MathsTalk **by AMSI Schools** (Episode 4)**:**

Talking Number: Fractional Thinking (Part 2)

Speaker Key:

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HB Helen Booth

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MG Welcome to MathsTalk by AMSI Schools, where conversations in maths become part of your professional learning practice. Today I've got Helen Booth and Nadia Abdelal with me, both of them fellow schools outreach officers. Good day Helen, how are you?

HB Fine Marcus, how are you?

MG And Nadia how you're going?

NA Not too bad thanks.

MG Now today I thought we'd continue to have a chat about a series that Helen and I began a couple of weeks ago on fractional thinking. We began to talk about fractions and decimals, and concentrated very much on both the relationship between fractions and decimals, and also some of the misconceptions that students had generally, but Nadia I thought we'd start with you today. You had a really interesting anecdote to share from one of the classrooms you worked in recently.

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NA I did. A couple of weeks ago I was in a Year 7 classroom. I always start my class with number talks and this day was no different. I walked in and I gave them the faction twelve thirteenths plus seven eighths is closest to what number. I had four options, the first one was one, the second one was two, the third was 19, and the fourth was 21. After some thinking time many of the kids said that the answer was either 19 or 21. In fact not one student recognised that the answer was closest to two. I thought that was a really interesting lesson. What do you think about that Helen?

HB I think it's something that is obvious in the way that we teach fractions. We talk about different constructs for fractions. There is identified five different constructs for fractions. We talk about fractions as part and whole, the relationship between part and whole. But as part of that part and whole is also the concept of magnitude.

What I would say is those students are not recognising that twelve thirteenths is nearly one, and that seven eighths is nearly one. They're not actually thinking of them in relation to the whole. We've got nearly one and a nearly one, so that's going to be nearly two. They were thinking, oh, I need to do a procedure on them, so they're busy trying to add them together and that hasn't worked because they obviously haven't known how to turn them into equivalent fractions.

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NA That was really interesting and the fact that many of them thought the answer was 19, which was just the numerators added together, or 21, which is just the denominators added together. They're didn't even recognise that perhaps they needed to find a common denominator and then add the numerators, they just saw it as whole number addition.

MC That kind of thinking often surprises us as teachers because we think what is going on in your head, why are you thinking this way? So what I thought we'd do today is basically work through a series of ways that kids can think about fractions. As Helen just mentioned, we're going to call them fraction constructs. Let's have a start with that idea that you've just mentioned, Helen, the idea of fractions as part of a whole, and then we'll have a chat about a few other fraction constructs, in other words other ways that kids can think about fractions and hopefully unpack some of that confusion that students have. Do you want to have a chat firstly about what we mean by thinking about fractions as part of a whole?

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HB Fractions, when they first are introduced are introduced as part of a whole. So in a lot of ways when you go right back to Year 1 class, and we're introducing this concept of a fraction as part of a whole or a collection, students are not understanding that the concept of a half, so at Year 1 level we introduce a concept of a half, they're not understanding that a concept of a half is dependent on the size of the unit or the whole that we're talking about. So when we first introduced the concept of a half in Year 1, we're introducing it as half of a whole or a collection, or particularly of a whole, but students don't understand that the size of the half is dependent on the size of the whole. So you ask a student to show me a half and they are still applying their natural number bias to that understanding by thinking that a half is static. Like one is always one and six is always six, they think a half is always a half, but the size of a half is totally dependent on what the whole is.

NA Now, Helen, you actually said something really interesting earlier today when we were having a conversation, and that was whenever you ask the question of what is a half, what's the question you always ask?

HB If I ask you what is a half, show me a half, often with adults, even with adults, they will show you half a circle, or they'll draw a circle and then shade it in half because we have standardised the half. Whereas the question you should be asking me is, actually, what's a unit? Because if I ask you the question what is a half and I'm talking about a dozen eggs, which is a lot different from if I have an apple and ask

you what is a half with relation to that apple. So the size of the half is totally dependent on what the unit is I'm talking about.

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MG So that confusion between a fraction and the fact that a fraction varies depending on the unit, that often then translates as far out as adulthood. Nadia, I'm going to pull you back into the conversation here and ask you about a situation that you were in when you were shopping for skincare products quite some time ago. Do you want to tell our listeners about that because I thought that was a great story and I've used it a few times when I've been talking to teachers in professional learning sessions? Explain to the listeners what happened and how that reflects that confusion between a unit and fraction of a unit.

NA You're talking about my percentages one. Yes, I made a whole unit or lesson on this interesting story. I was in a very well-known skincare store. I was shopping for some skincare with a particular ingredient in it. I was speaking to the lovely customer service representative and she was talking to me all about the ingredients that are in it. This particular product had a certain percentage of this ingredient called hyaluronic acid. She was talking to me about how much they both had together, so there was serum and then there was a moisturiser.

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The serum had 75% hyaluronic acid and the moisturiser had 30% hyaluronic acid. She was basically saying in the end that if you use both of these ingredients together you're going to get more than 100% hyaluronic acid on your skin. It's a very common misconception that people have with regards to this. She's just taken 75% of one and 30% of another and not looked at it in terms of the proportions that are in each of the products and just basically added together and got 105%, which you can't really have.

MC So there's an adult that’s confused the idea of a fraction and not understanding that a fraction depends on the unit, like it's a proportion of a whole unit of something. You can't just add static fractions together.

HB Yes and that actually relates to the next construct, which is the construct about fraction as a measure. In that case, when we’re taking about 30% of a certain amount, it's 30% of whatever the total fluid amount is of that particular product, which is going to be different for the 75% of the fluid amount of the other. In one it might be, and this is where if you talk about all of the concepts, the constructs around fractions, you are not only just bringing in, you can't separate them out. You can't separate, oh, we're only going to do this. Yes, when we first start talking about fractions we introduce the concept of fractions as in the part whole relationship.

But when you look at the other types of constructs, so you have constructs of fractions as a measure, that’s also to do with the number line. So not only is it about quantities, it's also about where does it sit on a number line, which again

relates to the magnitude of it. How do we compare these particular fractions to one another? We have to put them on a number line. We need to know if I'm going to compare thirteen fourteenths and twelve thirteenths, where do they sit on that number line. Which is bigger? Which is going to be closer to one? And being able to think about them as one another and knowing that seven eighths means we're only one eighth off a whole and thinking of it in that way.

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NA And also that if you have a number line from zero to one, half will sit in a different place than it would sit if you had a number line from zero to two.

HB Yes, and that's the other problem that often happens is we end up teaching fractions only between zero and one, so kids end up thinking that's where fractions exist. Whereas, as you said, if I've got a line, and this is where going back to this concept of a unit, if my unit is three apples and I ask what is a half, it's different from when I've got one apple, because a half of three apples is one and a half apples. So we've got to look at fractions in lots of different ways, not just in one way.

MG It comes back to that importance of using number lines. I know we've mentioned that in two previous podcasts, the importance of showing things in number lines because that can be quite confusing. Half can be a point on a number line, as in zero point five, so it can be a particular point, or it can be a position on the number line depending on how much we're talking about. So it goes back to Nadia's comment just before about whether a half is at one because it's half of two, for example, or whether a half is that particular point on the number line.

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Let's move on and have a think about fractions as operators. This also is quite surprising for a lot of students to work out that in fact the vinculum in a fraction, so that little line between the numerator and the denominator, is really just operating as a division sign.

HB Yes, and it comes back to we were talking about the language that you use when you're talking about fractions as an operator, and when we're talking about the whole concept of multiplying by a fraction. A lot of kids learn the process. We have a half times a third, they learn all they've got to do is multiply the numerator, multiply the denominator, and they get an answer of one sixth. They're got no idea how they get there because they're not understanding what that multiplication sign really means. They simply see it as I've got to do an operation, I know what to do. There's always a lot of confusion around them when they think why is it getting smaller, because when they apply the whole number bias to it, any time you multiply two numbers together, the number gets bigger. But when you multiply two fractions together the answer gets smaller, and there's a lot of confusion around that when they just learn the procedure.

NA Yes, there's a huge amount of confusion and misconception around that isn't there? What would you recommend?

HB The easiest way to explain it to adults and students, and I found this is when I've been doing professional learning with a group of teachers, and I've put up on the board three times four. They all know the answer and I'm going, well, how else could you talk about three times four? We could say three multiplied by four, but what does it actually mean?

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Often it's that thing of what does it actually mean, that's the important thing. What does three times four mean? It means three groups of four. I like to get rid of the group altogether and just three of four, so we've got three of four. If you use that language, when you then apply that to two fractions that you're multiplying together, so you have a quarter times a half, if you put the language of in there instead of multiply, you've got a quarter of a half. Now as soon as you reword it like that it becomes pretty obvious that the answer's going to be smaller, because you want a quarter of a half, so you can visualise it. Whereas saying a quarter times a half, it's quite difficult to visualise that.

MG Immediately the student is thinking of the half as a unit of something. So we're going what's a quarter of, for example, half an apple, or what's a quarter of half a kilometre, so it gives them a unit context for working out the fraction.

HB It goes back to what we were talking about before on a number line. We were talking about the half can be in a different place. Sometimes a unit isn't a whole number, the whole is actually a fraction, which is what we're doing now. We're saying we're just dealing with a half and we want a quarter of it.

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NA Putting it in a story sometimes makes it a little bit easier as well. When I deal with teachers and when I do it with students I'll say let's say mum has baked a cake. She's cut the cake in half and she's saved the half for dessert. She's allowed us now to share half of the cake between me and my three siblings, so the four of us. How much of the cake are we going to get each? I'm sharing a quarter of half the cake, so immediately they start putting it in context, and whenever you put anything in context it starts to make a little bit more sense.

MG So here's a doozy then, we then end up in a really interesting situation when we start dividing fractions together. I'll give you another little story, a true story. I was running something I called Fraction Bootcamp with some parents at a school I was teaching with. What had happened was I taught a unit of fractions, I wasn't happy with how Year 5 and 6 had done in that unit test. So I thought you know what I'll do, I'll get the parents in and I'll get the parents to sit with the students and we'll do a fraction bootcamp in the morning.

Now I must admit that what I had been doing is teaching a little bit of process rather than understanding to the students. So I had taught them that great trick of if I have a half divided by a sixth, for example, what you do is you flip the second fraction and you change the divided by to a multiplied by, and, hey presto, you get the answer right. Purely process driven I'm ashamed to say.

NA Stay, change, flip.

MG Stay, change, flip, that's the one. Now, the parents then said, so why does that work, and I came completely unstuck and found myself tripping over my tongue. I had to then say I'm going to get back to you on that, and then spent half an hour working it out on paper and emailing the parents so I could give them a solution. But do you want to explain how we can also think about dividing two fractions together by using that same careful use of language to make the meaning more clear?

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HB Yes, and often what you've just talked about is something that happens I would say in probably 60% of classes where division of fractions is introduced. They get taught the process. One class I went into it was being called KCF, instead of KFC it was KCF, so, Keep Change Flip. But if you again go to the language and you think of let's start with whole numbers, so 12 divided by four. We know the answer is three, but what does that division sign actually mean and how could we reword it to make it make sense. One of the ways to think about it is to either say how many groups of four in 12, or, one way I like it is where you say 12, how many fours. If you consider that language and then apply it, for example, to a half divided by a quarter, you were saying, a half, how many quarters. Now when you think about that, what we're therefore asking is how many quarters are there in a half. Half, how many quarters.

Again, we can look at it as we can do a physical model of that, and again when we put it in that language of, a half, how many quarters, you again can visualise, okay, I have this half, I want to know how many quarters I can fit into it. When the answer comes out as two, and that's what you get when you do your keep change and flip, again kids are confused because they go, okay, when we do division in whole numbers, the number gets smaller.

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But that doesn't work when we do fractions, so what's going on? It comes back to the fact that we don't talk about what the unit we're talking about. So a half divided by a quarter, the answer is two, but it's two quarters, there are two quarters in a half, not two whole numbers, two quarters.

NA That's where the confusion is a lot of the times when they say when I'm dividing two fractions, because usually when I divide two numbers it gets smaller, but when I'm dividing two fractions suddenly I'm getting a whole number value and that's confusing. They don't know where that comes from, but having that unit at the end, that language of there are two quarters is really important. The other question I had for you is when you have one where it comes out as a fraction, so let's say a quarter divided by a half

HB Yes, so, again, if you think again about the language and you put it into the language of how many, so instead of this time, a half, how many quarters, we're going, a quarter, how many halves. If a quarter is only this size, so let's think of it in terms of a metre, so we're thinking of a metre, and a quarter of a metre is going

to be 25 cm. If I want to know what is a half, where does a half sit into that, well, we can see that, a quarter, how many halves, is going to be a half, it's equal to only half a half because I can only fit, and I'm using a physical model here, I can only fit a half into a quarter.

MG It's a great demonstration of the fact that division is not commutative in the same way multiplication is.

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So a quarter times a half is the same as a half times a quarter, but of course thinking about the language that you've just described there, of course, a half divided by a quarter is not the same as a quarter divided by half. We can show that with a physical model and say to students if you're confused, because, division, we can't just reverse things in division you get the same answer, here's why, and showing them a physical model of that.

HB Yes, and that's really important for them to understand that.

MG You're listening to MathsTalk but AMSI Schools and we're having a chat today with Helen Booth and Nadia Abdelal about fractional thinking and unpacking some of the ways that students think about fractions or what we like to call fraction constructs. If you'd like to check out our website, calculate.org.au, there's lots of modules and resources on there and also professional learning articles for teachers to help in the classroom. Also you'll find our podcast notes and some other references regarding fractional thinking as well. Helen, we were talking about firstly fractions as an operator and then fractions as a quotient. Do you want to keep going on that topic, fractions as a quotient, so thinking about fractions as one number divided by another number?

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HB This is a really important concept for kids to understand. We talked about it in the earlier one where we talked about when we were going to turn a fraction into a decimal, and the concept that the vinculum is the division line. If you remember that, that applies, I always used to have so much trouble when I would be doing higher level maths and you'd be doing some stuff and calculus where you would have a fraction divided by a fraction. So it was written as a half over a half, or a half over a quarter, and it was like every time I'd look at it my brain would just go into some sort of seizure mode as I couldn't understand what it was wanting. If you consider that the vinculum means a division line, then simply it is saying a half divided by a quarter. So instead of getting confused by the visual representation of it, which looks quite clunky.

MG Yes, it does look clunky.

HB It does look clunky when you have a fraction over a fraction. If just remember what that line in the middle, the vinculum, actually means then it goes, oh, hold on, that's just a half divided by a quarter, rather than going into a panic over what do I do with all these numbers that are stacked on top of one another.

MC So just to recap, we've talked so far about the following constructs of fractions. We've talked about fractions as part of a whole and we've talked about the importance of stipulating the unit to explain what that whole is, whether it's a single unit of something or a collection. We've talked about fractions as both an operator and also fractions as a quotient, so looking at the importance of the vinculum as a divided by sign. We've talked about fractions as a measure and the importance of showing that lots on number lines. Let's have a chat about fractions as a ratio.

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Fractions can be used to describe sets, and that's what we mean by ratios, but often there's some confusion about whether we're talking about one part of the set to the whole of that set or one set to another set. Nadia, you had a really interesting anecdote about that, about a situation you found yourself in where that confusion wasn't made clear for you.

NA Yes, that's right. This happened when I first started teaching. I remember there was an assessment question. The question was you had a solution of cordial to water, and it was one part cordial to four parts water. It said express this as a fraction, so that was the question, express this as a fraction. So my confusion was what part of water to cordial am I expressing? Am I expressing it one part water to four parts cordial, so a quarter, or am I expressing it as part of the solution, so one part water to five parts of the total solution? It wasn't really made clear in the question itself, so, of course, the kids were confused. I got half the class having the answer of a quarter and the other half having the answer of a fifth. So really it wasn't the answers that were incorrect, it was the question.

MC Helen, there's a little bit of unpacking we can do here. Do you want to explain to listeners why it's so important in a question like that to make it clear what we are talking about when we are talking about ratios?

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HB When we talk about a ratio we can be comparing two different sets. So, for example, we might say the ratio in a class is three girls to five boys. So therefore we're saying that for every three girls there are five boys in the class. That's the part of what Nadia was talking about of the ratio of one to four, so for every one part there were four of whatever else was in the solution. So we compare one set with the other set. The other way of looking at it is if we are comparing it as the whole thing. If we're comparing the girls and the boys, we've got three and five, so that's a total of eight. We can talk of the girls as being three eighths of the whole set and the boys being five eighths of the whole set.

Whereas in a ratio we tend to be comparing one set with another set. Determining whether you are comparing one set to another set, or determining whether you are comparing one set to the whole amount is the important concept around ratios. It's why everybody, a lot of people have a lot of difficulty understanding how ratios work.

MC We end up in probability with a similar problem, don't we, for example if we've got a spinner and there's five spaces on the spinner, two of them are blue and three

of them are yellow, the probability is not two out of three of getting blue, it's two out of five. So understanding the way probability works is a similar concept, isn't it, understanding what ratios really mean.

HB I think that's the other thing. When we look at stuff like probability and we apply it across, it's that link of when we're teaching probability, we are also teaching fractions, decimals, percentages and ratio.

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It's one of the things that I find really frustrating, I guess, with a lot of teachers in that they want to isolate everything. They want to teach things in isolation. We're going to do probability now and later on we're going to teach fractions, decimals and percentages. That's the problem, it's often probability doesn't even get included because we've run you of time and we haven't got time to do probability. Well, hold on, you can make fractions, decimals and percentages far more contextual and far more sensible by teaching it through probability. Then the students can talk about probability in fractions, decimals percentages and in ratios of that probability, what is the probability.

MC The moral of the story there, as teachers we need to make it very clear which we're talking about. I thought we'd wrap up today by looking at a teaching idea to help kids understand fraction concepts and fraction constructs a little better and that's the idea of fraction talks. Nadia, do you want to talk a little but about what a fraction talk is because you're saying you're very keen on number talks as a lesson starter?

NA A fraction talk is done very similarly to a number talk or a dot talk. Basically it's a warm up routine for kids. You get the students to sit there with their fist to their chest, no discussion, no calling out and no hands up when they've got an answer, but the idea is that you can do farcin talks in a couple of ways. You can place an image up on the board, and there's a website that has got some great images, it's called fractiontalks.com.

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We'll put the link in our show notes, but they've got a lot of images on whole parts and also discreet values that are broken up in lots of different ways in lots of colours. The idea is that you're getting the students to look at the fraction and maybe ask them a question like what fraction is shaded blue. Once you get the students discussing, it comes out that students see these fractions in lots of different ways. The discussion is supposed to promote conceptual thinking about fractions and also their spatial thinking about fractions and it's a really great way of doing it.

The one that I spoke about earlier on is another way of doing it where you're getting the students to estimate the value of a fractional operation, because that's also a really important part of students learning, to be able to look at two fractions name be able to say it will approximately be equal to this amount.

HB There's a couple of other websites that we could put up there as well that also talk about fractions and using fractions. There's fractiontalks.com, but there is other ones that talk about the use of fractions and visual form to get kids to get used to looking at fractions in visual forms and then making an estimation of what it is or coming up with different ideas. There's a really great activity called Bryony's Triangle, which is on an Nrich website, which we'll also put up. That's a good one with a little bit of paper folding, but the object of it is they have to determine the size of a triangle that they colour in once they're finished doing all of the paper folding, which makes kids think about how are they going to work it out.

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MG One of the things I love about fraction talks and number talks and dot talks as a teaching routine is it becomes a great little formative assessment task for kids too because you can record it. You can either point an iPad with a video as kids share their responses, or jot down some notes on a whiteboard as to how kids are responding, capturing their thinking in the moment there which is a lovely way of recording kids thinking as we go in the classroom. You've been listening to MathsTalk by AMSI Schools. My name is Marcus Garret and today we've been talking about fractional thinking. Thanks a lot for coming in Helen and Nadia.

HB Our pleasure.

NA Thank you.

MG We will continue to unpack fractions and fractional thinking as we talk further about rates and ratios in another podcast, and then onto decimals and some of the trickier aspects of teaching decimals and percentages later on.

For our show notes and information on this podcast and more, please head to AMSI Schools teacher support webpage at calculate.org.au.

Thanks today to our sound recorder, Nadia Abdelal, who has also been on the show as a speaker. Thanks of course to Helen as another guest. Thank you to Kristin, who has looked after our marketing, and Laura Watson, who has looked after our media. We'll be with you next time.

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