32$ Alternatively, applying the order of operations rules, the equation could be written as: $(4 + 4) \times 8 = 32$
DID THE LESSON ACHIEVE ITS PURPOSE? (WAS IT SUCCESSFUL?)	Although the focus of this lesson was to increase students' familiarity with the order of operations the other purpose was to help students work collaboratively. Mathematicians are known for collaborating on projects. The process on building on each other's solutions should not be seen as copying, instead it should be seen as a valid step in the solution process. Students can learn from one another, just as they can learn from the teacher. The fact that students could work as a collaborative unit to find solutions to more than 30 numbers (in just less than one hour) shows the successful nature of this lesson. A short reflection to reinforce the power of working together at the end of this lesson and in other lessons of this type would help reinforce this concept.