MATHSTALK **by AMSI Schools** (Episode 2)**:**

‘Multiplication Matters’ (Part 1)

Speaker Key:

MG Marcus Garrett

LM Leanne McMahon

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MG Welcome to **MATHS**TALK by AMSI Schools. Where conversations in maths become part of your professional learning practice. My name is Marcus Garrett. I’m an AMSI Schools Outreach Officer and today I’d like to introduce my colleague Leanne McMahon. Hi Leanne, how are you?

LM Hi Marcus, very well thank you.

MG Now, Leanne do you want to tell us a little bit before we start about what you do at AMSI Schools and where you work?

LM Well, much like you I’m an Outreach Officer. I spend most of my time though in central Queensland in Mackay working with teachers and students to try and improve mathematics teaching and learning.

MG So shifting between sunny Melbourne and even sunnier Mackay. That must be interesting sometimes.

LM It’s lovely.

MG Now, today is the first part in a series called Multiplication Matters, that we’re going to bring to you. We’re going to focus on what we call multiplicative thinking. That is, why it’s important to be able to understand the mathematical concept, not just of multiplication but a way of thinking multiplicatively. Why it’s important and how it underpins so much of our mathematical understanding.

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I had an interesting situation quite a few weeks ago Leanne, when I was talking to a group of parents in a school in the Hunter Valley in New South Wales and there was a grandfather there who was very passionate about his grand-daughter’s education. And he was very concerned because he said to us, you know, back when I was at school it was just a matter of learning our times tables, I don’t think kids learn their times tables properly, I don’t understand why it’s all so complicated these days.

LM Oh yes. That's something that I think all teachers have come across and recently too, it's not just grandfathers that believe that, learn your tables and you know your maths. I don't think that's enough though.

MG No, it's something that's common from parents and even sometimes students I think as well. So, what is multiplicative thinking? Why is it not just times tables?

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LM Yes, multiplicative thinking is a lot more than just times tables. We need our students to be thinking flexibly and efficiently, and we need them to be working with a large range of numbers. We want them to understand the relationship between the numbers and be able to use those relationships to solve problems.

MG So, it's not just about following a process, is that what you're saying? It's not just about learning something by rote but it's a way of thinking and being able to solve problems flexibly.

LM That's exactly right and you will hear people say, just learn your tables by rote. And that's one of the things that I think is really important, that distinction between learning by rote, which is learning by repetition. Three twos are six. Three threes are nine. You need to be able to use that in context. Yes, we do need to memorise it but we need to have an understanding of the multiplication facts.

MG Yes. It's often not really an either-or, is it? I think knowing number facts is useful because it reduces students mental load when they're solving problems but they have to be able to use that understanding to solve the problem in the first place.

LM Absolutely.

MG So, what then, if we're thinking about multiplicative thinking as a way of thinking mathematically. Where does that start? What is the starting point for students when they begin to think multiplicatively?

LM Well, I guess there's two ways in which students need to understand multiplication. There is a conceptual progression and then there's a pedagogical progression. So, in any, when we learn anything, we go through a pedagogical progression. We start off with concrete, move through representational to abstract. And I'm going to give you the example of a baby who points to an apple, they want an apple. Okay?

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That is an absolutely concrete example. They then move to saying the word apple when they want an apple, which is representational. And finally, by the time they're about five, they can write the word apple. So, that's an abstract concept. Now, we need our students to be doing this with multiplication. So, the concrete, they're moving their counters around, they're actually seeing that three groups of five counters ends up with 15 counters.

Representational, they can then draw, erase or form equal groups drawing them, without actually moving counters around. And finally, the abstract is writing. Three times five equals 15. And actually, understanding that, that could mean three groups of five, or it could mean three rows of five, or it could be an arrangement in some other way.

MG Have you ever come across students who, so I'd imagine that most students would be thinking abstractly in terms of multiplicative thinking by the time at least they're in, say Year 5 and 6. Hopefully a little bit earlier. Have you ever seen examples of students who perhaps haven't gone through that pedagogical progression as you call it? Even by the time they're at high school? Is that something you have encountered before?

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LM Definitely. I am secondary trained and so have a lot of experience teaching Year 7 and 8. And I think sometimes as a secondary teacher, we have the expectation that when our students come in, they are thinking abstractly. We don't really provide them with a lot of opportunity to use concrete materials. In fact, there's only one school that I've taught at, that actually had concrete materials available for the maths classes at Year 7. Sometimes we go for representational models, but mostly we expect them to be thinking abstractly. So, yes. There will be, I would say there would be two to three students in every class that are still on the concrete stage and need those sorts of concrete materials.

MG Yes. I'm thinking of your average Year 7, 8 textbook. You know, the beginning of the chapter, we might have a diagram or a picture or something representing, for example, if it's the seven and eight fractions and decimals topic. Which is quite a significant chunk of that early high school curriculum. So, it might have a diagram and a picture. and then the rest of the textbook really is just those symbols and numbers, the abstract.

LM Oh Marcus, we don't have time for that sort of nonsense. We've got to get through the curriculum.

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MG Yes that's right. So, you mentioned the conceptual progression. Let me put it this way, in English, so if we're learning English from an early age at school. We start with learning individual words. So, we learnt to write and read individual words, we begin to make progress there and then we're putting words together in sentences. So, the sentences begin to make sense. We're combining sentences then into paragraphs.

And then we're actually using those paragraphs to create text and to create meaning. So, we're describing situations or we're writing an information report about something. That is a progression of ideas that we're gradually building up, bit by bit, until we can read and write fluently in English. Is there a way of describing

the development of multiplicative thinking in a similar kind of way? Of developing the concepts for multiplying from an early age?

LM There certainly is Marcus. It's something we discussed at the beginning. When we're talking about the conceptual progression, we start with subitising, or subitising.

MG Tomato, tomato.

LM Yes. It's something that we start very early and children start before they even begin school. Where you can recognise numbers without actually counting them. That is so important. It's important in trusting the count. And if students are going to think multiplicatively, they have to actually realise that, I know that that's a group of three so I don't have to count it. So, that's really important. If students don't have that, there's no point going on.

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MG So, there'd be a lot of prep or kindergarten, or foundation teachers who when they're doing subitising or subitising exercises with their students, they're actually commencing the process of multiplicative thinking.

LM Yes. All of those early years teachers who are doing that sort of thing, they are making very, very strong in-roads into student's understanding of multiplicative thinking. So, there you go. We then go on to repeated addition and that occurs very early too. That tends to occur in the prep years, where you can see it on number line, going up by ones initially, because ones do have multiplication facts. And then twos, threes, fives, tens.

MG So, I'm doing some skip counting at this stage. I probably would be doing that in class?

LM Skip counting is very, very important. It's important for a number of reasons, just see those patterns. If students are doing skip counting, they can see it as repeated addition. Sometimes students can actually visualise it as multiplication before they actually understand what multiplication is.

MG Really?

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LM Yes. They can actually see that they've made three jumps of five and it's 15. They get to 15. There's students that are thinking a lot more, in a lot more advanced way than others but.

MG And I can see that these two things relate to one another. So, just to jump back and talk about, you know, you were mentioning the concrete, representational and abstract way of thinking about this. I can see there then that it would be important for teachers to both show that skip counting happening. So, here's a group of three

counters, I'm going to repeat it four times. How many have I got? I've got three, I've got six, I've got nine, I've got 12 counters.

But then to also show that representationally on a number line and show that the same thing works when I do jumps on the number line.

LM That's it. I don't think number lines are used enough. Personally, I'd be using number lines for everything and skip counting along the number line is a fabulous way of showing that your moving up by three, five, whatever. Getting bigger, making the same size jumps every time. If I go backwards what am I doing? You can start thinking about that sort of thing.

MG Yes. So, consistent increments. So, we can see how that relates to measurements.

LM Yes, that's right.

MG And I'm going to pre-empt you a little bit for a couple of podcasts time when we can even show that division can be repeated subtraction.

LM That's exactly right. And to see that, to understand what that is. And to show that they are equal jumps, meaning that they are equal groups. And that goes on to that next model that we are looking at, of arrays. Where we have, we move from those equal groups to rows of. And that is very important then in the area model understanding. So, they understand that there are five rows of three. And then you can look at that as those five rows of three take up an area of 15.

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MG I've seen a routine in one of the schools I work in that a teacher uses to help her students actually remember their number facts but she uses grid paper, and has the children colouring in those arrays or those areas. So, they can show, for example, that six rows of seven are 42 because they've coloured in 42 squares.

LM There's a great game you can play with that, the multiplication game, where you throw dice and colour in the areas. And you can make up your rules as you go along. You can say, all right, well if I have five rows of ten, can I have two rows of 25? That sort of thing, and that comes to the next step in our progression which is using the arrays, and the area in partitioning numbers.

MG Right. So, that's really important isn't it? And I often think and I realise too that when I move from high school teaching into primary school teaching, why that was so important. So, my original background was an economics teacher and teaching Year 11 and 12 economics and income expenditure analysis. Where we were using algebra, and the idea that when we multiply numbers out, we can actually break those numbers apart and then we have to add the parts back together. That's often missing from children's understanding when they move from primary school to secondary school. It's that bit that they're missing, isn't it?

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LM That's right, and often because they have been taught very well their basic multiplication facts. So, the single digit numbers but what about when they get to the double digits and they're taught the process? So, they get taught an algorithm and they don't actually understand that if they're multiplying five times 17, they're multiplying five times ten, plus five times seven. By using that algorithm that's what they're doing but it's not very clear when you see that algorithm. Whereas if you use the area model and arrays, it is far clearer that that's what they're doing.

MG Yes. So, we then, you can then understand we get to two digit by two digit. And this is where it really gets to come unstuck for a lot of kids, I think you know where I'm going to go with this. So, we understand that say 23 times 36 for example, is 20 times 30, plus three times six, plus 20 times 6, plus 30 times three. All those four components need to be added.

LM That's right. Whereas 26 times 32, why isn't it 20 times 30 and six times two? Or that sort of thing? The old A squared plus B squared isn't the same as A plus B, all squared.

MG Yes. That's right. A plus B all squared is A squared plus 2 AB plus B squared. And there's actually, it's interesting because a lot of adults when you show them, that you say, so if I've got A plus B in brackets and I square that, what's the answer? A lot of adults will tell you it's A squared plus B squared, missing of course two components of what we call the commutative relationship of multiplication, where we can pull it apart but if we pull it apart, we have to multiply each part by each part. And then of course if we miss that and we hit algebra, in about Year 8 or 9, we're all at sea.

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LM Unless you have someone like Mr Donahue who told me A squared plus B squared is A squared.

MG So, you've had someone drill it in.

LM Drill it in to me.

MG There you go. So, so far when we're talking about the conceptual progression, we've gone from subitising to repeated addition, into understanding multiplication though arrays. We've moved that into the area model, where to next?

LM Well after we've really got that commutative law happening, we need to move on to fractions because fractions and fractional thinking are a very, very big part of multiplicative thinking. We cannot understand fractions unless we do understand that multiplicative thinking because a fraction really is division, which is the opposite of the multiplication process. So, we really need to understand fractional thinking as a part of multiplication.

MG Yes, and that's probably a topic for a whole new podcast series.

LM Definitely. I think we can call that 'Of' Means 'Times'.

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MG Yes. That's right.

LM And then lastly rates and ratios that include percentages and the areas where you really apply a lot of this multiplicative thinking to problem solving.

MG And we'd be normally well into high school by then.

LM Well into high school. We try and look at ratios in Year 7, not always successfully. Once they get a bit of algebra behind them, ratios become a little bit easier, I believe. So, as we're understanding this, don't forget that students do need to understand that division is intrinsically related to multiplication. There's another idea for a podcast for you Marcus.

MG Good.

You're listening to **MATHS**TALK by AMSI Schools and we're having a chat today in our first part of a series called Multiplication Matters. I've got Leanne McMahon with me today. Don't forget too, if you are interested in some of the materials supporting these podcasts, don't hesitate to check out our 'Calculate' website – http://calculate.org.au . On the website are lots of teacher support materials, specifically there are some modules.

So, if you'd like to check out some content modules about multiplicative thinking, go to professional learning, the tab on the website there, and you'll find some teacher support modules. Also, there's lots of classroom resources, like games and activities and units of work, if you go to classroom and then go to the classroom resources tab. So, Leanne, going back to why kids really, some kids really struggle to develop multiplicative thinking and a full set of multiplicative thinking skills.

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I remember a couple of years ago I was in a classroom in a Victorian school and we did that very classic Jo Boaler number talk, 18 times five. And it was a Year 5 class and probably about four of the six kids that I asked who got the question correct. When I asked them to demonstrate how they did it, they literally counted five, 18 times. So, it was a repeated five, ten, 15, 20. So, that they did that 18 times.

So, clearly those kids were still thinking additively, they were doing multiplication by repeated addition. So, how can we then help kids by understanding ourselves, the variety of skills and structures around multiplication?

LM That's a really good question. One of the reasons that students struggle to move from additive to multiplicative thinking is that they haven't been given structures, or the different structures. They might have been given groups of, they might have been given arrays but there are a number of multiplicative structures that we can use to help students understand multiplication on the whole. I'm using Ann Downton's work from 2008 to explain this.

Now she suggests that there are five different multiplicative structures. So, starting off with equivalent groups and that's something students have a lot of experience with. They start actually by sharing and they know about the concept of equal groups.

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MG They get that don't they because if they're sharing out a bag of lollies, they want to make sure everyone's got the same.

LM And that's why equal groups are so important, and children have an innate sense of justice and fairness.

MG Yes, they do.

LM So, that equivalent groups is a really good way to start. So, the example there are six bags of lollies with seven lollies in each. How many lollies altogether? And that is your classic, they can write six times seven, they can move very clearly, what I have just said is representational. You could actually put the lollies on the table which is concrete. Then you've got the representational and they can come up with the abstract, six times seven.

So, then you could do the allocation and rate which, one pack of cards costs 50 cents, how much would eight packs cost? And so, in that case, they're multiplying eight times 50 but it's different to groups of. It's not eight groups of 50, it's a different structure that they're using.

MG Yes, I see. I can see how conceptually I'd think about that in a different way because I'm thinking of amounts not groups. Does that kind of make sense?

LM Yes, and it's a little more abstract. 50 cents, you don't have 50s, not these days anyway. So, yes, it is not so much a group as a number. The rectangular arrays, we've mentioned quite a few times. The example that I've been given is that, I planted 12 rows of lettuce, there are six lettuces in each row. How many lettuces did I plant altogether?

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MG And that's really essential for when we get to area and understanding the measurement concept of area.

LM That's exactly it. And if we teach arrays properly then when we get to teaching area we don't have to teach a formula, we don't have to teach them length by width, because they actually understand that conceptually. They understand it because they understand about multiplication. Then there's times as many. Jasmine has eight dollars, Rhaya has three times as much, how much money does Rhaya have? And finally, the Cartesian product which is something that comes in a little bit later. Shareef has five shirts and three pairs of pants, how many different outfits can he wear?

MG So, that's combinations really, we're talking about combinations and of course we start to unpack that when we get to probability. So, they all link, don't they really? The other strands of maths that teachers will be aware of, measurement, geometry, probability, statistics. That multiplicative thinking and being able to understand multiplication in a variety of different ways is really important to understanding in those other areas too.

LM Very important and it's that transfer, being able to transfer those concepts into those other areas, that's absolutely vital. And if we just teach them five twos are ten, five threes are 15, five fours are twenty, how are they able to get that transfer? Whereas if we're teaching them these structures and enabling them to apply them to different situations then the use of the multiplication facts will help their memory.

We do want them to memorise it but that rote learning is not going to help them in these, unless they actually understand these different structures.

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MG So, it really, that's why it's multiplicative thinking and not just learning to multiply.

LM That's right. That's absolutely right. We're thinking, they have to think.

MG I reckon next time Leanne in our next podcast, we can have a chat about some of those, in more detail about those some of those conceptual steps that kids take. And start to unpack those a little bit further.

LM I think that's really a good idea, we've glossed over them.

MG Okay. So, thanks very much for coming in today. That's been a pleasure having a chat to you and getting your knowledge and expertise on multiplicative thinking.

LM It's my pleasure Marcus, thanks for having me.

MG You've been listening to **MATHS**TALK by AMSI Schools. My name's Marcus Garrett and today we've been talking about Multiplicative Thinking. The first in the series called Multiplication Matters. For our show notes and information on this podcast and more, head to our AMSI school's teacher support webpage at calculate.org.au and we'll see you next time.

Thanks today to our sound recorder Michael O'Connor, our editor and producer Nadia Abdelal and of course to our special guest today Leanne McMahon.

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