Labellings of combinatorial graphs
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I spent my AMSI-funded vacation scholarship studying labellings of combinatorial graphs. A total labelling of a graph G with v vertices and e edges is a one-to-one map that associates the integers 1, 2, ..., v+e with the vertices and edges of G. Such a labelling is called edge magic if the sum of all labels associated with an edge equals a constant, independent of the choice of edge, and vertex magic if the same property holds for vertices.

I commenced my scholarship by developing Matlab code to search for vertex and edge magic labellings of all "small" graphs, being graphs with 7 or fewer vertices.

The primary focus of my scholarship was considering graphs that are simultaneously both edge magic and vertex magic. Such graphs are very rare, and computational searches are very slow. However, it is possible to develop more effective algorithms, by reducing the labels modulo small integers (such as 2 or 3).

I then developed Matlab code which implemented efficient algorithms for testing graphs to see if they are simultaneously vertex and edge magic. I was able to use this software on all graphs with 8 or fewer vertices (there are 12,345 such graphs).

I anticipate that this scholarship will lead me to undertake further work in this area.