Year 9 Indices – Content Summary, Part 3: Converting Indices into Numerical Form

- **Write given indices in their number form**

We can also easily express numbers (and variables) in index form as numbers (and variables) in numerical form. This process is known as 'evaluating' indices.

Remembering our index laws can help us here:

Index laws

Index law 1

To multiply powers of the same base, add the indices.

$a^m a^n = a^{m+n}$

Index law 2

To divide powers of the same base, subtract the indices.

$\frac{a^m}{a^n} = a^{m-n}$ where $m > n$ and $a \neq 0$

Index law 3

To raise a power to a power, multiply the indices.

$(a^m)^n = a^{mn}$

Index law 4

A power of a product is the product of the powers.

$(ab)^m = a^m b^m$

Index law 5

A power of a quotient is the quotient of the powers.

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ where $b \neq 0$

<u>Source</u>: Brown, P., Evans, M., Gaudry, G., Hunt, D., McLaren, R., Pender, B. and Woolacott, B. (2011), *ICE-EM Mathematics, Year 9, Book 1*, Chapter 8, p.264. Cambridge University Press : Melbourne, Victoria.

Let's look at a few examples, using a calculator to assist.

(a) **3^2 x 3 x 3^4**

Remember, that expanding this expression would give us 3 x 3 x 3 x 3 x 3 x 3 x 3. So, that's actually 3^7, which is an easier thing to plug in to a calculator to solve.

3^7 = 2,178

(b) **$4b^2$ x $3b^2$**

Multiplication is commutative, that is, it doesn't matter what order we multiply numbers or variables – we'll get the same result.

So let's start with the numbers and then after we'll look at the variables.

$4b^2 \times 3b^2 = 12 \times b^2 \times b^2$

Now, the variables in this expression are the same – that is, 'b'.

Simplifying this expression by multiplying the common variables raised to powers together gives us the result:

$12 \times b^2 \times b^2 = 12b^4$ …Thus, $\mathbf{4b^2 \times 3b^2 = 12b^4}$

(c) $\dfrac{(6ab)^3 \times 2a^7b^4}{(2ab)^4 \times (3a^2b)^2}$

This looks complicated, so let's start by expanding all the terms out of the brackets so we can see what we've got. We'll use Index Law 4 from the above to help us with what to do with the brackets:

$= \dfrac{(6ab)^3 \times 2a^7b^4}{(2ab)^4 \times (3a^2b)^2}$

$= \dfrac{6^3a^3b^3 \times 2a^7b^4}{2^4a^4b^4 \times 3^2a^4b^2}$

Multiplying the numbers out first, and following our index laws (especially Law 1) we get:

$= \dfrac{216 \times 2 \times a^3b^3 \times a^7b^4}{16 \times 9 \times a^4b^4 \times a^4b^2}$

$= \dfrac{432 \times a^3b^3 \times a^7b^4}{144 \times a^4b^4 \times a^4b^2}$

$= \dfrac{432 \times a^{10}b^7}{144 \times a^8b^6}$

Let's use both our knowledge of division and index laws (especially Law 2) to simplify this expression.

As it turns out, $432 \div 144 = 3$, so:

$\dfrac{432 \times a^{10}b^7}{144 \times a^8b^6} = 3 \times a^2b$

Thus, $\dfrac{(6ab)^3 \times 2a^7b^4}{(2ab)^4 \times (3a^2b)^2} = \mathbf{3a^2b}$

(d) $\dfrac{\mathbf{5^7}}{\mathbf{5^{10}}}$

Remember, that's the same as saying '$5^7 \div 5^{10}$'. Following Index Law 2:

$5^7 \div 5^{10} = 5^{-3}$ *(…because 7 – 10 = -3)*

Evaluating this, $5^{-3} = \dfrac{1}{5^3} = \dfrac{\mathbf{1}}{\mathbf{125}}$

(e) $\dfrac{\mathbf{7a^2b^{-3}c^{-4}}}{\mathbf{21a^5b^{-7}c^{-9}}}$

Let's deal with the numbers first, and then the variables in their index form.

$\dfrac{7a^2b^{-3}c^{-4}}{21a^5b^{-7}c^{-9}} = \dfrac{1a^2b^{-3}c^{-4}}{3a^5b^{-7}c^{-9}}$

Now, let's divide each common variable (a, b and c) and deal with their powers as per Index Law 2:

$= \dfrac{a^{-3}b^4c^5}{3}$ *(…because $a^2 \div a^5 = a^{-3}$; $b^{-3} \div b^{-7} = b^4$; and $c^{-4} \div c^{-9} = c^5$.)*

Remember, a base number (or variable) to a negative index can be expressed as the **reciprocal** of that same base number (or variable), raised to the power of the same (positive) index.

So, $a^{-3} = \frac{1}{a^3}$ Therefore,

$\frac{a^{-3}b^4c^5}{3}$

$= \frac{b^4c^5}{3a^3}$ *(…See what we have done with the 'a^{-3}'? We've expressed this as '$\frac{1}{a^3}$', and so moved a^3 to the bottom of the fraction.)*

Thus, $\frac{7a^2b^{-3}c^{-4}}{21a^5b^{-7}c^{-9}} = \frac{b^4c^5}{3a^3}$

Main Reference:

Brown, P., Evans, M., Gaudry, G., Hunt, D., McLaren, R., Pender, B. and Woolacott, B. (2011), *ICE-EM Mathematics, Year 9, Book 1*, Chapter 8. Cambridge University Press : Melbourne, Victoria.