

Year 9 Indices – Content Summary, Part 1: Integer Indices

1. Apply index laws to numerical expressions with integer¹ indices:

- (a) Understand the meaning of negative indices for numerical bases, e.g. by looking for patterns, such as

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}
27	9	3	1	$\frac{1}{3}$	$\frac{1}{3^2} = \frac{1}{9}$

; and

- (b) Evaluate numerical expressions involving a negative index by first rewriting it with a positive index and then expressing it's 'reciprocal', e.g. $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

Remember that a **power is the product of a factors that are the same number** (or variable).

E.g., $3^4 = 3 \times 3 \times 3 \times 3 = 81$ is the 'fourth power' of 3.

- The number 3 in 3^4 is called the base.
- The number 4 in 3^4 is called the **index** or **exponent** or **power**.
- For any number x , $x^1 = x$.
- In general, $x^n = x \times x \times x \times \dots \times x$, where there are n factors in the product, that is, we are multiplying n 'x's together, over and over. (Here x is the base and n is the index)

Let's have a look at a table of values showing the base of 3 raised to the power of 4, then 3, then 2... so that the power (or index) is reducing by one each time in the table. Look carefully for a pattern in the result (bottom row of the table), to see if you notice what's happening:

Base $(3)^n$	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Result	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

...Did you notice that every time the power (index) went down by 1, the result reduced to **one-third** of the result before it?

Now, have a look at the base (top row) pattern from 3^1 onward. We remember that any number to the power of '1' is itself (so, $3^1 = 3$); and that any number to the power of zero = 1 (so $3^0 = 1$). One (1) less than zero (0) is -1; one less than this is -2, and so on.

Have a look also at what happens when we continue the 'one-third of the previous result' pattern from this point in the 'result' (bottom row):

- $3^1 = 3$;
- $3^0 = 1$ (which is one-third of 3);
- $3^{-1} = \frac{1}{3}$ (which is one-third of 1);
- $3^{-2} = \frac{1}{9}$ (which is one-third of $\frac{1}{3}$);
- $3^{-3} = \frac{1}{27}$ (which is one-third of $\frac{1}{9}$)...

Pretty cool, huh?

¹ Remember, an **integer** is a whole number, that is, a number that's *not* a fraction. Integers can be positive or negative.

Grab some paper and a calculator (if you need it) and see what happens when you do the same thing with 5 as the base. Start with 5 to the power of 5, and reduce the powers from there, like in the top row of this table. Continue the pattern you find into the results with negative indices – **what do you notice is going on?**

Base $(5)^n$	5^5	5^4	5^3		5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}
Result	3,125									

Here's the rule that comes out of all this pattern making:

... that for any base of x raised to a power of a negative index (e.g. ' $-n$ '): x^{-n} , the result is always ' $\frac{1}{x^n}$ ', or what we call the **reciprocal** of x^n . So,

- $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$;
- $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$;
- $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$... and so on!

Your turn!

Open the attached 'GeoGebra' file called 'Visualising Powers'.

(To do this, you will need to have GeoGebra loaded onto your computer or device. GeoGebra Classic 6 software downloads can be accessed here: <https://www.geogebra.org/download?lang=en>)

In this GeoGebra file, you may need to click and hold, and slide, the 'Spreadsheet' menu on the right hand side so you can see the table of values for both the 'Power (P)' and the 'Base to Power' result column (x^P).

Try entering a base – such as '2' or '3' or '5' – and drag the slider to change the power or index to which your chosen base is raised. You'll notice the file allows you to set a minimum index power of -4 , and a maximum index power of 10 .

Have a look at each of:

- The equation form result, which will give you the base raised to the power as an equivalent to a fraction form;
- The graphical representation of this equation form, showing the index value of your base on the x-axis, and numerical value of the result on the y-axis; and
- The table of values for your selected base, in the 'Spreadsheet' table on the right hand side.

(Notice that you can also change the scale intervals for the x- and y-axes with the small sliders underneath the graph).

Have a play with the Base numbers and the Index (Power) slider and see what happens to your results when you change the index power.

What do you notice about the numerical results, regardless of the base number, for all negative integer index values (i.e. for indices < 0 , where the indices are integers)?

Main Reference:

Brown, P., Evans, M., Gaudry, G., Hunt, D., McLaren, R., Pender, B. and Woolacott, B. (2011), *ICE-EM Mathematics, Year 9, Book 1*, Chapter 8. Cambridge University Press : Melbourne, Victoria.