AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Year 8 Percentages - Notes for Hearing Impaired Students

These notes support the AMSI Interactive 'Percentages', available at: https://calculate.org.au/year8/

This material relates to the following Australian Curriculum (Mathematics) Outcome/s:

Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187).

Where are Percentages used in the real world?

Percentages are used widely in the real world as a way of describing proportions of a whole of something. Remember, the word 'proportion' is just another way of describing a **fraction**.

For example, the percentage result you may get on a maths test indicates the proportion (or fraction) of the whole marks available on the test (if you got everything correct) that you were able to gain, given that you possibly made some errors.

The good thing about percentages is that they set a common scale that simply means 'out of 100' – and so this gives us an easy and commonly recognized way of comparing results or proportions of a quantity. Percentages can also be used to describe a proportion of increase or decrease in a quantity, again in an easy and commonly recognized way that allows us to make comparisons against an original amount.

Percentages are used widely, in finance, in the pricing of goods and services, in taxation, in the growth and decline of things like populations, disease and illness, in education and the assessment of learning, in science such as chemistry and pharmacy… If you look carefully, it is highly likely you'll encounter percentages used somewhere on any given day during everyday life.

Click on the images below to explore just a couple of the practical applications percentages have during the course of an average day.

Weather Forecasting

When you wake up in the morning, you may check the weather forecast to see the temperature and whether or not it's likely to rain. We often describe the probability of events using percentages. Meteorologists (weather scientists) use a range of atmospheric data and observations to predict the probability of rain in a location. A percentage is often used to describe that probability. A high percentage (e.g. 90%) means a very high likelihood of rain while a low probability (e.g. 5%) means that it's highly unlikely rain will fall.

News Bulletins and other Information Reports

You jump on the bus and flick open the news feed on your phone. There's an article there talking about people catching public transport in Melbourne on the first day of the lifting of Covid-19 restrictions. The article says that the train network is carrying 'about 18%' of its pre-Covid-19 passenger load.'

Percentages are often used in the news, in business and government, in advertising and other information reports to compare a quantity and its increase or decrease over time. Used in this way, we often see percentages that are *more than* 100% - this simply indicates that a quantity is greater than the amount its being compared with, e.g. 'the number of solar panels on residential houses in 2020 is 140% of what it was in 2017…'

Assessment or Learning or Achievement

You head to your favorite class for the day – Mathematics – and your teacher hands out a test she has just marked for your class. You recall the test was out of a possible 60 marks, and you got 46 out of 60. The teacher has also written "77%" on the top of your test. You glance at your mate Tim next to you and notice that he has "78%". Tim turns to you and grins.

Percentages are often used as a grade score or point of comparison for achievement for something. Where comparisons are necessary (and they're really NOT necessary when it comes to Maths tests – so don't be so smug, Tim…), we can use percentages to quickly determine what was proportionally larger or smaller.

Returns (or losses) on savings and investments

You notice in your emails that your bank has sent you your latest statement. You click through to open it and notice that the long-term savings account you opened a month ago is currently earning you 3.5% interest 'per annum'. You currently have $500 in the account and are planning to add $100 a month to it from your part time job in a local coffee shop.

Percentages are used constantly in the world of banking and finance. They are a way of communicating to borrowers, lenders and savers how much their money is 'earning' (or costing!) through the lending and borrowing process. Percentages used in this way are known as 'interest rates' or sometimes as 'yields'.

Converting Fractions into Decimals and Percentages: "The Magic 100"

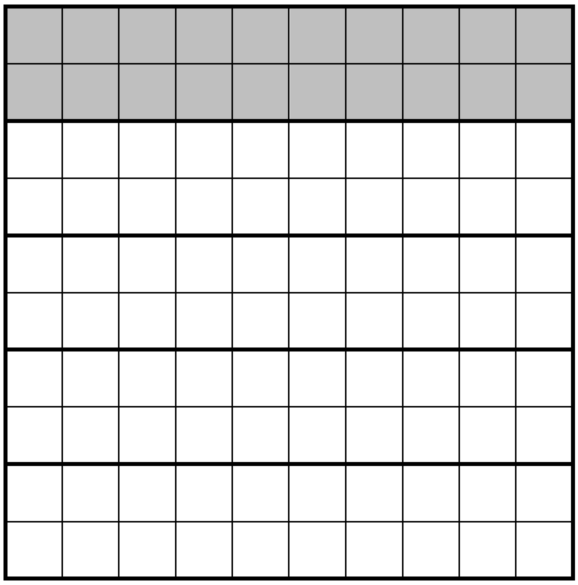
Remember we mentioned earlier that percentages are really just a way of describing a proportion or a fraction, out of 100.

So, when we're turning fractions into decimals and percentages, "100" is the number that will always help us out.

Let's look at an example. Consider the fraction $\mathbf{\frac{4}{5}}$.

We'll start with the denominator of this fraction, which is '5'. In other words, we're interested in 'fifths' of something.

Okay, then – let's let that 'something' be "100". So, we want to find $\frac{1}{5}$ of 100. Put another way, we want to find out 100 ÷ 5:



Looking at the ‘100’ grid square above, we have shaded $\frac{1}{5}$ of 100 squares, which is 20 squares. Of course, we could also work that out by going ‘100 ÷ 5 = 20’.

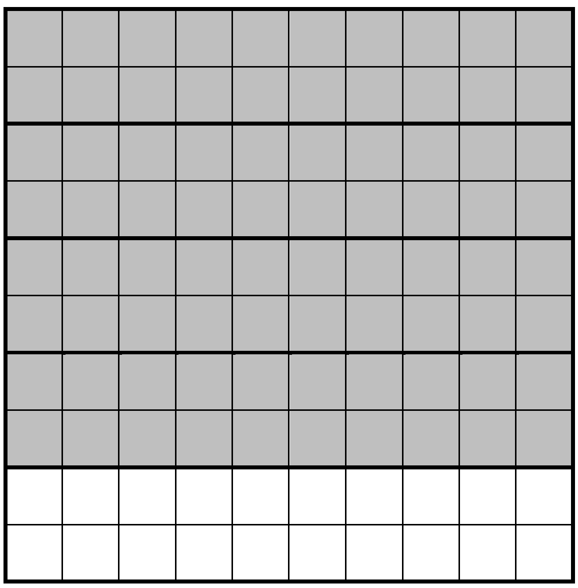
But… the fraction is actually $\boldsymbol{\frac{4}{5}}$, not $\boldsymbol{\frac{1}{5}}$.

So, now let’s look at the **numerator** of the fraction, which is 4. This tells us we want **4** lots of $\frac{1}{5}$ of 100; that is, 4 x 20:

$$4 \times 20 = \mathbf{80}$$

So $\frac{4}{5}$ of 100 is 80, as we can see illustrated in the grid square here.

It’s this ‘80’ (out of the Magic 100) that really helps us out. You may remember that a percentage is a proportion out of 100 (for example, if you got ‘100%’ in a maths test you would get the whole test right; if you got 50% in the test you would have got half the questions in the test correct because 50 is half of 100, and so on…)



So, if we now know that $\frac{4}{5}$ of 100 is 80, then we also know that $\frac{4}{5}$ **is the same as 80%.** It's as simple as that!

But what about turning this into a decimal?

Looking at 80 / 100 (eighty out of one-hundred) as a fraction, we have $\mathbf{\frac{80}{100}}$ which is the same as $\mathbf{\frac{4}{5}}$. the fis a decimal fraction, we can write this as **0.80**

That is to say, eight (8) tenths of 100, with zero (0) extra hundredths of a 100. *(Look at the grid square; each row of 10 is a tenth of 100, and there are 8 rows – so eight tenths!)*

(We can actually ditch the 'zero' on the end here as it literally means 'nothing' – that is 'no extra hundredths units at all'. So let's just write 0.80 as "**0.8**".)

So, the fraction $\mathbf{\frac{4}{5}}$ is the same as 80% of a whole and can be written as 0.8.

We say that $\mathbf{\frac{4}{5}}$, 0.8 and 80% are **equivalent** fractions, decimals and percentages.

100 is always the key to converting fractions into equivalent decimals and percentages!

Fractions whose denominators *aren't* factors of 100

Converting fifths into decimals and percentages is relatively easy because 5 is a factor of 100 (ie, divides evenly into 100): 100 ÷ 5 = 20; no remainders!

Other fractions – such as thirds, sixths, sevenths and ninths – don't divide evenly into 100 and so they are a bit trickier to work with. With these fraction-into-decimal conversions, it's okay to use a calculator, as long as you understand what you're doing.

And, the process is the same as it is for fractions that have denominators that are factors or 100 – we're just dealing with a few 'messy' decimal numbers in that process, that's all.

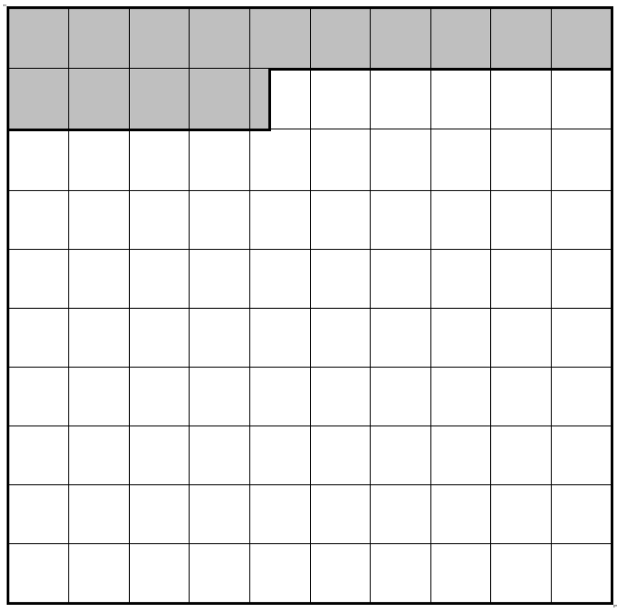
Let's look at the fraction $\mathbf{\frac{5}{7}}$.

We're going to use exactly the same working as we did in the case of $\frac{4}{5}$ above, using our Magic 100. However, we may need to use a calculator because 7 is not a factor of 100 and there will be a remainder.

To find $\frac{1}{7}$ of 100: 100 ÷ 7 = $\mathbf{14.\dot{2}8571\dot{4}}$ *(a recurring decimal)*

We can also write: 100 ÷ 7 = 14 $\frac{2}{7}$ [because (14 x 7) + 2 = 100]

This means that $\frac{1}{7}$ of 100 is about 14.3 (rounded to 1 decimal place). We could show it on a diagram like this:



But remember, we wanted $\frac{5}{7}$ of 100, which means 5 lots of $\frac{1}{7}$ of 100. So:

$$14.\dot{2}8571\dot{4} \times 5 = 71.\dot{4}2857\dot{1}$$

(using a calculator)

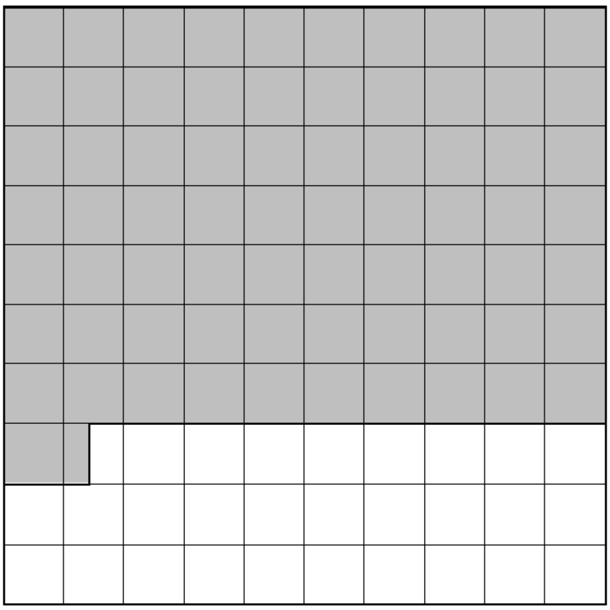
A short-cut would just be to calculate:

$$\left(\frac{5}{7} \times 100\right) = 5 \div 7 \times 100 = 71.\dot{4}2857\dot{1}$$

Rounding this answer,

$$\frac{5}{7} \times 100 = 71.4$$

On a hundreds grid, this would look like this:



Mathematically, we now have our answer that we can convert easily into a percentage.

$\frac{5}{7}$ of 100 = 71.4, so the equivalent percentage for $\frac{5}{7}$ is **71.4%.**

And there we have it!

By using a calculator, and with an understanding of recurring decimals and rounding, we can also easily express tricky fractions like thirds, sixths, sevenths and ninths as decimals and percentages!

Using Percentages to Increase and Decrease Quantities

Percentages are useful for increasing quantities, and for expressing an increase in a quantity. Using decimal and percentage equivalences is the easiest way with which to make calculations like this.

For example, if a bus has 60 passengers aboard and this number increases by 20%, this means that an additional 20% of 60 passengers come aboard:

20% of 60

= 0.2 x 60

= 12

Thus, an **additional** 12 passengers come aboard – there are now 60 + 12 = 72 people on the bus.

An easier way of calculating this would be to multiply the initial amount (6) by 1 plus the additional 20% of the amount. This is the same as saying '120% of 60'. The decimal equivalent of 120% (of something) is '1.2 times' that amount. So, 1.2 x 60 = 72. Much quicker!

To express an increase using a percentage, we simply find the difference between the initial amount and the new amount and work out what percentage this difference is of the original amount.

Thus, if there were 60 passengers aboard and after the next stop there are 72 passengers – that's 12 extra passengers - we could say that "the buses passengers increased by 20%." This is because:

72 – 60 = 12

12 ÷ 60 = 0.2 (or 20%)

Alternatively, we could do the following:

$$\frac{72}{60} = 1.2$$

Thus, at Stop 2 the bus passenger load increased 120% of its original load.

Note that we need to be careful how we express this!

Going from 60 to 72 passengers after a stop we would say was "an increase of 20% after the stop", OR we could say "after Stop 2 the bus had 120% of its Stop 1 passenger load".

We would NOT say that “the passenger load *increased* by 120%.” There is a big difference here! This latter statement actually means the bus went from 60 passengers to more than double this amount: 60 + (120% x 60) = 60 + 72 = 132 passengers!

We can express decreases in quantities as a percentage,
and use percentages to decrease quantities:

Simon ran the 100 metres in 12.3 seconds. His previous time was 13.1 seconds. What percentage was his new time of his previous time?

12.3 ÷ 13.1 x 100

= 93.9% (or 94% rounded off).

This will work the other way as well:

93.9% of 13.1 seconds

= 0.939 x 13.1

= 12.3 seconds

We can also express the *amount* of the decrease as a percentage:

Simon’s previous 100m track time was 13.1 seconds. Today he ran 12.3 seconds. What was the percentage that he shaved off his time?

13.1 – 12.3 = 0.8 seconds

0.8 seconds ÷ 13.1 x 100 = 6.1%

So, Simon shaved 6.1% off his time.

Did you notice something?

Simon’s new time was 93.9% of his old time.

He shaved 6.1% off his time…

93.9 + 6.1 = **100%**

The Unitary Method

The unitary method is a way of calculating percentage amounts or changes by ‘unitising’ the percentage of the amount to be changed or found. This simply means we first find out what 1% of the amount in question is, and then multiplying this 1% amount by the required percentage amounts.

In some cases, we will still need to use a calculator to avoid having to use long written algorithms or lengthy mental calculations. However, in many cases the unitary method can be used to calculate percentage problems mentally, in a couple of easy steps.

Let's start with a simple example:

What is 12% of $70.00?

Step 1: 'Unitise' the question by finding just **1%** of $70.00:

$70.00 ÷100 *(we do this because percentages are a proportion out of 100)*

= $0.70 *(or 70 cents)*

Step 2: If 1% of $70.00 is $0.70 *(70 cents)*, how much is 12% *(which is 12 x 1%)*

$0.70 x 12

= $8.40 (remember that 7 x 12 = 84, so 0.7 x 12 is 8.4)

So, 12% of $70 is **$8.40**

We can see here that the trick is to first find 1% of the amount, and then use this to multiply out the required percentage, to solve the problem.

Let's use the Unitary Method to solve a slightly harder problem. We'll need a calculator to help us for this one.

What if we have a 15% discount offered on all goods at the local thrift shop. A coat I'm wanting to buy is now $35.70 at the discounted price. What was the original price of the coat, before the 15% discount?

If there's a 15% discount, then goods in the store are (100 – 15 =) 85% of their original prices.

So, $35.70 = 85% of the original price.

Let's unitise now. What's 1% of the original price?

It must be $35.70 ÷ 85 = $\frac{35.7}{85}$

= $0.42 (or 42 cents).

So now, if 1% of the original price is 42 cents, what is 100% of the original price?

100 x $0.42

= $42.00

The original price for the coat was therefore **$42.00**

Percentages – The Quiz!

Ready to try some real-world problems involving percentages?

Q1: Find 83% of 11,900

A. 12,888

B. 9,877

C. 9,639

D. 8,311

Q2: Tick the **TWO** statements below that are TRUE:

- [ ] It's impossible to calculate more than 100% of a given quantity.
- [ ] A common use of percentages is to describe the passing of time.
- [ ] Percentages can be used to describe the probability or likelihood of an event occurring.
- [ ] Percentages are a useful way of comparing growth or change in a quantity over time.

Q3: At the start of the year I have $435.60 in my bank account.

At the end of the year I have earned a total of 2.5% interest on top of that amount.

What is the total in my account at the end of the year?

$ ______

Q4: Simon's hay crop weight this year was 5/7 (five-sevenths) of last year's hay crop. Last year's crop was 103,400 tonnes.

The correct working for calculating this year's crop is:

A. 103,400 tonnes / 0.74286 (rounded)

B. 1.714286 (rounded) x 103,400 tonnes

C. 103,400 tonnes - (0.714286 (rounded) x 103,400)

D. 0.714286 (rounded) x 103,400 tonnes

Q5: Match the change in quantity to the correct percentage change. *(Draw lines connecting the correct responses).*

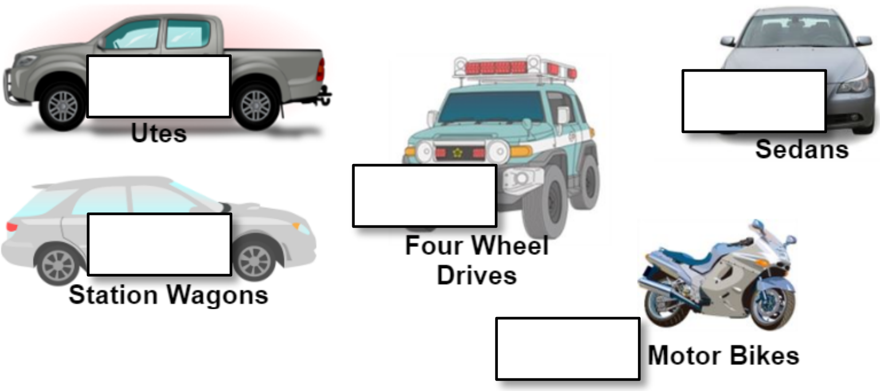
If I start with 825 and finish with 924, I have…	65% of my first amount.
If I start with 140 and finish with 91, I have…	25% less than my first amount.
If I start with 1.259 and finish with 1.58634, I have…	12% more than my first amount.
If I start with $85.60 and finish with $64.20, I have…	126% of my first amount.

Q6: A packet of 11 chocolate biscuits weighs 270 grams net (i.e., without the packaging). It is known that the biscuits contain 12% cocoa by weight. What is the total weight of the cocoa in one biscuit?

Approximately [ ] grams.

Q7: In the country town of Busted Waterbag, 28% of vehicles are utes, 38% are four wheel drives, 14% are sedans, 12% are station wagons and 8% are motor bikes. If there are **4,850** vehicles registered in the town, how many of each type of vehicle are there?

Select the correct solution for each type of vehicle from the list of numbers on the right. Write each solution into the blank space for the corresponding vehicle category.



Possible Answers:

388	462	582
679	1,054	1,358
1,402	1,843	2,185

Q8: The price of a vinyl album is $28.90 *including* Goods and Services Tax (GST). GST is charged at 10% of the original price. What is the pre-GST price of the album**?**

A. **$31.79**

B. **$26.01**

C. **$27.00**

D. **$26.27**

Q9: In a law firm, 45% of employees are female. The firm has 315 female employees. How many employees are there in total?

A. **700**

B. **800**

C. **630**

D. **750**

Q10: In a lottery, only 0.002% of tickets won a prize. If there were only 2 prizes, how many tickets were sold?

The Quiz – Answers

Q1. B.

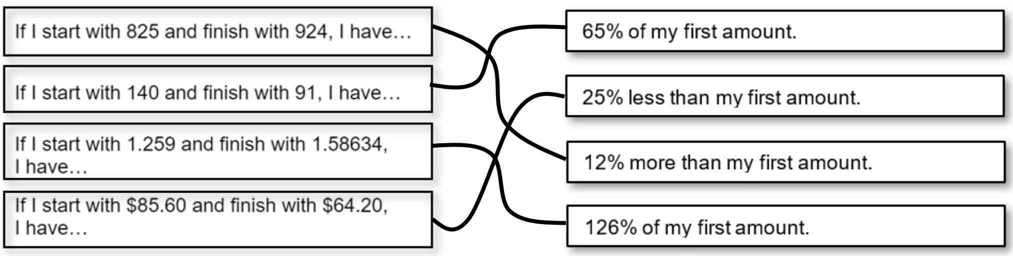
Q2. Only the third and fourth statements are TRUE:

- [ ] It's impossible to calculate more than 100% of a given quantity.
- [ ] A common use of percentages is to describe the passing of time.
- [x] Percentages can be used to describe the probability or likelihood of an event occurring.
- [x] Percentages are a useful way of comparing growth or change in a quantity over time.

Q3. $446.49

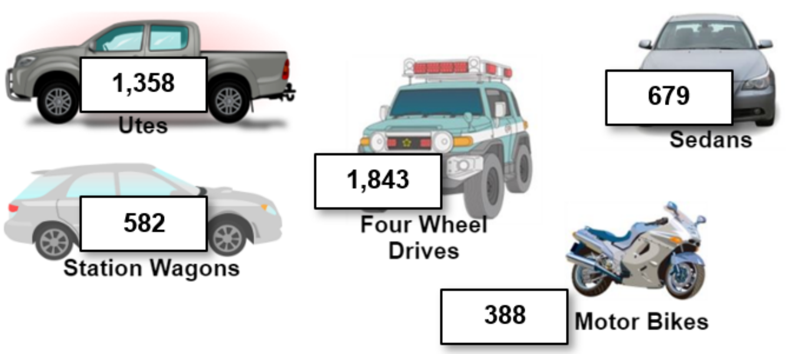
Q4. D.

Q5.



Q6. Approx. 2.9 (or 3) grams.

Q7.



Q8. D.

Q9. A.

Q10. 100,000