AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE
SCHOOLS.AMSI.ORG.AU

CHOOSE MATHS

Name: ______________________________

'Café Fractions'

A Rich Task in Operating with Fractions (Years 5-7)

This resource was developed in collaboration with staff from Greta Public School, NSW

BACKGROUND

You are the owner of a cake wholesaler which means you bake and supply cakes to local businesses to sell.

Every week your customers (the owners of the businesses) will send through their orders for the week.

You supply cake by the slice, and each of the cakes you make are cut into different number of slices.

The table below sets out each of the cakes you make, the number of slices you cut it into and the number of slices ordered by each local café in a week:

________________'s Wholesale Cakes – Cake Orders per week

Type of Cake	Number of Slices per Cake	Local Cafes			Total Slices	Total Cakes Needed
		Café 367	Meg's Place	Pecan Café		
Chocolate Mud	**8**	12	10	6	**28**	$= \frac{28}{8} = 3\frac{4}{8} = \mathbf{3\frac{1}{2}}$
Orange Poppyseed (GF)	**6**	10	8	4	**22**	$= \frac{22}{6} = 3\frac{4}{6} = \mathbf{3\frac{2}{3}}$
Key Lime Pie	**6**	8	8	4	**20**	$= \frac{20}{6} = 3\frac{2}{6} = \mathbf{3\frac{1}{3}}$
Salted Caramel	**4**	6	4	6	**16**	$= \frac{16}{4} = \mathbf{4}$
Strawberry Cheese	**5**	10	6	6	**22**	$= \frac{22}{5} = \mathbf{4\frac{2}{5}}$

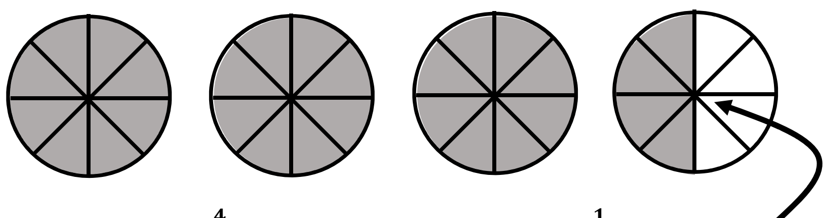
Part 1

Complete the Table above by calculating:

a) the **total slices needed for each cake** in the week, and
b) the **total amount of each cake you'll need** to make at the start of the week to meet your customer's orders.

The first cake (Chocolate Mud Cake) has been done for you. Here is the working out, explained:

- *The Chocolate Mud Cake gets cut into 8 slices;*
- *The number of slices needed in the week are 12 + 10 + 6 =* ***28*** *slices;*
- *As there are 8 slices in each cake, the total amount of cake needed is found by 28 ÷ 8 (or $\frac{28}{8}$ expressed as an 'improper fraction');*
- *This is 3 whole cakes plus $\frac{4}{8}$ (or '4 one-eighth slices') of a cake:*



- *You will notice that $\frac{4}{8}$ of a cake* ***is equivalent to*** *$\frac{1}{2}$ of a cake...*

  So, ***a total of $3\frac{1}{2}$ cakes will be needed for the week.***

c) As you only bake ***whole*** cakes (that is, you would bake 4 cakes, not '$3\frac{1}{2}$' cakes…), **state below how many of each cake you will bake of each type, then what fraction will be left over** to give away to your local food charity.

(For example, you would bake 4 whole Chocolate Mud Cakes, and then there would be $\frac{4}{8}$ (or $\frac{1}{2}$) of a cake left over to give away to the food charity.)

Total whole Chocolate Mud cakes to bake:	4	*Leftover fraction:*	$\frac{1}{2}$
Total whole Orange Poppy Seed cakes to bake:	4	*Leftover fraction:*	$\frac{1}{3}$
Total whole Key Lime Pies to bake:	4	*Leftover fraction:*	$\frac{2}{3}$
Total whole Salted Caramel cakes to bake:	4	*Leftover fraction:*	0
Total whole Strawberry Cheese cakes to bake:	5	*Leftover fraction:*	$\frac{3}{5}$

Part 2

In one particular week, the final cake slice orders were recorded, however, the number of slices for each cake, and the slices ordered for each café, were left blank. The Table below records the cake orders for this week.

a) Calculate how many slices were needed of each cake by looking at the 'Total Cakes Needed' column and the 'Number of Slices per Cake' column, and then **fill in the 'Total Slices' Column.** *Can you explain and show how you worked these out?*

b) Decide the number of slices needed by each café of each cake and then **fill in the columns for each café in the 'Local Café Orders' column** *(there will be many solutions to this so you decide… but be careful of your addition here!)*

c) Looking at the total cakes needed column, assuming you will make whole cakes, **complete the 'Fraction leftovers from Whole Cakes' column**.

___________________'s Wholesale Cakes – Cake Orders per week

Type of Cake	Number of Slices per Cake	(b) Local Café Orders			(a) Total Slices	Total Cakes Needed	(c) Fraction Leftovers from Whole Cakes
		Café 367	Meg's Place	Pecan Café			
Chocolate Mud	8	*Answers will vary but must add to 35*			35	$4\frac{3}{8}$	$\frac{3}{8}$
Orange Poppyseed (GF)	6	*Answers will vary but must add to 41*			41	$6\frac{5}{6}$	$\frac{1}{6}$
Key Lime Pie	6	*Answers will vary but must add to 34*			34	$5\frac{2}{3}$	$\frac{1}{3}$
Salted Caramel	4	*Answers will vary but must add to 30*			30	$7\frac{1}{2}$	$\frac{1}{2}$
Strawberry Cheese	5	*Answers will vary but must add to 18*			18	$5\frac{3}{5}$	$\frac{2}{5}$

Use this space to explain your working out for one of the 'Total Slices' answers you gave in the above:

Working out should demonstrate (i) conversion of the Total Cakes mixed numeral into an improper fraction;

(ii) conversion of this into an equivalent fraction with the same denominator as the 'Slices per Cake' (eg.

'$\frac{15}{2}$' = '$\frac{30}{4}$'; then (iii) recognition that the numerator of this final fraction will be the 'Total Slices' needed.

Part 3

You have changed your cake menu!

This time, you decide that in order to reduce wastage and 'leftovers', you will slice each of your new creations into exactly the right number of slices to ensure there is no wastage, based on your café customer's orders.

The first one – the Triple Choc Deluxe – has been done for you.

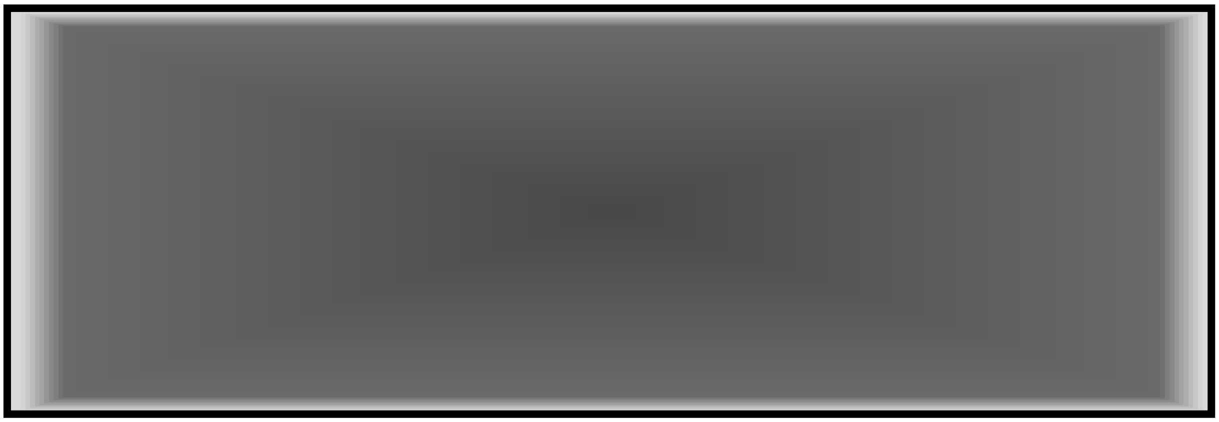
a) First, **complete the 'Total Slices' column** by adding together the local café orders for each cake.

b) Then, work backward to decide upon how many cakes you could bake altogether (**'Total Cakes' column**) and the number of slices you could cut each cake into (**'Number of Slices per Cake' column**), ensuring that there are ***no leftover slices at all.***
(This means that all your numbers in this table must be whole numbers – no fractions!).

Type of Cake	Number of Slices per Cake Eg.	Local Café Orders			Total Slices	Total Cakes Eg.
		Café 367	Meg's Place	Pecan Café		
Triple Choc Deluxe	**5**	6	8	6	**20**	**4**
Banana Nut Slice (GF)	**11**	12	4	6	**22**	**2**
New York Cheese	**6**	7	7	4	**18**	**3**
Lemon Meringue Pie	**4**	6	8	10	**24**	**6**
Battenberg	**7**	7	9	5	**21**	**3**

Part 4

You have introduced chocolate fudge brownies to your menu. Your brownie tray is rectangular shaped, like the following:



Your café customers have asked for the following slice fractions of this rectangular brownie tray:

- Café 367 - $\frac{1}{3}$
- Pecan Café - $\frac{1}{4}$
- Meg's Place - $\frac{1}{6}$
- Sweet Dreams Cafeteria - $\frac{1}{12}$
- Café on Bridge – $\frac{1}{8}$
- Butterworths - $\frac{1}{24}$

(a) Cut out the Brownie slices on the next page and **show that you can fit one of each of these fraction slices into one single brownie tray**, with no leftovers.

(b) Demonstrate numerically (ie, using numbers) that these fractions do in fact add up to one whole (1).

__Hint:__ To do this, you will need to use __equivalent fractions__. You will need to show your working carefully.

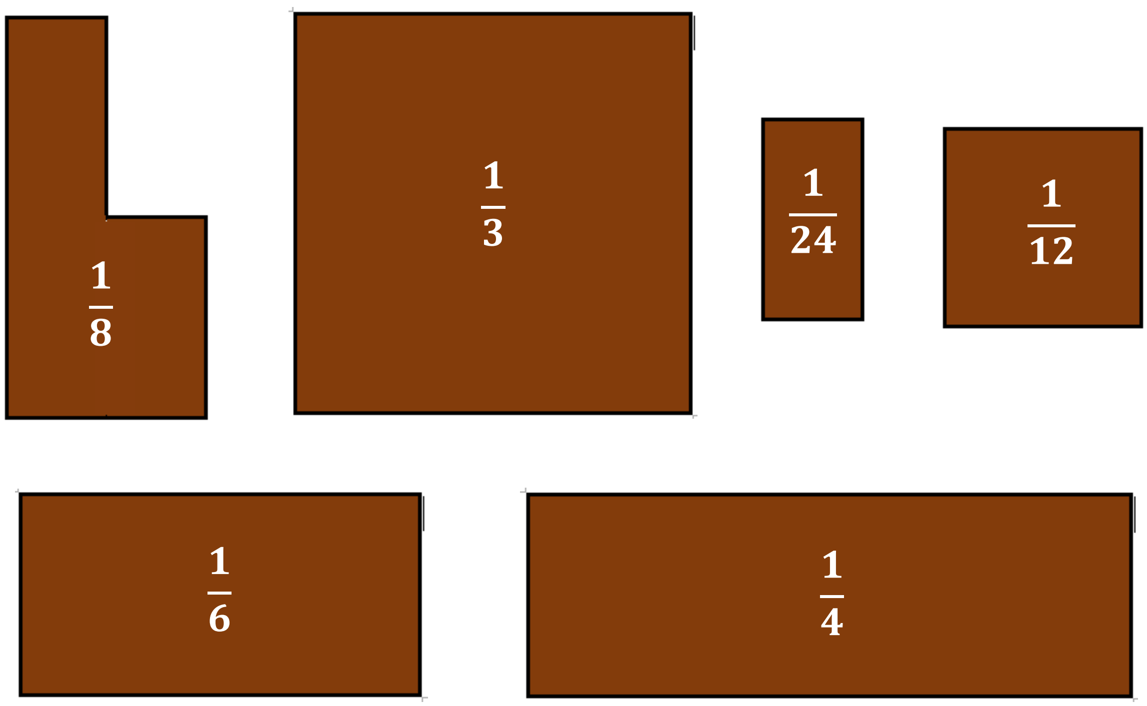
(c) Decide on some dimensions (ie, width and length, in cm) for the brownie tray featured above, and work out the area in cm^2.
Given your chosen brownie tray size, **what is the *smallest* fraction size** (eg. sixths, eighths, tenths…) **you could cut your brownie cake into if you wanted to make sure your brownies were no smaller than 15 cm^2?** *Again, show your working carefully.*

(d) **(Extension!)** Can you ***prove*** that **each of the slices below accurately shows the fraction written on it?** For example, can you show that the slice marked with '$\frac{1}{3}$' is actually one-third of the entire brownie tin (on page 5)?

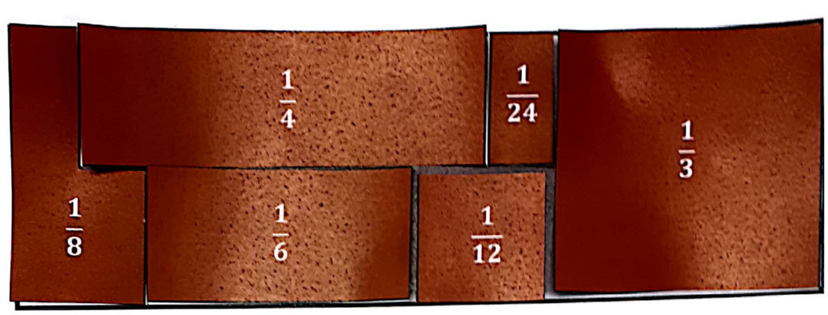
Hint: *Use a ruler and your knowledge of equivalent fractions to show and prove this for each of the fraction slices below (*$\frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}$ *and* $\frac{1}{24}$*).*

Brownie Slices

Cut out the following slices of brownie to use in Part 4 (a) of this task.



Possible Solution to Part 4 (a):



Your solution and working for Part 4 (b):

Solutions and working out should show something similar to the following, although there may be a range of ways in which students demonstrate the solution:

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{12}+\frac{1}{24}$$

$$=\frac{8}{24}+\frac{6}{24}+\frac{4}{24}+\frac{3}{24}+\frac{2}{24}+\frac{1}{24}$$

$$=\frac{24}{24}$$

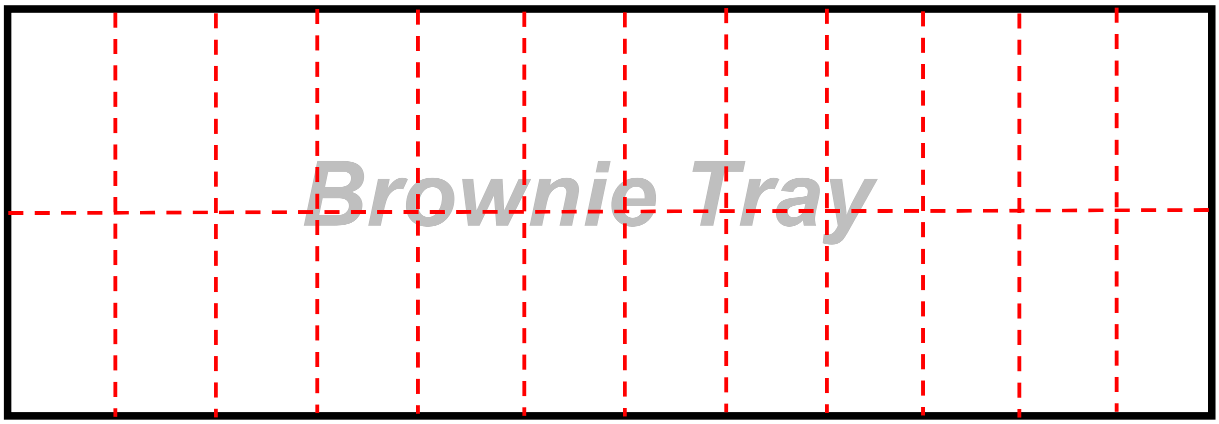
= 1 whole tray.

Your solution and working for Part 4 (c):

Solutions and working should:

- **Correctly calculate cm^2 area from chosen width and length dimensions;**
- **Divide the total area by 15 (cm^2) to find the number of slices (and therefore fractional proportions) into which the brownie should be sliced;**
- ~~**If the answer is fractional (ie, is a non-whole, decimal number), round down to the nearest whole integer; and**~~
- **Express the answer in context as a fraction - eg. *"The brownie cake will be 380 cm^2 and so it should be sliced into at least 25ths (25 slices) to ensure each slice is at least 15cm^2."***

Your solution and working for Part 4 (d) (Extension):

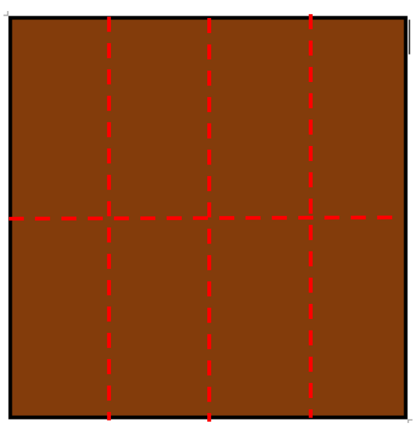


Note that a student capable of clearly demonstrating this proof will most likely be working at a Comprehensive level. Working and reasoning will show that the entire shape can be divided into 24 equal parts of the same size as the fractionpiece marked '$\frac{1}{24}$' . Students should then show, for each other fraction, that the number of one-twenty-fourths that fit into that shape will make up an equivalent fraction that is the same as the fraction on the shape.

For example:

In the '$\frac{1}{3}$' shape, we fit 8 x $\frac{1}{24}$ *(see below).*

8 x $\frac{1}{24}$ = $\frac{8}{24}$ = $\frac{1}{3}$ *(because 24 ÷ 8 = 3).*



Similarly, the '$\frac{1}{8}$' piece fits 3 x $\frac{1}{24}$ pieces:

3 x $\frac{1}{24}$ = $\frac{3}{24}$ = $\frac{1}{8}$ (because 24 ÷ 3 = 8)… etc.

Café Fractions 'Mathematics in Context' Investigation Project:

Grading Rubric, Years 5 & 6 *(Australian Curriculum, Mathematics)*

Achievement Grade	Achievement Performance Description
A Comprehensively working at above Grade 3-4 level	• Uses multiplicative strategies to operate with quarters, halves and thirds, including with mixed numerals, to determine an overall quantity • Uses visual and numerical strategies, and converts fractions to find common denominators, to operate with any fractions within and beyond fractions of a whole • Investigates equivalent fractions used in a context, simplifies proper and improper fractions and converts efficiently between improper fractions and mixed numerals • Uses accurate mathematical terminology and symbols to describe and represent maths operations using fractions of a whole in more than one way • Selects and uses several efficient mental, numerical and visual strategies to solve problems and demonstrate proofs involving more complex fractions and area of a rectangle • Applies fractional thinking to an area problem, proves the accuracy of solutions and comprehensively explains the reasoning used
B Thoroughly working at Grade 3-4 level	• Adds and subtracts quarters, halves and thirds, including with mixed numerals, to determine an overall quantity • Uses both visual and numerical strategies to accurately add and subtract fractions (including 1/2, 1/4, 1/3, 1/5 and their multiples) within and beyond fractions of a whole • Investigates equivalent fractions used in a context, simplifies proper and improper fractions and converts efficiently between improper fractions and mixed numerals • Uses clear terminology to describe, and symbols to represent, maths operations using fractions of a whole • Selects and uses efficient numerical and visual strategies to solve problems involving simple fractions and area of a rectangle • Checks the accuracy of a statement and explains the reasoning used
C Satisfactorily working at Grade 3-4 level	• Satisfactorily counts by quarters, halves and thirds, including with mixed numerals, to determine an overall quantity • Uses either visual or numerical strategies to satisfactorily add and subtract fractions (including 1/2, 1/4, 1/3, 1/5 and their multiples) within fractions of a whole • Investigates equivalent fractions used in a context and uses these to simplify proper and improper fractions • Uses some correct terminology to describe, and symbols to represent, maths operations using fractions of a whole • Selects and uses sound numerical or visual strategies to solve problems involving simple fractions
D Working at basic Grade 3-4 level, sometimes with support	• Counts by quarters, halves and thirds using visual prompts to determine an overall quantity within a whole • Uses either visual or numerical strategies to add and subtract fractions (including 1/2, 1/4, 1/3, 1/5) within fractions of a whole with variable accuracy • Considers equivalent fractions used in a context and uses these to simplify proper and improper fractions with some support • Uses limited terminology and some symbols to represent maths operations using fractions of a whole • Uses either numerical or visual strategies to solve problems involving simple fractions with support
E Experiencing difficulty / working with support at or below Grade 3-4 level	• With assistance, counts by quarters, halves and thirds using visual prompts • Uses visual strategies to attempt to add and subtract fractions (including 1/2, 1/4, 1/3, 1/5) within fractions of a whole, with limited accuracy • Considers equivalent fractions used in a context, however, requires assistance to use these to simplify or operate with fractions • Requires support to use correct terminology and symbols to represent maths operations using fractions of a whole • Requires assistance to apply either numerical or visual strategies to solve problems involving simple fractions.