

## **MATHSTALK by AMSI Schools (Episode 1): Talking Number – Fractional Thinking**

### **‘Speaker Key:**

MG Marcus Garrett,

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MG Welcome to the AMSI Schools maths teaching and learning podcast. Where casual conversations in maths become part of your professional learning dialogue.

I'm Marcus Garrett. And today, with me, I have my colleague and fellow AMSI Schools outreach officer Helen Booth. Today, we are talking number.

Good day, Helen, how are you?

HB I'm fine, thanks, Marcus.

MG We exchanged a couple of emails a few weeks ago, after we were doing some lessons with some people out on the road. At the end of a lesson I had on decimals and metric measurement, a young Year 6 girl, you might remember, wanted to know how, since there are technically an infinite number of numbers or decimal places between zero and one, because they... As there are between all integers, how did we ever get from zero to one, she wanted to know.

And I thought that was a fantastic question because it actually showed me that she really understood the concept of decimals. She reasoned we can't count to infinity, so how do we ever get between integers with that number of decimal places.

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I told her that was the type of great question that makes maths teachers even happier than finding an answer.

HB Yes, I had a similar example with a Year 5 student who, when we were doing work with fractions, rather than with decimals itself, we were... And I was using a concrete material play dough. And what I had them doing was taking a piece of play dough; we cut it into ten even size pieces, so we made tenths. And then they had to take one of the tenths and cut it into ten pieces, so we made hundredths. And then they took one of the hundredths and cut it into ten pieces to make thousandths.

And I had this young Year 5 boy come you to me and go, oh, you could keep doing that forever and ever and you'd get smaller and smaller and smaller, but you'd never get to zero.

And I said, oh, so what do you mean you'd never get to...? He said, because when you put them all back together, you'd have one.

So, again, that was really good understanding that we get closer and closer and closer, but we'd never actually get to zero. So, the actual opposite to what your girl was talking about, never getting to one, you also never get to zero.

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MG Isn't it lovely when you can see the light coming on with kids like that showing they really understand some of the more abstract concepts on maths, like... And particularly in fractions and decimals. We know proportional reasoning is something that's a real struggle for a lot of kids to wrap their head around.

But questions and comments like that show that kids really have worked out how things work and they're beginning to apply reason. But, I guess we know that a lot of kids don't really get to that level of thinking. Many of them, not even into adulthood.

HB No, they don't.

MG So, I thought we'd have a chat today about what it really is that helps kids move into a genuine understanding of decimals and how they work, and how they can... And fractional concepts too, and how they can begin to apply those into things like unit conversion and measurement.

So, what do you think some of the key [big ideas in maths](#) are that are prerequisite to students getting decimals and decimal understanding?

HB I think the most important is they have to understand the connection between fractions and decimal fractions. I think there's a big misconception among a lot of students and, interestingly enough. I've discovered between the people that are supposed to be teaching them, or helping them, that decimals and fractions are something completely different.

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And yet, a decimal is actually a decimal **fraction**. In other words, it is a fraction with a denominator of a tenth of 10; 100; 1000. So any multiple of 10, those are decimal fractions as a denominator, but kids don't see that connection. And so, they see decimals and fractions as two completely different things.

And so, one of the things that I say to teachers is you need to teach tenths first and hundredths first, and then show them how a tenth becomes a decimal 0.1. So, how do we turn a fraction into a decimal so that they can see that... Have that, hopefully, ah-ha moment of oh. So, if I'm looking at a decimal 5/10... Sorry, if I'm looking at a fraction 5/10, if I put that into a calculator and go 5 divided by 10, I get 0.5. Oh, that's where it comes from.

MG Yes.

HB What happens if I put in  $\frac{2}{4}$ ? 2 divided by 4? Oh, I get 0.5. So, there's that connection that a half, it doesn't matter what it looks like, as a fraction, is always going to be 0.5.

MG Yes, it's one of the interesting connections that I'm often quite shocked about when you make that connection for students and say, actually, the vinculum, so the line between your numerator and your denominator, it's just another way of saying divided by.

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HB Yes.

MG And when you explain that to a lot of kids, I'm often really surprised that they go, oh, really, it's that... Is that just... It's another way of saying divided by, yes.

HB Yes. One of the things I do, and I don't know whether it's mathematically correct or not, but I certainly find that it makes a point to kids, is I'll write up a fraction. And I'll go, well, what happens if I rub out the numerator and replace it with a dot. And then I rub out the denominator and replace with a dot, what sign have I created? Oh, you've made a division sign. Yes, because that's what a fraction is. It's the numerator divided by the denominator.

MG That's what it means. Oh, I like that.

HB Yes, so it just gives them that visual, oh, again, that ah-ha moment, yes.

MG So, what do you think are some good teaching tools and strategies teachers can use to help kids understand that connection between tenths, hundredths, thousandths as fractions, and understanding the decimal number system? Because, of course, the way that we write a fraction looks different. If it's written up on the board, for example, it looks different to the way that we'd write a decimal. And sometimes the numbers are different too. So, for example, two over four as a fraction, we would write as 0.5 as a decimal. And often, that confuses many students.

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So, what are some good teaching tools you could recommend to help teachers build that understanding?

HB I'm very keen on having concrete where you can have concrete. So that's why I love using the play dough. By getting them to actually have that... I find play dough is even better than having other concrete material, because there's another sense with it. So, you not only have the visual and the oral, you also have that tactile component to it, and the kids get to make it and they're comparing things.

So, I reckon if you can make it... Particularly when you first start to introduce the concepts of decimals, so tenths and hundredths in Year 4, giving them that opportunity to have that tactile component, I think is really important.

I think the other thing is the decimats, which you can find on the Calculate website, there's... I think they're another really good concrete tool to use.

But, I think there's also the need that when we are talking about decimals, that we're not only reading it as 3.1, but we're also saying, which is 3 and 1/10. So there's always that connection between how we... What it looks like, but, really, when we read it, it should be read as... So, 3.25 should be read as 3 and 25/100. Not as 3.25.

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MG Yes.

HB There's that connection that... How do we know it's hundredths? Well, because there's two digits that are making up the decimal components. So, therefore, it must be over a hundredth.

MG Yes. So, using language and making that language very explicit to kids when we first introduce decimal notation and reminding them that decimal notation, literally, is talking about a fraction of a tenth, or a hundredth, or a thousandth, depending on the number of decimal places.

HB Yes.

MG Helen just mentioned there, too, for those people listening, the [Calculate](http://Calculate.org.au) website. So... And she mentioned a tool called decimats. So, if you're interested in that, just pop onto the Calculate website, which is just calculate.org.au, and go into classroom, and then planning. Or just do a search and you'll find the unit on decimats, which is a great teaching tool.

Helen, what do you think some of the common misconceptions that kids and some adults have about decimal numbers? So what are the common things that you've noticed kids and students get wrong? And how can teachers help correct those common misconceptions?

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HB I'll give you a story that I had just recently, where I was talking with a teacher, Year 4, 5 teacher, about place value. And we brought up the concept around language and how we tend to be very sloppy with the way we describe things. And one of the things we are sloppy with is when we refer to decimals, decimals fractions, as just decimals. And I made this comment to her and was talking about how we need to be precise about our language so that students understand that decimals are fractions.

And there was a SSO in the room at the time, in the staffroom, and she was like, they are? I didn't know that. So... And this is one of the big misconceptions I think kids have, is this thinking that decimals and fractions are two completely different number systems.

MG And clearly, not just kids but adults too.

HB Yes. Yes. And so, I think that's a really important thing, is making sure that there's this connection between them. And this is why it's really important that... When we're doing stuff like counting, we talk about counting down in the early years. But we should be getting to count in different ways all the way through. So, even if they're in Year 5, they should be counting; they should be counting in decimals, but they also should be counting in decimal fractions.

So... And let's... So, today, we're going to start counting at three and we're going to go through to five, and we're going to count by tenths – let's go.

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MG And would you show that, also, visually on, say, a number line as well?

HB Oh, absolutely on a number line. Number lines are absolutely vital. The development of number lines, we have... One of the most basic mathematical understanding we have is around the mental number line. We all carry mental number lines in our heads. Now, if we don't develop those mental number lines beyond whole numbers, kids are going to struggle.

One of the things that has been shown through research, and we can certainly put the research that I'm talking about, here, on to the bottom of... Into the webcasts, so people can find... Look it up for themselves. But they're saying that the two main predictors of success in algebra, algebraic thinking, and in general mathematical performance at a higher level is if kids understand fractions and division in Years 5 and 6.

MG Wow.

HB So that whole thing around the number line is... And it's... The other thing about fractions, and this includes decimals, is when we think about fractions, we tend to teach about the part-whole relationship, mostly.

MG Yes.

HB But the other important aspect of fraction understanding, fractional understanding, is the magnitude. So, it's how the size of these different fractions relate to one another. Just as when we are doing whole number magnitude, that we know that 100 comes further up the number line than 24, or that 765 thousand comes a lot further up the number line than 74... We also need to understand where, in magnitude, these fractions exist in relation to one another.

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Where decimal fractions sit an relation to one another. If I'm going to talk about 3 thousandths, I know that's a lot less than 3 tenths.

MG And... Yes, so it's interesting, isn't it, because we often... So, in... Early in middle primary school, will give kids lots of activities, where they're bundling units into ten and then bundling tens, and they can see that magnitude when they see how much more ten is than one, and how much more 100 is than 10, and so on and so forth.

But we don't necessarily do that in late primary, with fractions, by showing how much smaller a hundredth is than a tenth, and how much smaller a thousandth is than a whole, for example.

HB Yes.

MG So what you're saying is giving kids an understanding of magnitude when we're talking about fractions and decimals, is just as important as giving them an understanding of magnitude in early primary with whole numbers.

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HB And it's probably even more important because what the research does show... As I said, the research shows... And they've looked at everything, addition, subtraction, multiplication, division, as well as in whole numbers and in fractions what it does. The only two areas that are direct predictors of future mathematical achievement is fractions and division.

So... And the other thing is, if you look at the curriculum, by the time you get to Year 6, more than 50% of the curriculum is around fractions, decimals, and percentages. And I would actually ask teachers, at Year 6, to really look at their curriculum and be honest about how much time they're actually spending on fractions, decimals, and percentages.

MG It's often a shock, particularly for Year 5 and 6 teachers, isn't it? It's often a bit of a shock because there is so much content at that level around fractions and decimals, yes.

HB Yes. And the other thing is these... They go, oh, yes, but it's such a crowded curriculum; such a crowded curriculum. But the other thing that goes along with that is, okay, it's such a crowded curriculum, so how do you make these links across the strands within the curriculum? And my response is you look at measurement; measurement in Year 6 is all about conversion between units, which is...

MG Yes. Fractions and decimals.

HB When you look at probability, it's about probability exists between zero and one, which is...

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MG Fractions and decimals, yes.

HB Yes, so it's about finding those links. Instead of trying to teach things in isolation, particularly fractions, decimals, and percentages, we need to be teaching it in context, so the kids see the purpose of what they're doing.

MG Yes, definitely. And that actually brings us to the last question for our little podcast today. How would you, then, know that a student has a good solid understanding of decimals? So, at, say, Year 5, 6 level, how would you know, yes, this kid has

got it; they understand it? Besides questions like those that we talked... And anecdotes kids give us... Those we talked about at the beginning of the podcast. How would you know, yes, this kid has nailed it?

HB One of the easiest, which is already out there, and has been tested and so moderated, would be the decimal comparison test to actually find out whether kids really do understand that.

But the other thing to remember is, just because kids can write the numbers and even read the numbers in logarithmic way, doesn't actually mean that they really understand it in a qualitative... Quantitative way. That often takes a little bit longer.

So, if you look at a Year 2 student, where... Well, even a Year 1 student, where we're expecting them to be able to work with the numbers up to 100, what the research, again, is saying that they don't actually understand that and being able to think about the magnitude of those numbers until they're in Year 4.

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MG Wow.

HB So, it's the same sort of thing with decimals. We can get them to become familiar with how to write them and read them and that sort of thing, but they're not necessarily going to understand the magnitude of those numbers until they're in Year 7 or Year 8.

And I think that's also what the decimal comparison test looks at. If you look at the research that went into that, you can actually see that the misconceptions kids have around decimals in Year 5, 6 begin to change and disappear, because they're no longer putting their whole number bias onto the decimals. When they get into 7 and 8, they begin to make that connection about the magnitude of these numbers. But it takes a couple of years for that understanding to come through.

MG And I guess exposure to using decimal numbers in things like measurement and unit conversion... Geometry; even if you think about angles and understanding angles, that's about breaking things up into proportions and using proportional reasoning.

HB And I'd say to... Particularly when they get into Year 7, where kids... Part of what they need to do in measurement is around formulas and using formulas to work out area... Area and so forth, perimeter, different shapes. Those kids, you can differentiate. So those kids who are struggling with using decimals, so they measure in millimetres. Those kids who are able to do decimals and work with decimals, they measure in millimetres and convert them to centimetres, and then give the answer in centimetres. Or, kilometres, or metres, or whatever.

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So, you're getting them to think about how are they changing that number.

And I think the other thing that often happens is we don't give students enough opportunity to work with decimal numbers. We let them work with whole numbers all the time, but instead of going, okay, today we're going to add two decimal numbers together, or three decimal numbers together, we're always giving them whole numbers. And when they can do two whole numbers plus another... Sorry, a three-digit whole number plus another three-digit number, we move them on to four digit by four digit, or five digit... If you know how to do that, why make it just bigger and bigger? Why not give them decimals? Why not give them more than two, so they're actually working with these numbers.

MG Yes, not just overcooking whole numbers, but moving into decimals.

HB Yes. And also going with the measurement. Give them an odd shape that they've got to measure everything and work out what it is that they... What's the perimeter of the shape, by adding all these different decimals together.

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MG Helen, thank you very much for coming in and giving us your insights into teaching fractions and decimals.

HB My pleasure.

MG Again, folks, if you want to check out the [Calculate](http://calculate.org.au) website and the teaching resources that are available there to assist you with some of those strategies, it's just calculate.org.au. And you'll find plenty of classroom teaching, and resources, and professional development articles there to help you out.

Thank you very much.

HB Thanks, Marcus.

MG I'm Marcus Garrett, and you've been listening to the AMSI Schools podcast.

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