AMSI SCHOOLS PROBLEM SOLVING STRATEGIES

General Tips: When introducing a problem-solving strategy to students choose an activity or problem that helps to highlight that strategy. Also, think about choosing a problem where the maths is at a level that is easily accessible to students. If students are confused by the content of the problem, they may not see the relevance of using that particular strategy. Once students become more familiar with the different strategies the level of difficulty of the problems selected can increase.

This online series by Blake Problem Solving suggests a number of activities to help students to become more familiar with the various problem-solving strategies:
http://ey.westside66.org/wiki/projects/mathblakeproblemsolving/MathBLAKE_PROBLEM_SOLVING.html

STRATEGY	EXPLANATION	POSSIBLE ACTIVITIES
Guess and Check	• A natural strategy that children will use from a young age, e.g. what does the block taste like? Can I poke the cat? • Used commonly every day, e.g. is this food too spicy? Can I lift this box? I think my bill will be about $50 • Teachers may need to encourage the guess part (or estimation) as students will often develop a fear of being wrong and become reluctant to guess • An estimate or predication can be thought of as an informed guess, i.e. I use my prior knowledge to help me estimate information, e.g. I think the person driving the car is about 20 not 5	**Activity 1 – How many?** • Tip a pile of counters on the table – ask students to estimate (guess the amount) • Check the amount by counting • Arrange the counters in such a way so it would make it easy for another person to check the amount • Share strategies used, e.g. students out the counters into lines, pairs, groups of 5, bundles of 10, etc. • Talk about which strategies make it easier to check and why **Activity 2 – Folding fractions** • Give students different length size pieces of paper, e.g. strips, squares, circles, rectangles, etc. • Ask students to fold their paper to show thirds (guess) • Use a strategy to CHECK if they are correct • Students may choose to cut and compare their pieces, i.e. equal pieces will have equal areas • Students could use a ruler to measure the size of their paper and calculate the exact distance, e.g. if the strip is 33 cm in length each third would be 11 cm
Make a model	• This strategy encourages students to use hands-on materials to help explain their thinking • This strategy is often a precursor to drawing a diagram or making a table in order to more formally record their thinking • It is important for students to have opportunities to "play" with materials, e.g. touch, turn, manipulate, etc. this process will help students form a visual representation of that material in their head	**Activity 1 – Show me** • Provide students with some materials, e.g. counters or just their fingers and ask them to show you a particular number, i.e. Show me 7 • Students must show the number then explain their thinking, e.g. if I hold up one hand and two fingers that is 7 as I know I have 5 fingers on one hand • Encourage students to show the number a different way and then explain their thinking **Activity 2 – Cube Nets** • Provide materials (such as geoshapes) so students can make a traditional cube net, i.e. a net that looks like a crucifix – you could just give them a paper cube net and ask them to cut and re-stick it • Challenge students to move the location of some of the squares in order to make a cube net that looks different from the original but still can be folded in order to form a cube • NCTM has a related task https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Cube-Nets/ • There are in fact 11 unique nets of a cube http://mathforum.org/library/drmath/view/54682.html

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Draw a diagram	• The importance here is to emphasise the different between a detailed picture that a student may draw in art class and a simple diagram to represent mathematical information • A diagram must be clear and easy to read, different sized dots or stokes can soon become confusing for students and others • Students could use a key (or legend) to explain what their diagram represents, e.g., one bar represents a 10 unifix block tower • As students increase in experience, they often display a preference for using symbols rather than simple diagrams • It is important to present students with problems where drawing a diagram will assist students to find possible solutions	**Activity 1 – Heads and Legs** • A heads and legs style problem (see https://nrich.maths.org/924) can help encourage students to see the difference between a detailed picture and a diagram that can help to represent their thinking • Ask students to draw a diagram to help them solve the problem – do not tell them not to use too much detail • After giving time for students to grapple with the problem, then talk about how drawing fully detailed chickens with beaks and wings, or pigs with snouts and tails does not really assist with solving the problem • Instead drawing circles to represent the heads and lines to represent the legs is a more efficient way to use a diagram to represent the information **Activity 2 – Brian's Pegboard** https://nzmaths.co.nz/resource/brians-pegboard-i • This problem involves students finding pathways on a 3 x 3 array of dots • Given that most students will not have access to a pegboard and string, drawing possible pathways becomes the preferred strategy for this collection of problems (see also https://nzmaths.co.nz/resource/brian-s-pegboard-ii)
Act it out	• This strategy is a way of physically modelling a problem, either with people or objects • It is particularly useful in problems where students are trying to identify the order of events or the location of objects • Students will often need to use this strategy in conjunction with another strategy in order to record and explain their thinking	**Activity 1 – Demonstrating location words** • Make a list of location words with students (ensure to include words listed in the curriculum like clockwise) and record these on flashcards • Give each student an object, like a beanbag, cone or hoop, then show the class a location word and have all students demonstrate it • Select students to explain their thinking, e.g. I am standing **next to** the beanbag; I am standing **between** the beanbag and the table • Students could also get into small groups to demonstrate the location words or go outside and use play equipment to help demonstrate the different words **Activity 2 – River Crossing Problems** https://nrich.maths.org/11175 • River crossing style problems (such as the one that involves a farmer, a fox, a chicken and some grain) involve following a set of rules in order to move people (or animals and objects) from one side of the river to the other – if the rules are not following the solution is not possible • Other similar problems that will encourage students to move about why they attempt possible solutions include Frogs (https://nrich.maths.org/1246), Handshakes (https://nrich.maths.org/6708) and Seating Arrangements (https://nrich.maths.org/966)

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Make a list	• This strategy encourages students to record what they know about a problem • It can be a precursor to making a table • Whereas a table is often recorded in order, to assist students to look for patterns and predict results, a list is simply a record of everything known • A list is often re-written in order when the final solution is presented in order to help identify patterns or to better explain thinking • A list is often useful as a pre-assessment to find out what students know about a topic or number, i.e. items or examples that are missing from the list may help to identify misconceptions	**Activity 1 – Target Number** • Choose a number and ask students to record everything they know about the number • Use prompts to help direct student thinking, e.g. include a model, diagram, number sentence or number line • Students' responses can be used to highlight misconceptions, e.g. students who are unable to locate a number on a number line may have difficulty interpreting numbers or using multiple representations • The target number task could be modified to by giving students an expression, e.g. 3 x 4 and asking students to list everything they know about it • Alternatively, you could simply ask students to list everything they know about a certain term such as length or volume **Activity 2 – Factors** • Although there are a number of ways to find the factors of a given number, the solution to the problem is commonly recorded as an ordered list • For example, to find the factors of 15 a student could use their knowledge of multiplication facts and record 1 x 15 = 15; 3 x 5 = 15 • The identified factors would then be recorded as an ordered list: 1, 3, 5 and 15 • This process of recording an ordered list helps others to quickly check if all the factors have been recorded
Make a table	• This strategy is probably one of the most useful problem-solving strategy and is often used in conjunction with other strategies • A table is a systematic way to display information • It can be used to identify patterns, find missing results and accurately predict rules and future results • Interpreting the information in a table is a valuable life skill – tables are used to communicate information, such as through timetables, bills, bank statements and various statistics reported in the media • Being able to identify and understand the different elements in a table will help students be able to better interpret the results	**Activity 1 – How many?** • Provide students with a random pile of coloured counters • Ask them to find the how many or each colour they have and to use a table to display their results • Encourage students to label their table and ensure the information within the table is clearly presented • Share tables as a class before developing a guide (anchor chart) as to the features of a table (for this problem the table would have 2 or 3 columns, one for the colour, one for a tally and one for the totals) • This guide should include: a heading for the table, titles for each column and totals **Activity 2 – Finding All type problems** • Making a table is a useful strategy where students are required to find all the possible solutions to the problem • Modifying a problem to make it more open (i.e. have more than one correct solution) can help students to see the value of recording their thinking in a problem • A Heads & Legs style problem can often be adapted to encourage students to record the possible solutions in a table, rather than naming the number and heads and legs, simple identify the number of legs – students will then be required to find all the possible solutions • Problems that involve continuing patterns can also help students to see the benefits of using a table in order to look for possible patterns (see Sticky Triangles https://nrich.maths.org/88)

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Look for patterns	• This strategy encourages students to look for patterns in order to help them identify rules or missing or future terms in a sequence • If a pattern can be found the problem can often become easier to solve as the number of steps required to find the solution may be cut down • If numbers are recorded in a vertical list or table, the pattern is often easier to identify	**Activity 1 – Number Trails** • Ask students to fold their page into 4 columns – this will help students to record their numbers and identify any patterns • Choose a starting number and a number to count by (forwards or backwards), for example, start at 6 and count forward by fives • Students should then record in their book (writing down the column) the numbers in the sequence • After a given time (this will depend on the numbers chosen - say around one minute or enough time to record ten or more numbers in the sequence) ask all students to stop and place a line under the last number recorded • The teacher now selects a student to identify the first few numbers in the sequence and records these in a column on the board • The teacher now chooses another student and records a few more numbers – enough until the pattern is repeating – stop recording numbers at this stage, but do ask if any student managed to get higher than what is written on the board – acknowledge that this student is awesome • As a class, talk about any patterns the students can identify in the sequence and use this information to record future terms **Activity 2 – Pattern Problems** • Problems that involve using materials to build an object and then identify the amount of materials needed for larger objects or amounts can often be solved by identifying patterns • Examples of these include Up and Down Staircases (https://nrich.maths.org/2283), Cannon Balls (https://nzmaths.co.nz/resource/cannon-balls) and Triangular Numbers (https://nzmaths.co.nz/resource/triangular-numbers)
Work backwards	• This strategy can often be useful to solve problems where the result is known, but the steps involved in reaching the result or the initial numbers used in the problem are unknown • To use this strategy, students need to be encouraged to identify what they know about the problem and what they still need to find out	**Activity 1 – Start with the Result** • Present students with the steps and the result, for example, Cass now has 10 cherries, at recess she ate 5 cherries. How many cherries did she have at the start of the day? • Present students with function machine type problems where students will know the initial and final number, but need to identify the rule **Activity 2 – Age Problems** • Problems that involve finding the age of the people in the problem often require students to work backwards • Examples of these include, How Old? (https://nzmaths.co.nz/resource/how-old-0) and My Son is Naughty (https://nzmaths.co.nz/resource/my-son-naughty)

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Solve a simpler problem	• Often this strategy is used by teachers to assist students who may be having difficulty – commonly the teacher is the one who modifies the problem to help support the student • For students to use this strategy they will need to learn how they can modify a problem in order to use this information to go back and solve the original problem • Solving a simpler problem can be particularly useful when it may not be able to model or draw the whole problem due to the amount required	**Activity 1 – How Many?** • Play the How Many? activity as suggested in the Guess and Check strategy with students • This time place a greater emphasis on the strategy used by the students to organise the counters in order to check the count • Discuss which strategies are more effective than others – is counting by ones or counting by fives make the problem simpler? **Activity 2 – Ratio and Pattern Problems** • Problems that involve ratios and patterns are often good examples of how solving a simpler problem can make finding the solution a less complicated process • Consider problems that involve modifying recipes or building objects • For example, imagine we wanted to find out how many counters we could fit in 10 cups, we could find out how many counters could go into one cup then use this information to find out how many would be needed for 10 cups • Problems that require finding say the 10^{th} term in the sequence can often be made simpler by finding earlier terms and looking for a pattern or rule (see Matchsticks https://nrich.maths.org/10)
Seek exemption (Identify what doesn't work)	• This strategy is particularly useful when students are presented with a list of possible solutions to a given problem • By using logic and reasoning students can often eliminate possible incorrect solutions and hopefully be left with only the correct solution • Students must be able to clearly commutate the reasons behind why particular solutions cannot be possible • If their logic is flawed, it is possible that they will eliminate a potentially correct solution and then incorrectly identify the actual answer	**Activity 1 – Give Students the Solution** • Pose a problem to students by suggesting a that there is a certain number of solutions (this may or may not be true) • Students must then convince you that this person either is or is not correct, e.g., John says there is only three ways to fold a rectangle piece of paper into two equal halves – is he correct? • Provide students with a solution to an equation, e.g., Mary says that 100 – 63 is 37 – is she correct? **Activity 2 – Multiple Choice Problems** • Present students with a multiple-choice problem, e.g. NAPLAN question, and rather than asking them to choose the correct answer, students must convince you whey the other solutions cannot be correct • For example, 385 x 5 =? A) 2000 B) 1925 C) 1927 D)1952 It can't be C or D as any number multiplied by 5 results in the answer that ends in 0 or 5 It can't be A as 400 x 5 = 2000 as I know 4 x 5 = 20 Therefore, the answer must be B