MATHSTALK **by AMSI Schools** (Episode 3)**:**

'Multiplication Matters' (Part 2)

Speaker Key:

NA Nadia Abdelal

LM Leanne McMahon

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NA Welcome to Maths Talk by AMSI Schools, where conversations in maths become part of you professional learning practice. My name is Nadia Abdelal, and I will be your host for today. With me, once again, is fellow outreach officer, Ms. Leanne McMahon. How are you Leanne?

LM I'm very well, thanks Nadia.

NA That is amazing. Leanne, you've just gotten back from Alice Springs, haven't you? Can you tell us a little bit about what you were doing there?

LM Alice Springs was amazing, and quite surprising to me. There was a career expo for students. So, we spent two days talking to students right from primary level, about grade five, all the way up to year 12, coming to see what careers are open to them. I think one thing that we're hearing less and less is, oh, I'm not a maths person, I can't do any of that.

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So, our careers campaign seems to be hitting all the right marks, and for me Alice Springs was a great way to round it off in a nice warm environment, with great people and a campfire at night.

NA Well done. I saw those photos, they were wonderful. All right, so back to talking about multiplication, because this is the second part in our series of multiplicative thinking, called multiplication matters. Why do you think it's such an important, Leanne? Why such a big focus on multiplicative thinking?

LM As we discussed in the last episode, we need our students to be thinking flexibly and efficiently, and working with a large range of numbers. They need to understand the relationship between the numbers, and be able to use those relationships to solve problems.

Now, if they are learning number facts without that deep understanding of number, then they're not going to be able to solve problems, they're not going to be able to use those numbers flexibly. Multiplicative thinking allows them to understand how numbers can be used flexibly. It gives them an introduction into proportional reasoning, which is the underpinning of everything, fractions, decimals, all of that.

NA Ratios.

LM Ratios.

NA Algebra.

00:02:21

LM Absolutely.

NA And we often find that when we talk to kids and when we talk to teachers, a lot of the problems that are inherent in our students as they get in to high level of maths, is that they've not really developed that ability to think multiplicatively at the year four, year five level. And so, it transfers and translates into problems with those higher maths levels.

LM Absolutely. And you've got a perfect example of a year ten student I think.

NA Yes, I did. It was a year ten student. I work closely with this student, and she is a real struggler. She thought she was a struggler in maths. She did struggle. Her confidence was not very high. She didn't have a lot of confidence in her ability to do maths.

And as I sat with her and we looked in to the reasons why she was struggling, I always go back to a couple of points. I'll always go back to a couple of points. When I'm doing any intervention with students, if they're younger, I'll look into their ability to quantify. So, trust the count. Then I'll look in to their place value. And the older students, I'll do those things, but I'll focus a lot more on the multiplicative thinking.

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And so, with this student, I did give her a question. We were sitting there doing a session, just having conversation towards the end. And I said to her, what's 18 times five? We mentioned 18 times five last week. Classic Jo Boaler. 18 times five. It's one of my favourites, because there's just so many strategies of getting the answer to 18 times five. And this student was struggling. In the end she said, I think it's about 80.

And I knew that this young girl suffered from maths anxiety quite severely. And when I said to her, how did you get 80? She said, I counted by fives. Five, ten. But because she was under so much pressure, she didn't quite count enough. She didn't have the right timing.

LM If you watch the Jo Boaler video on 18 times five, I think it's called number talks, and you see these Stanford University students. So, these aren't dumb kids, trying to work out what 18 times five is, some of them have this look of sheer panic in their face. And you can see, there is not much cognitively going on there.

NA No.

LM Because there is so much panic, they've been overwhelmed by it, and I can imagine.

NA And I am a person that totally relates to that feeling of panic. Going through school, I had immense maths anxiety. And it highly impacted my ability to do maths, and

perform in maths. And in fact, teachers, when I was younger in the primary levels would say, she's just not a maths person. And of course, now I do this.

LM You can fool some of the people all of the time. Or all of the people some of the time.

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NA That's right. So, anyway, back to this young girl. When we started digging deeper, I realised that she hadn't actually developed her ability to think multiplicatively. She had no idea about the distributive properties of multiplication.

And when I showed her how to do it using an open array, and how to split her numbers and to distribute the numbers equally, she was blown away. She had never seen it done in such a way. And this was a girl who'd obviously been just exposed to the multiplication facts. Here are the multiplication tables, learn it by rote.

LM How was she with those?

NA She knew her multiplication facts really well, but she just didn't know how to apply it to a larger… She needed a pen and paper. She said to me, because we didn't have a pen and paper at the time, and she said to me, oh, I need a pen and paper to work that out.

And I said, no, I want you to work it out in your head and see how you go. And she was in such a state. But I'll tell you what, once she started grasping those ideas, she actually said to me, why has nobody ever shown me this before?

LM I know.

NA I love number talks. I could talk about number talks for the entire podcast.

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But whenever I go into a school, without fail, if I do a number talk regardless of what it is, the students always walk out going, that was the best. That was my favourite part of the lesson. They absolutely love the number talks, because suddenly they're exposed to things that they understand. It's not just about learning them off by heart, it's about the deeper understanding of what's actually happening. And they develop strategies, and they start to figure out that there's not only one way.

LM That's it.

NA I've been taught this way, but Johnny over there is doing it that way. And what an efficient way of doing it.

LM I think you've hit the nail on the head too, with Johnny over there doing it. It's not the teacher saying, now you can do it this way or you can do it that way, or you can do it another way. It's the people in the class, and the teacher actually saying, yes, your way is valid. Even that, as you said, that 18 times five. If you count by fives 18 times, you will get the right answer.

NA That's actually a strategy as well.

LM It is an accurate strategy.

NA Yes, a valid strategy. But when we talk about strategies, we often talk about efficient strategies.

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LM Yes.

NA And I think that's what is important when we do, A, number talks, and two, think multiplicatively. Because it's about solving problems in the end. It's about using what you know and what you've learnt, to apply them to situations that allow you to work out the problem efficiently. Because you can sit there and you can solve a problem, and it can take you five hours, or you can use a more efficient strategy and solve it in an hour or ten minutes, or however long it is.

LM That's right, yes. So, these students with the less efficient strategies are still getting the right answer, but they're learning from their colleagues that, oh, my brain didn't work that way, but if I look at it another way, I might be able to do it.

NA Yes, that's fantastic. I touched on something when I was speaking earlier, and that was quantity. And I want to take it all the way back, because often people say, what is the very basis of multiplicative thinking? What is it that we teach first? And our students come in with certain levels of understanding, and can you explain what those levels are?

LM Yes. When students come into schools, they have a basic understanding of certain things.

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And I have to give credit to Di Siemon for these ideas, because I've heard her speak about multiplicative thinking, and she says that before children come to school, they usually know what it means to get more. That's addition. They know what it means to have something taken away, or have less than someone else. We all know what children are like when they've got less than someone else. So, they actually understand taking away, or difference.

They certainly should know what it's like to share equally, especially if they've got brothers and sisters. They don't have a lot of experience…

NA You'd hope that they would know how to share equally if they have brothers and sisters.

LM Children have a very innate sense of fairness.

NA Yes.

LM However, what they don't have is the ability to make and count equal groups. It's not a natural part of their everyday experience. So, as teachers we have to provide that experience for them. What we tend to do in those very early years is reinforce those three ideas of addition, subtraction, and division, or sharing. We need to look also at making and counting equal groups, making that as a part of their experience.

NA But before we get to equal groups, there's one very important concept, isn't there? And that's subitising.

LM Yes.

00:10:58

NA I think you touched on subitising in the last episode, but I want to talk a bit about subitising, because it's such a crucial part of student development. And I don't think it's actually done enough in schools. What do you think?

LM Absolutely right. Subitising underpins much of the quantity mathematics that students need to know in those early years. Now, subitising, what it is is the ability to recognise a quantity without counting. Simply put, there are two areas of subitising that are important. Firstly, that recognising a number, and that is really important in trusting the count.

So, you're looking at numbers, basically to five, or familiar patterns such as dice patterns, to six, or ten frames. The ability to see a number without counting, that enables students to trust the count. So, to believe that there is five without counting allows them to say, I have five, I want two more. Five, six, seven. Counting on. A student that doesn't trust the count, can't do that because they don't actually believe that there are five there in the first place.

NA Yes, so they have to always count the collection again.

LM Yes.

NA So, if a student doesn't know how to trust the count, and that's one of the big ideas that Di Seamon talks about, is trusting the count, that's actually the very first big idea.

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And so, if students don't know how to trust the count, they will always need to go back and count the collection, even though they've already counted the first collection. And subitising allows them to be able to recognise that this is five, without actually having to go back and count that it's five.

LM That's right. And I must say, when I do demonstration classes with preps and I need to add, that's not my forte. I am secondary trained.

NA As am I. Preps scare the bejesus out of me, can I just say.

LM They terrify me too. But I have developed a great lesson that works all the time, and the learning intention for that lesson is we understand that we can know a number without counting.

NA Yes.

LM And so, we make beads, strings, we do all sorts of things. And it is basically subitising. It's lesson after lesson, after lesson. Teachers need to be careful of thinking, oh, I taught that, because there's a difference between teaching something and learning it. So, you can do lots. You can do it day after day, after day.

NA Now, I had a question from a teacher. We were talking about subitising and the importance of subitising, and her question was, do we give them the quantities in the shape of the dice, how they see it on the dice? And I said, the idea of subitising is that they're not always going to be presented with the quantities in the pattern that's on the dice.

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LM That's right.

NA So, they really need to be able to look at any random collection, whether they're counters or whether they're cars, or whether they're people, and they should be able to say, that is six, or that is nine. Now, numbers past five is a little bit more tricky.

LM Absolutely.

NA So, what strategies do you think teachers can use to help students to do that?

LM Numbers past five require a totally different strategy, and I guess they rely on being able to group. And that's where subitising is the introduction to multiplicative thinking. So, that first one that I just talked about is trusting the count, recognising your number by just seeing the number of dots. Just about the dot patterns, I say, yes, go for it. Go for it with the dot patterns, but mix it up.

So, have a five the way that the dot pattern is on the dice, but then mix it up. Have a five another way. Have a five in a ten frame. Because it shows that this number is five, again, trusting the count. This number is five, no matter which way you'll arrange it.

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Okay, so back to the second point, when you've got above five. And that's a neurobiological fact I guess, that human beings aren't capable of automatically subitising above five. So, we need to group. I'm thinking of a dot pattern. If you think of two arrows, one on each of a line, but in dot patterns. So, what you tend to see in that way is a three and a three, and then one in the middle. So, automatically, students are thinking I've got two groups of three. That's your introduction to multiplicative thinking.

NA And that's also really good for partitioning numbers, because when you're learning quantities, really the very first thing that you want to do is you want to help students to develop a really deep understanding of numbers from zero to nine. And how to partition the numbers, break them apart and put them back together in lots of different ways. So, the idea that nine is seven and two. It's also five and four. It's ten minus one. It's all of these different ways.

And I think there are a lot of different ways of doing that. I think dot talks are really good ways of doing that as well. I love dot talks, because everybody may get the sense that I'm a big number talks, dot talks fan. Because I just think that it just gives kids an opportunity to be able to not only to see things from their own perspective, but also to communicate and verbalise what they're actually learning.

LM That's right.

NA And communication is a huge part of the students' learning and development.

LM And reasoning. Communicating their reasoning.

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NA And it's not an easy thing to do for kids, especially the younger kids. It's very difficult to communicate their understanding. And often when students solve problems, they'll just say, I just knew. But these days it's not about I just knew, you need to talk about why you did that. So, dot talks are really good at developing that ability to partition numbers, and how did we see them, and also work on their spatial skills and spatial reasoning, and things like that.

LM That's right. I actually think that is a whole other podcast.

NA Yes.

LM I think we really need to get into that.

NA Yes, we really could spend a whole lot of time talking about those things.

LM With regard to the subitising though, what you're doing is, even if you put six in a dice pattern, you are showing those children that that is three rows of two. So, you introduce these groups of all rows, whatever the arrangement is, without introducing multiplication facts or anything like that. They learn that two groups of two makes the number four, by, as you said, the partitioning. And partitioning is that very basis of number sense, and that's all grouping is. Partitioning equally.

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NA So, after subitising, what is the sequence of multiplicative development? Where should students go? Where should teachers direct their students in terms of their learning?

LM Once they've got that understanding of number and the subitising, and just being able to recognise groups within a number, we need to look at that concept of equal groups. Now, as I said before, students and children when they come into school, have that understanding of sharing equally.

So, this is just working the other way, making equal groups. And as long as we keep talking about those equal groups, students can understand that. And from there, we go into the skip counting and looking along that number line, jumping along with those equal groups or equal jumps along the number line.

NA So, you mentioned groups of, and skip counting. And that's the very basis of beginning our multiplicative thinking. But we really want to move from groups of, into arrays, don't we?

LM That's right. The skip counting teaches us or teaches the students, about repeated addition. Okay, so before you actually get to the arrays, they do need to understand this repeated addition. This is all additive thinking. This isn't multiplicative. Students need to understand additive thinking before they actually get into the multiplicative.

NA Absolutely.

LM So, it's once you get to the arrays that you can start to think multiplicatively, because you're looking at area. So, the arrays show multiplication as repeated patterns in rows or columns. And then, you can manipulate those arrays to give the same area.

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NA That's right.

LM And that's where your 18 times five is the same as nine times ten comes from.

NA And also, 18 times five is the same as five times 18. So, we can use the commutative law, we can also use the distributive law with arrays too. Which allows students to see that you can split the numbers in lots of different ways and still get the same answer, because the area is not changing, it's just the two numbers that you're actually multiplying.

LM That's right.

NA So, if we're looking at area, it would be your length and your width, or the number of rows or the number of columns that we're multiplying to get the total number within our grid. So, how do you start students off with working with arrays and things like that? What's the progression?

LM There's a great little activity that you can actually use. We call it the cupcake activity, where you have pictures of cupcakes, cut them out, and give them an A4 piece of paper. And that's your baking tray, and you have to arrange the cupcakes on the baking tray.

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There are lots of ways of arranging. 24 cupcakes is the best number I believe.

NA Yes, it's got a lot of factors.

LM Yes, that's right. And you can arrange them randomly. You can arrange them into groups. That's a way of doing it. But of course, you're always going to get kids that are going to arrange them into rows. Rows and columns.

NA Because, of course, if you've got a tray, most of the baking trays will be rectangular.

LM And you've got muffin tins that you can actually…

NA Which are also rectangular.

LM That's right. And they are in arrays already. So, children have the experience. Cooking is the best mathematical experience they can ever have.

NA It really is. Yes, if you want to teach your kids a bit about any type of maths or maths in context, get them in the kitchen.

LM Yes. There's another podcast we can do.

NA Absolutely. Maths in the kitchen.

LM Yes, that's right.

NA I like maths, but I don't really like getting in the kitchen.

00:23:08

LM You and me both, sister. Okay, so I love that activity, because it does get them to think about arranging these cakes in a certain order. Some people have one long row of 24, two rows of 12.

NA And so, then the question you would ask would be what is the most efficient way of arranging the cupcakes?

LM Yes. What's the way it's going to fit in…

NA Into the pan?

LM Now, you could arrange it in one row of 24, but it's not going to fit on the tray, is it?

NA Yes.

LM So, two rows of 12 aren't really going to fit in the tray. And it's not efficient in a normal oven. Three rows of eight. And so, eventually they'll find an efficient way, but then that question that we always ask, can you do it another way?

NA Can you do it another way, exactly.

00:24:02

LM Oh, I've got a really long, thin oven, what can we do? Those sorts of things. Questioning is so important as a teacher. You'll get kids that'll say, done it.

NA So, you could then ask the question I guess, just for a little bit of an extension activity, if my tray didn't fit that many, how could I divide it or split my numbers so that I would still have 24 cupcakes? But then what would they look like? So, then that's leading them into those distributive properties.

LM That's right.

NA So, you can take those activities and you can really take them in to a higher ceiling type format, for the kids who have gone, I understand this. So, inadvertently you're going to get those students in your class that need that extension. And rather than giving them larger numbers to work with, you give them different situations to problem solve with.

LM That's right. The other thing you can do with your cupcakes is when they're out of the oven, you put them on plates. How many people have we got coming? So, you share them.

NA And then that moves it back from arrays to groups of, and sharing, and division.

LM That's right, so they can see the relationship between those arrays.

NA What a great activity.

LM I have to thank Anna Bock for that one.

NA Anna Bock, she's a legend. She's another one of our outreach officers. She's been avoiding the mic till now.

00:25:32

LM But we can always drag her in, can't we?

NA Yes, we will.

LM No, that's one of hers. And yes, I've found it really effective in introducing arrays.

NA You're listening to Maths Talk by AMSI Schools, and today we're having a chat to Leanne McMahon in our second part in a series called Multiplication Matters. Don't forget, if you're interested in some of the material supporting these podcasts, don't hesitate to check out our calculate website at calculate.org.au. On the website, there are lots of teacher support materials. Specifically, there are modules and also links to these podcasts, and some of our show notes.

All right, so back onto the progression. So, we started with subitising. That's the very first concept. Then we move into skip counting, and then into groups of, and then into arrays. And the arrays where we initially begin would be arrays using counters, or in this case cupcakes. So, what next?

LM As we discussed in the last podcast, we're going to go from concrete to representational, to abstract. So, the concrete was the counters, the cupcakes. So, representational, we're going to do girds. So, it's going to be still the area. We're still going to be able to see the area, but we're going to use grids, not counters.

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Then we move on to the open array, or the area model where it could be representational of the area, but we don't have counters or we don't have squares or anything like that.

NA And why is the area model so important? Because it is important to take the kids through that step when they're learning multiplication. But we want them to get to the area model or the open array. Why?

LM The area model is vital in understanding place value, the value of the places. That is, when I'm multiplying 18 times five, I'm multiplying one ten by five, and eight ones by give. I'm not multiplying just 18 ones by five. That area model shows very clearly that you are multiplying certain numbers and whole numbers with value, and it shows you the value of those numbers that you're multiplying.

NA And it's really important, isn't it? When they start multiplying two digit numbers and three digit numbers.

LM Very.

NA Because when we start students off with multiplying using the vertical algorithm, which happens a lot, the students don't actually understand the value of the place that they're multiplying.

LM No.

NA They don't know that 18 times five is eight times five, and ten times five. It's not eight times five and one times five, and then we add a zero to the end.

00:28:51

LM Adding a zero.

NA When we add a zero to anything, it doesn't change the value of it. So, we don't say add a zero. We like to get the students to understand that it's eight times five, and then ten times five. And I think using the open array gets student to actually see what's happening when they do the vertical algorithm. And there's nothing wrong with vertical algorithms. It's a perfectly legitimate strategy.

And interestingly, somebody actually told me that the vertical algorithm was actually developed because when we first started printing on paper, paper was very expensive. And printing took a long time and was also expensive.

So, they needed to take some shortcuts. So, then in order to save time and save money, they shortened the calculation of multiplication, and they've made it into the trading or the carrying of the numbers. And not really looking at it in terms of their whole place value parts.

LM That's right. And so, we often get parents saying, why can't you just teach it the way that we learnt it? It's not the most efficient way.

NA No.

LM It's not. With all the research that's been done on student learning, they've found that it's not the most efficient way.

00:30:12

NA Or the most effective way, I guess.

LM Effective, that's right. I just want to put in here that professor Doug Clarke, in 2005, argued that students should not be taught conventional algorithms until they're able to add and subtract two digit numbers in their head. So, let's come back to why the area model. Why the area model? Because it makes sense. It is in fact the algorithm in a much more simple form.

NA Yes.

LM Simple in that it uses easy numbers.

NA It shows all of the parts as well.

LM That's right.

NA It makes everything explicit.

LM That's right.

NA So, thank you Leanne. I think we've covered a lot today and in a lot of detail.

LM We could have gone on for another three hours I think.

NA We just seem to ramble on, don't we?

LM We do.

NA This is why Marcus needs to be in this chair.

00:31:09

LM Yes, he can rein me in.

NA Okay, so we have quite a number of podcasts coming up, and quite a few ideas. I think we wanted to do one on dot talks and number talks. We've got a bit planned on maths anxiety, and some on problem solving as well. So, thank you very much for joining us today.

LM Thank you.

NA Thank you Leanne for today. And hopefully you guys got something out of this. Join us next time for our final episode in the Multiplication Matters series. We'll see you then.

00:31:49