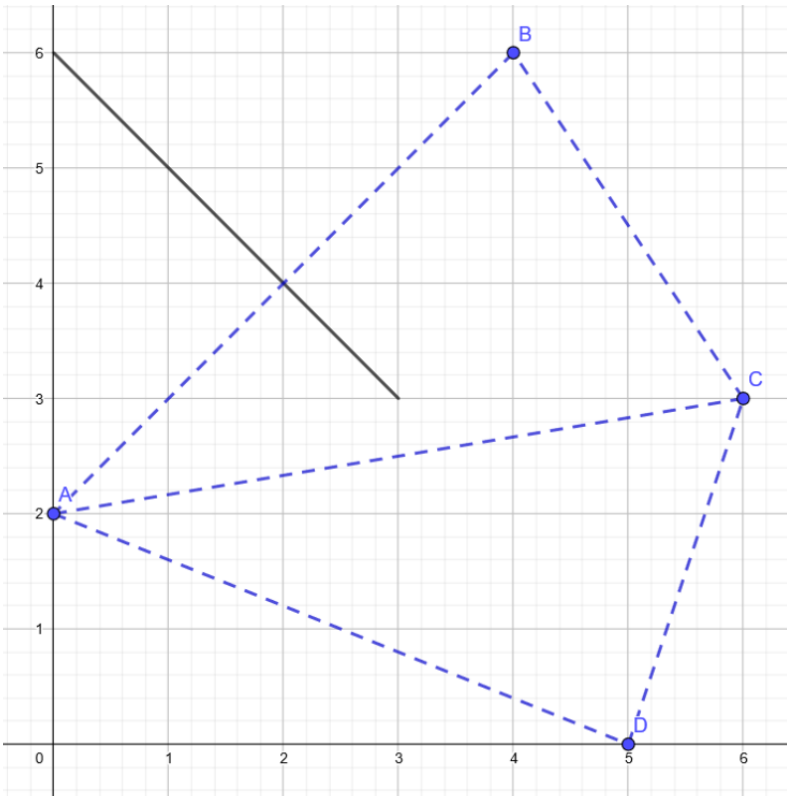
Exercise 1: Perpendicular Bisectors

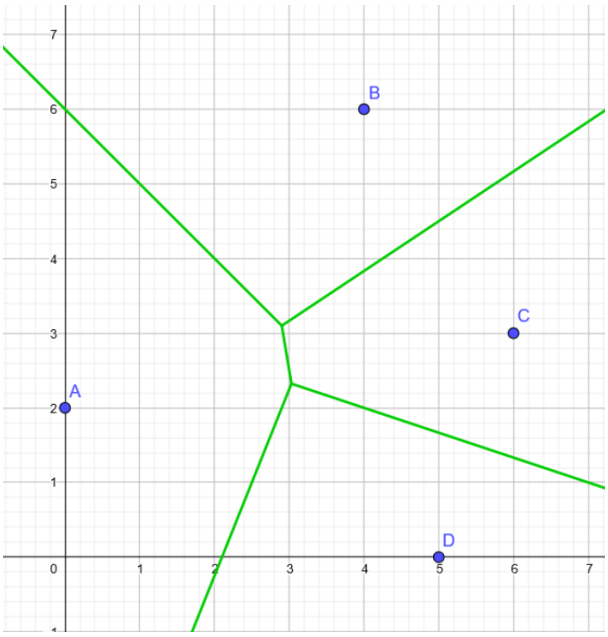
Complete the questions in the table and draw the perpendicular bisectors for each of the line segments on the graph below. The bisector for line $\overline{AB}$ has already been done for you in the previous activity.



1. **Line *AC*** i. Gradient: _____ ii. Midpoint: _____ iii. Perpendicular Gradient: _____ iv. Second Point: _____	**2.** **Line *BC*** i. Gradient: _____ ii. Midpoint: _____ iii. Perpendicular Gradient: _____ iv. Second Point: _____
3. **Line *AD*** i. Gradient: _____ ii. Midpoint: _____ iii. Perpendicular Gradient: _____ iv. Second Point: _____	**4.** **Line *CD*** i. Gradient: _____ ii. Midpoint: _____ iii. Perpendicular Gradient: _____ iv. Second Point: _____

Exercise 2: Using Your Voronoi Diagram

Once the Voronoi diagram has been constructed, we can now use it to determine the seed that is closest to any given point on the map. The following questions provide simple examples on how these diagrams work.



1. Using the Voronoi Diagram, determine which seed *A*, *B*, *C* or *D* each of the coordinate points is closest to:

i. (2, 3) _____

ii. (6, 5) _____

iii. (2.5, 3.5) _____

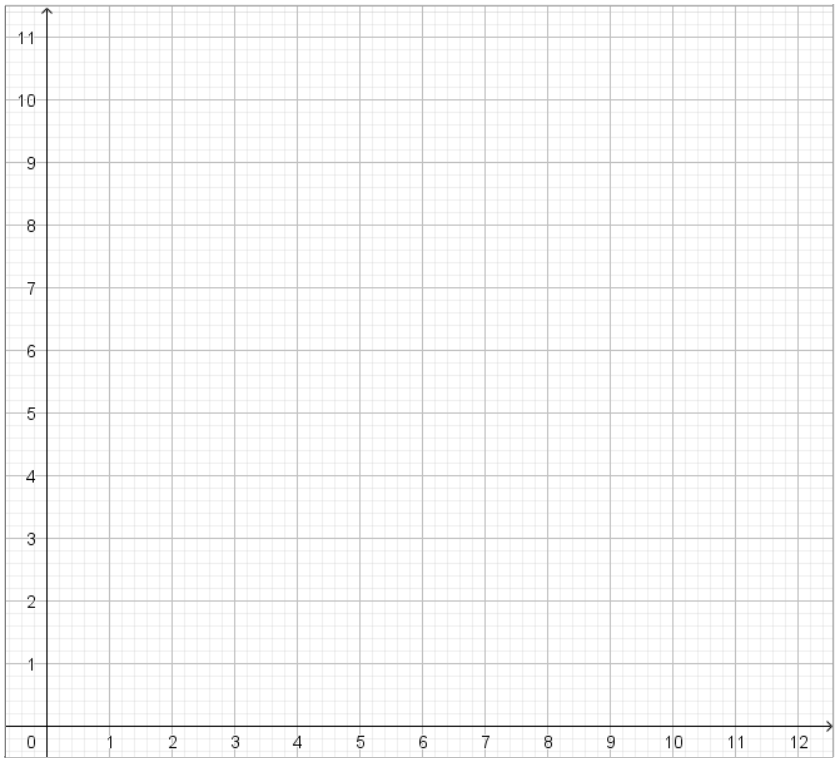
iv. (7, 6) _____

v. (4, 2) _____

Exercise 3: Constructing a Voronoi diagram by hand

1. On the Cartesian plane below, plot the following coordinate points:

   (0, 2), (5, 7), (5, 0), (6,3), (10, 4), (7, 8), (1, 8), (3.5, 9), (3, 5)



2. Using a lead pencil lightly join each point with a straight line to create the Delaunay Triangulation. Try to ensure that the triangle constructed between three points has the smallest area possible. This may take you a few attempts.
3. Once you're happy with your Delaunay Triangulation, Find the perpendicular bisector for each of its line segments by following the steps in Part 2. *Note: When drawing in your bisectors it may be easier to draw the lines so that they only bisect the one line and stop in the middle of each triangle. This will reduce the confusion when tracing the edges of your Voronoi cells. Make sure however that you make them long enough so that three bisectors can intersect at a given point within each triangle.*
4. Trace over each line segment in a different colour, stopping at the vertex where three bisectors intersect. Continue this process with each bisector until each coordinate point is sitting within its own cell on your Voronoi map. You now have a completed map, ready to use for your analysis.

You may find the following clip helpful in following these instructions:
https://www.youtube.com/watch?v=bmaUtG4CbOs

i. In the above video, the designer constructed his Voronoi diagram using only a ruler. What would the disadvantages of such a method be?

Exercise 4: Creating a Voronoi Diagram Using GeoGebra

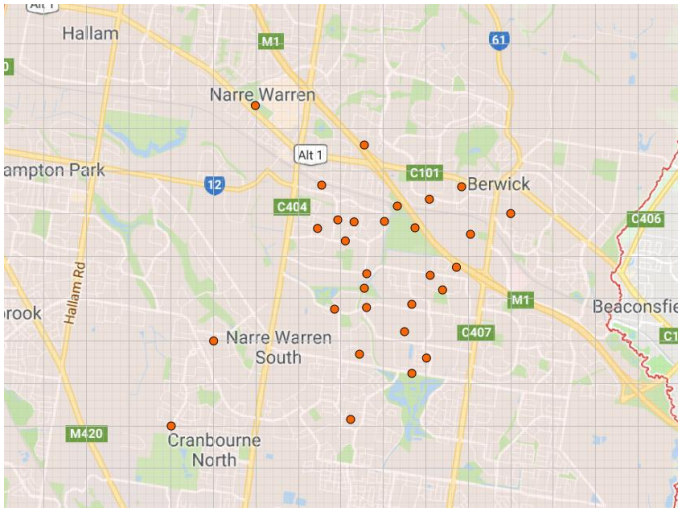
As you have just experienced, drawing Voronoi diagrams by hand can be time consuming. In this section, we will be incorporating the use of a Dynamic Mathematical Software (DMS) called GeoGebra to make help make this process much simpler and more useful for larger scale applications.

One of the many applications of Voronoi Diagrams is in the investigation and containment of an infectious disease where the outbreak source is not clearly apparent.

John Snow famously demonstrated the power of using these diagrams when identifying the source of the 1854 cholera outbreak and in the next exercise you will put these same skills to use by creating your own Voronoi diagram using GeoGebra.

Investigating the Source of an Epidemic

The City of Casey has been struck down by a mystery disease. There have been 127 reported cases so far and the rate is climbing. As a Public Health Official, it is your job to identify the source of the disease before more people are infected. The figure below shows a map of the City of Casey with several reported cases highlighted in orange. The GeoGebra file can be accessed at the following address: https://www.geogebra.org/m/bhcrhvwa



The Department of Public Health has collected enough data to identify a few potential sources of the outbreak; **Schools**, **Childcare Centres** and **Healthcare Facilities**. However, they need to narrow their field even further in order to pinpoint the exact location of the infection source. It has been decided that a Voronoi Map would provide the best way of doing this. The program GeoGebra will be used to help construct these maps accurately for each of the locations.

Part 1: Plotting the Coordinate Points

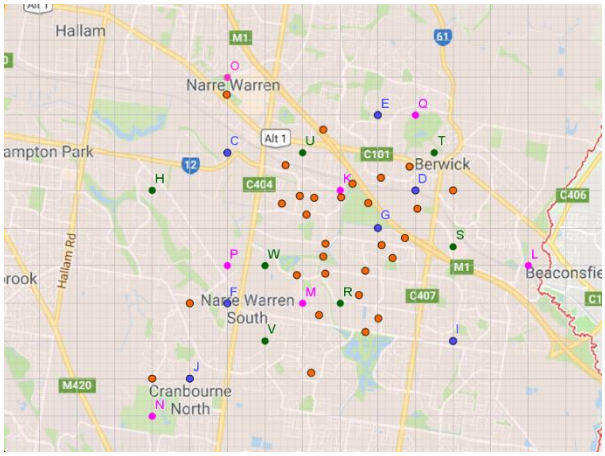
In the input panel of GeoGebra, enter the ordered pairs specified for each of the sources below or simply select 'point' in the top left-hand corner of your page and click on each point location on your cartesian plane:

Schools: (9, 7), (14, 5), (8, 4), (4, 1), (6, 10), (6, 5), (11, 9)

Child Care Centres: (6, 8), (11, 7), (10, 9), (6, 4), (10,6), (12, 3), (5, 2)

Health Care Facilities: (4, 7), (9, 4), (12, 5.5), (11.5, 8), (8, 8), (7, 3), (7, 5)

You should now have a map that looks like the one shown.

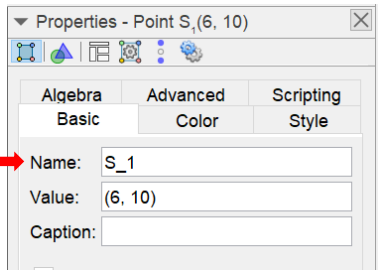


Having all the locations on the map does not allow for suitable analysis or successful use of a Voronoi diagram. For our purposes, it will be much more beneficial to work with each the facilities individually. Separating these on the graph can be done in GeoGebra by inserting checkboxes into your document using Boolean values.

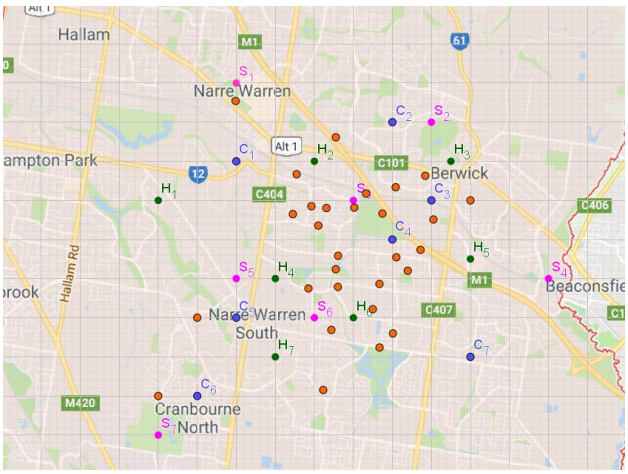
Formatting Points

It is a lot easier to identify your points for each facility by assigning them a *colour* and *reference name*. This can be done from the 'Algebra' panel or straight from the points on the map by right clicking on each point and selecting *'object properties'*. A toolbox like the one shown below should appear to the right of your page.

Change the name of each point

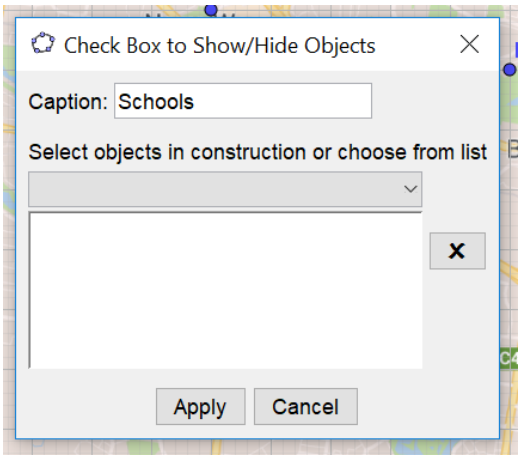


On the *'Basic'* tab you can change the name of the point and under the *'Color'* tab, the colour of the point. Assign the series of points from each facility with the associated names and the same colour to make your map easier to read. The map below shows one where these steps have already been completed.

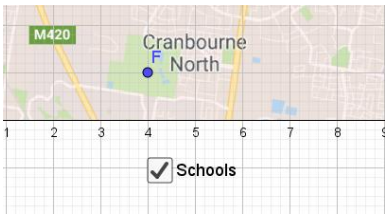


Part 2: Inserting Checkboxes

1. Select the 'checkbox' command on the toolbar at the top of your page and then click anywhere on your document. A dialogue box should appear as shown below. Label your checkbox with the facilities, you wish to group together. In this case 'Schools'.



2. In the dropdown box labelled *'select objects in construction or choose from list'*, select the coordinates that represent *all* the schools located on your map. A little checkbox should now appear somewhere on your document.



   If you click back onto the 'move' icon that looks like an arrow on your toolbar, you can now tick and untick this checkbox to make the schools visible or hidden on your map.
3. Repeat these steps for the Child Care Centres and the Healthcare Facilities to create checkboxes for each.

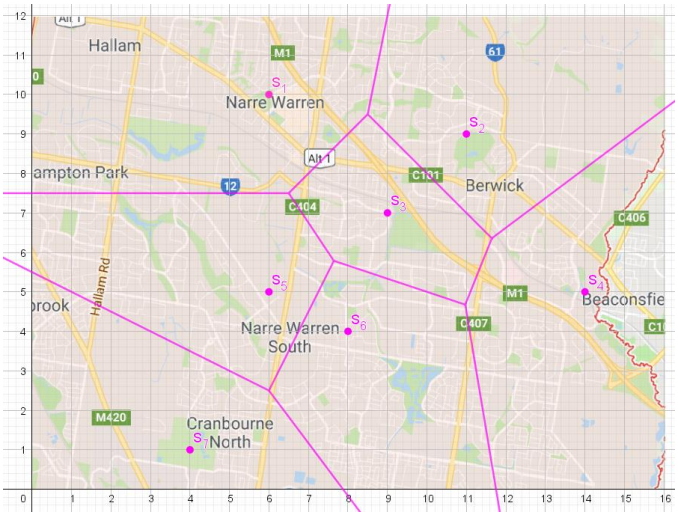
In order to identify the source of the outbreak, it is important to get a better look at a potential outbreak site. By using a proximity diagram like a Voronoi, we can determine whether a large proportion of the cases are located within a given cell and therefore within close range of a particular facility. To do this, we will need to create Voronoi diagrams for all schools, childcare centres and healthcare facilities separately.

Part 3: Constructing the Voronoi Diagram

Using GeoGebra to construct a Voronoi map is a much simpler process than constructing one by hand. The software has a tool already part of its programming that completes to procedure quickly and accurately.

1. In the input panel at the bottom of the page, simply type 'voronoi' and the following line will appear:

   Where it states <LIST OF POINTS>, enter the letters representing each coordinate point you wish to map. For your schools, these will be, (S_1, S_2, S_3, S_4, S_5, S_6, S_7). As you enter these points, you will begin to see your Voronoi map take shape. Hitting enter after you're done, will complete the process and your Voronoi diagram for your schools will be visible on your map.



2. Repeat this process for your childcare centres and your healthcare facilities to create three Voronoi diagrams. You can change the colour of each map by right clicking on your Voronoi, selecting 'object properties' and then 'colour'. Having different colours for separate maps will make the analyses of your cases much easier.
3. Inserting 'checkboxes' for each of the Voronoi diagrams will also aid you in your analysis because this will allow you to work with one Voronoi at a time or even two together. The instructions for how to do this are the same as those outlined in part two of this exercise.

Now that you have all the information that you need on your map. It's time to start looking for some patterns and drawing conclusions from the data provided.

Lesson Ideas

1. Which is your closest McDonalds restaurant?
2. What school should you be zoned to?
3. Where is the closest bathroom/bin from your location in the school?
4. Where is your closest train station?