ACE Network Subject Guide

Complex methods for partial differential equations

Semester 1, 2025

Administration and contact details

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Host institution	University of Newcastle
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Subject details

Handbook entry URL	
Subject homepage URL	
Honours student hand-out URL	
Teaching period (start and end date):	**2025-02-24 to 2025-06-06**
Exam period (start and end date):	**2025-06-10 to 2025-06-22**
Contact hours per week:	**2**
ACE enrolment closure date:	
Lecture day(s) and time(s):	
Description of electronic access arrangements for students (for example, LMS)	

Subject content

1. **Subject content description**

   This course introduces a modern analytical method for solving partial differential equations that takes advantage of the power of complex analysis. Having solved boundary value problems for the heat equation using Fourier series, it is natural to ask how far separation of variables and Fourier expansion can be generalised. We will study one approach, invented in the last 30 years, called the unified transform method, which is particularly suited to constant coefficient partial differential equations. We will see how to solve several of the classical evolution equations of mathematical physics: heat, time dependent Schrödinger and wave. We will also see how to solve more complex problems for the which the classical methods fail. On the way, we will pick up a few new techniques and ideas from complex analysis and asymptotic analysis. Assessment will be through a portfolio of solutions to weekly problem sets.

2. **Week-by-week topic overview**

   1. Fourier transforms: definition, Fourier inversion theorem, solving PDE on the line
   Morera's theorem: proof, application to Fourier transforms
   2. Fourier series: Dirichlet, Neumann & spatially periodic problems for the heat & Schrödinger equations
   Asymptotic analysis: introduction, analysis of PDE solution formulae
   3. Asymptotic analysis of complex functions
   Contour deformation: Applications of Jordan's lemma & Cauchy's theorem to PDE formulae
   4. Time periodic problems
   5. Unified transform method for the Dirichlet heat problem: stage 1
   6. UTM for the Dirichlet heat problem: stage 2
   7. UTM for the Dirichlet heat problem: stage 3
   8. Zeros of exponential polynomials: argument principle, asymptotic locus
   Biholomorphisms: univalent holomorphisms, mapping polynomials to monomials
   9. Half line problems for UTM: time dependent Schrödinger with constant potential
   10. Interface problems for UTM: time dependent Schrödinger with step potential
   11. Higher spatial order UTM: Airy equation on the finite interval
   12. Second order in time UTM: the wave equation
   Mixed partials in UTM: linearized Benjamin-Bona-Mahoney

3. **Assumed prerequisite knowledge and capabilities**

   Elementary complex methods: Complex derivatives, Analytic functions, Contour integration, Cauchy's theorem, Jordan's lemma, Classification of singularities, Laurent series
   PDE: some experience using Fourier series methods for the heat equation in 1d
   Linear algebra: a typical undergraduate course
   Multivariable calculus: a typical undergraduate course
   Real analysis: a typical undergraduate course. Measure theory not required.

4. **Learning outcomes and objectives**

1. Solve the classical linear evolution equations of mathematical physics in 1+1d subject to initial and boundary conditions.
2. Solve third order linear evolution equations in 1+1d subject to initial and boundary conditions.
3. Describe the general unified transform method for linear evolution equations.
4. Apply complex analytic techniques including contour deformation and residue calculus to the solution of partial differential equations.
5. Find the zeros of exponential sums.

AQF specific Program Learning Outcomes and Learning Outcome Descriptors (if available):

AQF Program Learning Outcomes addressed in this subject	Associated AQF Learning Outcome Descriptors for this subject
Knowledge	K1, K2
Skills	S1, S2, S5
Application of knowledge and skills	A3, A4

Learning Outcome Descriptors at AQF Level 8
Knowledge
K1: coherent and advanced knowledge of the underlying principles and concepts in one or more disciplines
K2: knowledge of research principles and methods
Skills
S1: cognitive skills to review, analyse, consolidate and synthesise knowledge to identify and provide solutions to complex problem with intellectual independence
S2: cognitive and technical skills to demonstrate a broad understanding of a body of knowledge and theoretical concepts with advanced understanding in some areas
S3: cognitive skills to exercise critical thinking and judgement in developing new understanding
S4: technical skills to design and use in a research project
S5: communication skills to present clear and coherent exposition of knowledge and ideas to a variety of audiences
Application of Knowledge and Skills
A1: with initiative and judgement in professional practice and/or scholarship
A2: to adapt knowledge and skills in diverse contexts
A3: with responsibility and accountability for own learning and practice and in collaboration with others within broad parameters
A4: to plan and execute project work and/or a piece of research and scholarship with some independence

5. Learning resources

Lectures (live broadcast on Zoom and video recorded), lecture notes, regular problem sets.

6. Assessment breakdown

Exam	**0%**
Assignment	**100%**
Class work	**0%**

Assignment due dates	**Exam date (approximate)**
2024-06-12 (approx) Summative assignment	None

Institution honours program details

Weight of subject in total honours assessment at host department	
Thesis/subject split at host department	
Honours grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %

Institution masters program details

Weight of subject in total masters assessment at host department	
Thesis/subject split at host department	
Masters grade ranges at host department	
H1	Enter range %
H2a	Enter range %
H2b	Enter range %
H3	Enter range %