MATHSTALK **by AMSI Schools** (Episode 5):

'Multiplication Matters' (Part 3)

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LM Welcome to Maths Talk by AMSI Schools where conversations in maths become part of your professional learning practice. My name is Leanne McMahon. I'm an AMSI Schools Outreach Officer and today I'd like to introduce my colleagues, Nadia Abdelal and Marcus Garrett.

00:00:20

NA Hey, Leanne.

MG Hi, Leanne. How are you?

LM I'm very well. In the last episode we discussed the learning progression involved in multiplication and heading towards multiplicative thinking. Today's episode, for a start, the structure's going to be a little bit different because I'm not hosting. This is an egalitarian society. The three of us are working together. So, please feel free to jump in, Nadia and Marcus, any time you have something to say.

Now, we got up to arrays and grids and had a little bit of a chat about that but today I'd really like to get more into the arrays and grids and how we move from the grids to the arrays and talking about perhaps some larger numbers and what we can do there. So, Marcus, you had some thoughts about arrays and grids.

00:01:14

MG Yes, well, introducing the array say from Year 2 through to about Year 4 in primary school is a really important pre-step for kids to understand multiples, so multiples of a particular number, whether it's demonstrated, whether you talk about rows or columns. We've shown... In the last episode, I think you and Nadia were talking about the commutative property or the commutative law of multiplication, so we can show, for example, that three rows of five is also five columns of three. So, it's still 15.

But we can also use arrays to show the distributive law of multiplication too by partitioning numbers. So, there's a great little game that we play during some of our research lessons called multiplication toss where we give students 100 grid between a pair of students and a six-sided dice. A lot of teachers will have played this game before. So, the dice is thrown twice by each student and say a student throws a two and a four, they're going to find two rows of four or four rows of two and shade in eight squares on the grid. So, it's a nice little way for them to learn some of their number facts between one and six as well.

NM We'll put a link to that on our show notes.

00:02:39

MG Yes. Then what we do is we stop them after a while and we say, who's starting to run out of room on their 100-grid because, of course, you just have to have a couple of students throw some fives and sixes and we've got very big chunks of the grid taken up. So, how are we going to fit numbers in when we start to run out of room? We say, well, okay, let's imagine that... We've almost run out of room on the grid and then someone throws a three and a five. So, they've got to fit an array of 15 on the grid there. What could we do?

Well, we can actually partition that 15 into chunks. We can say, well, 15 is five times three but what if we said, well, 15 is three lots of three plus two lots of three? So, we show kids that on the grid and show that, okay, three lots of three is nine, two lots of three is six and, guess what, nine and six is 15, it still works.

NM And they can put those in any area on the grid.

MG That's right. They can find those two chunks somewhere else on the grid and basically split the number 15 into two smaller arrays. So, we're actually partitioning numbers and this is actually an introduction to the *distributive* law of multiplication. We can show that we can partition one of the numbers, multiply both those partitioned bits out, add them back together and we have the solution to the original multiplication.

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NA I had an interesting story about that multiplication toss game and this goes back to building that understanding in kids about what multiplication actually is and sometimes, when we focus on just the multiplication facts, we don't always get an idea of what the students understand about multiplication. So, I was in a class... I was doing this game with some students and the class was set out in a way... It was quite a small school, so they had a classroom from Year 3 to Year 5 and when the students started playing this game, it was really interesting because suddenly we were teasing out all of these misconceptions.

The students tossed the dice and when they placed their grid on the actual 100-grid, when they placed their rectangles on there or their arrays on there, they actually just did... So, let's say they tossed a three times five, they coloured in three squares as a column and then they coloured in five squares in a row.

LM So, in an L shape.

NA In an L shape, yes. So, they didn't quite understand or they didn't have the understanding that when you do a multiplication, it's actually the number of rows and the number of columns, so it has to create a rectangle. So, it was really interesting and we had to then step it back to the group solve that we were discussing last episode in the progression towards building multiplication, that you have to start with the groups of and then move into the arrays, so you show all of the different parts of multiplication.

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LM They obviously didn't have much experience at all with arrays.

NA Not at all, no, not at that point.

MG So, arrays are a really nice way then of introducing the concept of factors and multiples too. Because if you can take a large number of, say, counters and form them into a rectangular shape, by definition that number is composite because you... For example, take 15, you've got three rows of five or five rows of three. You can show that the number of rows or the number of columns are the factors of 15 and the fact that I can actually make a rectangular shape with it means that it's a composite number.

If it's prime, that is the only factors that the number have one and itself, then you can't make a rectangular with it. So, try and make a rectangle with seven. You can't, there's bits left over. You can make a straight row but you can't actually form a rectangle. So, it's a nice little way of showing kids what a composite number looks like in terms of arrays too.

LM And we can go back to our last podcast when we talked about the cupcake activity. That's a wonderful way of showing prime numbers, that they can only make that one row, they can't make a group.

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MG So, we then... We're taking it from arrays then into using grids or using grid paper and of course we can quickly show... It doesn't matter if they're circular counters or if they're squares on a piece of grid paper. Three times five or three rows of five or three columns of five is still 15. I can count the number of squares and we can play with our number facts until we're blue in the face and show that we can use grids instead of arrays. So, that would be the next step.

LM Is that...? Remember when we were talking in the first podcast about the CRA model? Are we then moving from concrete to representation?

MG Yes, that's exactly what we're doing. And you'll actually follow the progression that we're about to talk about. We can see that that's happening. Not only are we moving the conceptual progression for multiplication, that is moving from forming equal groups using arrays into representing that but we're also using the CRA model, so giving kids counters to start with and then moving them onto a piece of paper and showing them grids on a piece of paper.

LM Right. Well, I want to know now what about the larger numbers? We've looked at three times five and our old-fashioned listeners will be thinking, well, shouldn't they just remember it?

MG Well, as we've said before, remembering number facts is useful. It does reduce kids' mental load when they're doing problem-solving but showing kids what's going on when we multiply, particularly when we start to multiply large numbers, is really, really important because it's actually giving them skills for operating with numbers as far out as Year 9 and 10 in high school and even beyond.

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So, if we do start with larger numbers, and so at this point it's probably a good idea if you're listening, to pause the podcast. Grab yourself a piece of paper. Even better if you can grab a piece of grid paper because we're going to start to play with some larger numbers and look at what happens when we partition larger numbers and multiply them out.

LM A bit of a warning, if you're driving, don't do this.

MG Please don't do this if you're driving. So, let's think about the multiplication problem 17 times eight. So, if you were to show that on a grid, the one that I've got in front of me here in the studio, we've actually got eight rows of 17, so there's a grid area shaded with that number of squares. We show that to students and say, okay, this is 17 times eight. There's a lot of squares there. Who would like to have to count those one by one? Not me. In fact, if we do, we'll probably lose our place. So, what could we do to solve 17 times eight and make it a bit easier on ourselves?

And the answer is, just like we partitioned three times five equals 15 into two times three plus three times three equals 15, we're going to partition the number 17 into two chunks. So, we could do that by partitioning 17 into ten and seven, so using our good old place value units.

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LM And that goes back to what we were talking about, again, the last time about the relationship between place value.

MG Yes. But we don't have to partition it that way. Nadia mentioned earlier some kids are really good with their 12 times number facts, the 12 times table. So, 17 could also be 12 and five. This same little trick is still going to work, doesn't matter which way we partition 17. But let's just go with the ten and a seven for the sake of place value.

NA In fact, it's really important as teachers, as educators, to show students lots of different ways and allow them to choose the best possible approach because that's what we're trying to do, we're trying to build number sets in our students and giving them the option, saying, look, this is the way that I'm going to do it, show me lots of other ways that you can do it, and that could be an activity in itself.

LM I think one of the things that I find quite amazing then, at parent/teacher interviews, is when the parents come and say, you show them so many different ways, why can't you just show them the one way, like we did?

00:11:17

NA Let's get into that later.

MG Yes. Well, it's a... Look, it's a... I often say to parents that choose maths family nights, contrary to popular opinion teachers don't just hand kids a calculator and expect them to just use the calculator. We are actually teaching them lots of mental methods to work with number.

NA Because we're not trying to teach them the process. What we're trying to do is we're trying to build a deep understanding of number. Because what we want to do is we want them to be able to take their learning, to take whatever it is that they've learnt and to be able to apply it to a larger problem. Because when they're out there in whatever field they're doing, they are going to be solving problems and they are going to be solving problems and they are going to have to be applying some of the skills that they've learnt.

It's not about just putting it into a formula, putting it into a process and then... Because we've got calculators for that and we've got Google for that and we've got all sorts of computer programmes that will help us do that. But how are we going to apply some of the learning to different situations?

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MG Yes. It goes to the heart of what we mean by multiplicative thinking instead of multiplicative process. So, as a listener draw yourself a rectangle, a horizontal rectangle, with the side dimension of eight and the length dimension of 17 and break the 17 into ten and seven. So, if you've got grid paper, even better. And you'll see that now what you've got is you've got a rectangle made up of two smaller rectangles. One of those rectangles is eight lots of ten and the other rectangle is eight lots of seven. So, you've got two chunks of that rectangle.

And of course we notice that when we multiply those out, so ten lots of eight is 809 and seven lots of eight is 56. And we add those components together, we get the answer of 136. That is to say that 17 lots of eight is 136. In this case, I've done that by doing ten lots of eight plus seven lots of eight but students might do 12 lots of eight plus five lots of eight. They're still going to get 36.

LM Or something like eight lots of eight, which is a lovely square number, and nine lots of eight is a nice easy calculation to do.

MG Yes. So, all of a sudden you can say to kids, look at this, we've got the answer. We've made it easier because I know my ten times number facts, I know my seven times and eight times number facts. All we've got to do now is add those bits together, 136.

LM What about two-digit by two-digit. Will it work for that or is it just too big?

MG Not at all, but... And this is really the crux of it. We've got to remember to show students that when we partition two-digit numbers and we're multiplying two-digit by two-digit or two-digit by three or three by three, we have to multiply each component by each component and then add them together. That becomes incredibly important, and you'll see why.

00:14:25

NA Before you start, Marcus, I wanted to relay a little bit of a story. I'm all about stories. There's a million stories.

LM We all love a story, Nadia.

NA So, I was driving in the car one day with one of my daughters and gave her this question – what's 23 by 22. So, as you can imagine, we have lots of fun car rides.

LM Don't our kids just love it?

NA They hate my guts. My idea of fun is not really their idea of fun but, anyway. So, my daughter was younger at this point and she was trying to do the calculation in her head. I wanted just to see how she would tackle this. So, it was 23 times 22. So, she had learnt the split strategy by this stage.

So, she knew that if she was to multiply any two-digit number by another number, that she would have to split them into two different parts but where she was successful in multiplying a two-digit number by a one-digit number, when she tried to apply the same strategy of splitting the number and multiplying the tens and multiplying the rest of the digits, she did what is very common for a lot of students to do, and that is she multiplied the tens together and then she multiplied the ones together but failed to see the other distributive parts of the multiplication, which is what you're going to be talking about with this question, Marcus.

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MG That's right. And, again, that's why if you either have access to the podcast notes and you can see what we're doing in front of you, or you've got a piece of paper in front of you, that's going to be quite important. So, let's look at 14 times 23. So, this time draw yourself a rectangle. That's 23 across and 14 down. Now, if you can imagine that was a... if that was a piece of grid paper, there's a lot of squares there. Again, you don't want to have to count 14 lots of 23 one by one. How could we partition these numbers?

So, again, I'm going to use my place value units. I'm going to partition 23 into 20 and three and I'm going to partition 14 into ten and four. So, just draw some lines across your rectangle and you'll find there are four chunks, because you've got 23, you've got 20 and then another three columns on the end, and with your 14 you've got ten rows and another four rows down the bottom. You can imagine those two lines across your rectangle. So, now you've got four chunks.

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Let's multiply all of those parts out. I haven't changed the number of squares, I haven't changed the number of rectangles on the grid. Now what we have to do is we have to multiply ten by 20, then we multiply ten by three, so that's our 20 plus three times the ten part of the 14. Then we're going to multiply four by 20 and then we're going to multiply four by three. So, there are four chunks there, not as your daughter, Nadia, multiplied only two. So, what your daughter did was she did the equivalent of ten times 20 and four times three.

NA That's right, yes.

MG But forgot the other two bits and this is really common, not just for kids but for adults as well.

NA Absolutely. And when she did it, she realised that she had done something wrong but she wasn't sure what. Because the value that came out wasn't as high as what she predicted it would be, and this is actually... I was proud of her for doing that because that's a really important skill, that ability to reason the answer and relate it back to the question. Does it make sense? She knew it didn't but she wasn't sure why.

MG Yes. So, she's estimated and realised that her solution doesn't come close to the estimate, so Houston we have a problem. So, now we've got... If you add those four chunks together we've got ten times 20, that's 200, ten times three, 30, four times 20 is 80, four times three is 12. We're going to add those chunks together and we find that we get 322.

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LM It's interesting that you say, Marcus, just add them together because it's quite obvious, when you look at that grid, that those four numbers need to be added together. One of the things that isn't obvious in the algorithm is why you add these things together.

MG If you think about the way we teach the standard algorithm, kids are only adding two numbers together.

LM That's right.

MG Yes, not four.

NA Now, in the last episode we spoke about algorithms in some depth but not a lot. We referred to an article, and I actually read this article a while ago, and it made a lot of sense why teaching the algorithm has a detrimental effect on some of the students learning with multiplicative thinking.

So, the article's actually called Written Algorithms in the Primary Years, Undoing the Good Work and it's by Doug Clarke. It's a fantastic article and it goes into why. Now, there's nothing wrong with algorithms. I think that algorithms is a perfectly good strategy for teaching multiplication or even addition/subtraction. However, I don't think that the algorithm should be touched on at all until the kids have a really good grasp of the area model and arrays and have had a lot of practice in actually using those things.

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Because there is a connection that is made between the area model or the arrays and algorithms and, Marcus, you're going to talk a little bit about that. So, let's make our numbers even bigger now to do that. So, let's imagine 34 times 46. So, now we're really getting up there. A very, very large grid. Do we really need the grid part? Not really. If we know our number facts, then we can partition the number and get rid of all the grids. We don't need a piece of grid paper. We can just draw ourselves a box or a rectangle and we can divide the box into two by two because we've got two-digit by two-digit and partition that number into tens and units, or however we like.

NA Because the kids, by the time they get to two-digit by two-digit, should have a solid understand of their number facts.

MG They should, yes.

NA If they've been exposed to this sort of learning.

MG That's right. And that's what I mean by if you know your number facts, it's going to reduce your mental load. You don't have to check up on the times table chart.

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NA Absolutely. And because they're so visual, I find that... Given that our brains are comprised predominantly, I think I read somewhere... Now I may be wrong but it's something around 30% of our prefrontal cortex or our frontal lobe is predominantly visual neurons. So, you can see why conceptual teaching and visual teaching is so important. Because once they see this, they can then take this and start visualising it rather than having to put it on a piece of paper.

LM And then the algorithm makes sense.

NA Absolutely, yes.

MG Because you're seeing it. You're actually seeing, as you said, those four chunks. There's the four chunks. There's the bits that I need to add together. So, now I've got 34 by 46. Again, if you've got your piece of paper, draw yourself a rectangle, partition 46 into 40 and six and 34 into 30 and 4. Now let's multiply the parts together: 30 times 40 = 1200, 40 times 4 = 160, 6 times 30 = 180 and 4 times 6 = 24.

So, we've got all of those chunks. We're going to then add them all together, as we did for the previous example, and we've just performed quite a large two-digit by two-digit multiplication. The answer's 1,564. We've performed that using the area model rather than the standard algorithm and kids can see all of those four component parts because I partitioned each number two ways and so two times two is four, so we've got four chunks that we're going to add together.

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NA I think this is the point where you would start introducing the kids to the vertical algorithm with multiplication, because it leads quite beautifully into what happens with that.

MG Yes. So, I had a really interesting situation when we were writing this up into a professional learning course for teachers. I had my manager, Michael's, notes and he had presented the vertical algorithm in a flipped form. So, what he'd done is he'd presented 47 times 35.

So, if you set that out as if you were going to do the standard vertical algorithm and instead of multiplying the units in the first row and the tens in the second row, what he'd done is he multiplied the tens in the first row and the units in the second row and I said to him that's going to confuse kids and he said, well, it is if they've learnt the standard algorithm straight off but if you actually draw the area model next to that flipped version of the algorithm, and, again, you can check out the show notes and see how this relates. You can see straightaway that the top row in the area model matches the top row of the flipped algorithm.

So, 30 times 40 plus 30 times seven, which is 1200 plus 210 equals 1,410 and that's your 47 times 30. And then in the second row your 40 times five plus your seven times five or 200 plus 35 is your 235 which is your 47 times five. You can see a direct relationship there.

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Again, you probably have to have it in front of you to see what I mean here but the point is that all of a sudden we can see what is going on in that vertical algorithm and why we are adding bits inside the algorithm. All of a sudden we realise there's not two numbers, there's four chunks. All we've done in the vertical algorithm is summarise that and then added those summary bits together to get our final result.

NA In fact, I don't even go to that step. I myself find that a little bit confusing.

MG Yes.

NA So, I choose... Even when I'm doing my own calculations... Now, I learnt the vertical algorithm, so I know all about the carrying. So, if I'm in a hurry, I will carry but generally, if I'm multiplying two-digit or three-digit numbers, I will actually show all of the steps, so every single step in each of those rectangles I'll actually show. Because for me, I have a better understanding of what's going on.

It's just easier for me to see that and I think it's important to allow kids to kind of use whatever step works best for them and I think that's the whole idea of this developing number sets, of number fluency, is that the kids know, well, I'm allowed to use this method and this other person over there can use that one because that's what works for them but this one actually works for me.

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MG Yes, and, look, I think that's a great idea, the point being they want to understand conceptually before they understand a process so they can see what's going on.

LM You've been listening to **MATHS**TALK by AMSI Schools. My name's Leanne McMahon and today we've been talking about multiplicative thinking, the third and final in the series called Multiplication Matters. Well, we've been talking about a lot of squares. Let's talk about squares.

MG Square numbers you mean, I'm assuming.

LM I do mean square numbers, which are quite possibly my favourite numbers in the world.

NA You're such a nerd.

LM You know, when I turned, let's just call it a perfect square number, I spent the whole year telling people that I was a perfect square.

MG I don't think they would have believed you.

NA This is 64?

LM You are dead to me, Nadia.

00:26:25

MG Well, yes, obviously we call them square numbers because when we present them in an array, look at that, they form a square, so five lots of five or six lots of six. Square numbers are a really good way to show what we've just shown with the area model because we can start squaring larger numbers and show those component parts. And that's really important because we were talking just before the podcast about the look of surprise that we often get from people in professional learning when we remind them then, for example, that if we took 'a' plus 'b', all squared: $(a + b)^2$

So, if you can imagine A plus B, in brackets squared, so we're squaring that term (ie, the 'a+b'), the answer is *not* '$a^2 + b^2$'! The answer is: $a^2 + 2ab + b^2$! So, what we're about to show you is the *reason* that is. And if you think back to the area model that we just looked at, it becomes pretty obvious why, because we don't want to forget those two bits that Nadia's daughter left out. We don't want to forget the '2ab' (ie, 2 x a x b).

So, let's start with 28^2, the number 28 squared. We're going to partition it into 20 and 8; and 20 and 8. So, our 'chunks' on the area model, if you've got your bit of paper in front of you, are 20 times 20; 20 times 8; 20 times 8 again; and then 8 times 8.

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We're going to add *all* of those bits together - and we end up with 400 plus 160 plus 160 plus 64. So, we get 784 as the answer to 28^2 !

NA This is why arrays are so powerful. Because if students have been exposed to arrays and have spent a number of years working on arrays before they get to algebra, you can then start introducing the students to algebra in the form of an array.

So, when they have got a situation like x plus two multiplied by x plus one, they can see why it's $x^2 + 3x + 2$, *why* it's that 'a' spread rather than just going, well, you just use 'F.O.I.L.' and you just go First, Outer, Inner, Last, which a lot of students (just) use the FOIL method.

But showing them on an *array* actually allows them to see what's actually happening. So, then, when they're using the difference of two squares rule, the difference of two squares rule starts to make sense as well.

LM That's right. 'FOIL' is a tool.

NA Absolutely.

LM In the same way as algorithms are a tool.

NA Yes.

00:29:07

LM FOIL is a really good tool if you understand what the area model is. You can even go back. We said 28 squared. You could actually go all the way back to something like five squared (5^2). Five is the same as three plus two. So, five squared is not the same as three squared plus two squared. So, five squared is 25, three squared is nine, two squared is four. So, five squared is *not* the same as 14. You actually have to do that grid model!

NA That's right.

LM You go all the way back.

NA All the way back. And then the students start to... So, there's the method of completing the square, like why do we call it completing the square? Because that's what we're actually doing with that algebraic calculation. Students struggle with 'completing the square'. They don't understand what's actually happening. They understand the process but *why* is a big question... And whenever you are not understanding the process, then there's the great chance that you're going to not remember how to do it once the situation comes back and once it arrives.

LM That's right.

MG So, you can see there why teaching kids the area model in Year 4, 5 and 6 actually can have a big impact on their confidence in mathematics and their understanding of how algebra works in Years 7, 8, 9 and 10 at high school.

00:30:32

NA Absolutely.

MG So, take the number 28^2. What if instead of 20 and 8 we used the terms 'a' and 'b'? So, 28 is 20 plus 8. What if we said let's call the 20 'a' and the 8 'b'. So, now we've got our a plus b, all squared [ie, $(a+b)^2$]. Same thing! If you draw your grid,

In the top left-hand corner you've got a squared…

In the top right-hand corner you've got a times b.

In the bottom left-hand corner you've got b times a.

and in the bottom right-hand corner you've got b squared.

Add all the bits together and you've got a squared plus ab plus ba plus b squared (ie, '$a^2 + ab + ba + b^2$'). In other words:

a squared plus two lots of ab plus b squared (ie, '$a^2 + 2ab + b^2$')

There's your a plus b, all squared!

And, of course, if you say that to people, they look at you as if you've come from another planet. But as soon as you show it on the area model, of course it makes sense! You're completing the square!

00:31:34

LM Well, I think that's probably a good time to leave it, given that we're probably well over time and we've got lots of other things that we can discuss in future podcasts. We'd love to hear your thoughts. Please check the show notes and give us some feedback. You've been privy to one of our office conversations, basically. This is what we do. It's a great job.

I'd like to thank Marcus Garrett and Nadia Abdelal for coming in today and having one of our office chats with us. It's been an absolute pleasure having a chat and getting their unbelievable knowledge and expertise on multiplicative thinking and these areas that are so important.

MG Thanks for having us, Leanne.

NA Thanks, Leanne.

LM For our show notes and information on this podcast and more, head to our AMSI Schools teachers support web page at calculate.org.au. We'll see you next time. Thanks today to our sound recorder and producer, Nadia Abdelal; editor, Leanne McMahon; Marketing, Kristin Mariner; Media, Laura Watson.

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